NEW SYSTEM

OF

COMMERCIAL ARITHMETIC,

OR

GUIDE TO BUSINESS AND SCIENCE;

FOR THE USE OF SCHOOLS.

IN WHICH THE PRINCIPLES OF THE RULES, AND THE REASONS OF THE OPERATIONS ARE FULLY EXPLAINED;

ALSO,

Containing a Copious Selection of Questions, affording the means of Minute Examination on the Principles and Application of the Rules.

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INTRODUCTION.

Arithmetic, as generally taught in our Schools, has hitherto been a mere mechanical process, depending entirely on the memory, and not on the judgment. To give to this department of education that interest which its importance deserves, and to place the knowledge of it, on its natural basis, is the object of the following Treatise.

The several parts of the work are arranged in that order, which appeared to the Author best calculated to promote this desirable object. The definitions and rules are given with the utmost regard to perspicuity and conciseness, and are illustrated by appropriate examples. Copious notes are annexed, in which the principles of the rules, and the reasons of the several operations are made obvious to the youngest pupil. The notes also contain many useful contractions and rules for particular cases, with much information of importance to young men entering on business.

At the end of the volume, a collection of QUESTIONS is given for examination, embracing every part of the work. These QUESTIONS are so extensive, as not only to enable the pupil to elicit from the Teacher all necessary information, and to furnish the Teacher with the means of exercising the judgment and reasoning faculties of his pupils; but to put it also in the power of those concerned in the examination of Schools to ascertain minutely the various degrees of proficiency attained.

The plan of the work is entirely new, and every attention has been given to render the study of Arithmetic interesting, and acquirements in it substantial; but how far he has been successful, the Author leaves it with the public to determine.

Commercial and Mathematical Academy,
10, *Nicolson Street, Edinburgh,*
23*d April* 1830.

CONTENTS.

SYSTEM

OF

COMMERCIAL ARITHMETIC.

ARITHMETIC, as a SCIENCE, treats of the properties, and relations of numbers; and, as an ART, teaches to apply numbers in calculations.

The following are the characters used in arithmetical calculations, viz. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The 0 is called a cipher, and the other characters are called significant figures, or digits. By the various combinations of these characters, every finite number can be expressed.

The cipher has no value, either by itself, or in combination with other figures; its only use being to supply the places unoccupied by significant figures, and thereby remove them to their proper places.

From the following scales it will appear that every figure by itself or in units place retains the value expressed by its name, which is called its SINGLE VALUE; but, in combination, it acquires another value, varying according to the place it occupies, which is called its LOCAL VALUE. Thus 2, when standing by itself, or, on the right of any number, signifies two, but when standing on the left of a cipher, or in the second place of a number, signifies 2 tens, or 20, and on the left of three ciphers, or in the third place of a number, signifies 2 hundreds, or 200, thus acquiring a local value, ten times greater than its single value, by every place it is removed towards the left.

NUMERATION.

NUMERATION teaches to read any number expressed in figures, by assigning to each figure its local, as well as its single value, as explained in the following scales.

NUMERATION SCALES.

SCALE I.

1 ∾ Units.
2 ∾ Tens.
3 ∾ Hundreds.
4 ∾ Thousands.
5 ∾ X of Thousands.
6 ∾ C of Thousands.
7 ∾ Millions.
8 ∾ X of Millions.
9 ∾ C of Millions.
10 ∾ M of Millions.
11 ∾ X of M of Millions.
12 ∾ C of M of Millions.
13 ∾ Billions.
14 ∾ X of Billions.
15 ∾ C of Billions.
16 ∾ M of Billions.
17 ∾ X of M of Billions.
18 ∾ C of M of Billions.
19 ∾ Trillions.
20 ∾ X of Trillions.
21 ∾ C of Trillions.
22 ∾ M of Trillions.
23 ∾ X of M of Trillions.
24 ∾ C of M of Trillions.
25 ∾ Quadrillions.
26 ∾ X of Quadrillions.
27 ∾ C of Quadrillions.
28 ∾ M of Quadrillions.
29 ∾ X of M of Quadrillions.
30 ∾ C of M of Quadrillions.
31 ∾ Quintillions.
32 ∾ X of Quintillions.
33 ∾ C of Quintillions.
34 ∾ M of Quintillions.
35 ∾ X of M of Quintillions.
36 ∾ C of M of Quintillions.
37 ∾ Sextillions.
38 ∾ X of Sextillions.
39 ∾ C of Sextillions.
40 ∾ M of Sextillions.
41 ∾ X of M of Sextillions.
42 ∾ C of M of Sextillions.

SCALE II.

8	7	6	5	4	3	2	
Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Units.
137625,	839216,	431987,	542368,	932425,	364758,	697182,	931425

SCALE III.

10	9	8	7	6	5	4	3	2	
Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Units.
168703,	200451,	670004,	213110,	325000,	060783,	626012,	068721,	032467,	311111

ILLUSTRATION.—The 1st scale may be read by beginners thus: 2 two; 22, twenty-two; 222, two hundred and twenty-two; 2222, two thousand two hundred and twenty-two; 22222, twenty-two thousand two hundred and twenty-two; 222222, two hundred and twenty-two thousand, two hundred and twenty-two. The pupil being able to read the first six places, or period of units accurately, he will find little difficulty in reading the two first periods at next step, thus, 222222,222222, two hundred and twenty-two thousand two hundred and twenty-two million, two hundred and twenty-two thousand two hundred and twenty-two. The above scale is read thus: Two hundred and twenty-two thousand two hundred and twenty-two sextillions, two hundred and twenty-two thousand two hundred and twenty-two quintillions, two hundred and twenty-two thousand two hundred and twenty-two quadrillions, two hundred and twenty-two thousand two hundred and twenty-two trillions, two hundred and twenty-two thousand two hundred and twenty-two billions, two hundred and twenty-two thousand two hundred and twenty-two millions, two hundred and twenty-two thousand two hundred and twenty-two.

When the pupil can read the 1st scale, extended to 60 places; the 2d and 3d, or similar scales may be given, extended to sixty places, or so far as the teacher may think necessary.

It will be observed, that the scale is divided into periods of 6 figures each, beginning on the right, and counting towards the left, and that it is read from the left to the right.

The pupil having made himself master of these scales, may proceed to point, and read the following numbers.

EXERCISES.

3765. 9873. 52031. 818032. 71546876. 10121141. 514232. 1234567890. 30405678425. 6542312131OO. 126456741O2. 45636100293445. 901932142321303. 76214236724340650. 1210036373524987000616872l4. 52670132008760213423654. 30110234213436213423106. 24241234567298724321634503O. 210004502980713687241O4. 126072456790872503041218 76. 6682010432401682032000. 49542130058260126003760413 7. 900825600005236746801230744678943184509256214900000.

NOTATION.

NOTATION teaches how to express in figures any number proposed in words.

RULE.—Write down the figures expressed by the words, beginning at the left, and writing towards the right; then

consider what place in the numeration scale, each figure should occupy, and write them anew, supplying vacant places with ciphers.

EXAMPLE.—Express in figures, nine hundred and five millions, sixty thousand, four hundred and two. The figures expressed here, are, 9, 5, 6, 4, 2; but in reading the number again, we find that 9 must occupy the 9th place in the scale, or hundreds of millions, that 5 must occupy the 7th place, 6, the 5th place, 4, the 3d place, and 2 the first place in the scale; thus leaving the 2d, 4th, 6th, and 8th places to be supplied with ciphers in this way 905060402, which is the number required.

EXERCISES.—Write the following numbers in figures.

Ninety-six. Seven hundred and thirty. Six thousand and fifty-three. Forty-seven thousand three hundred and twenty-nine. Five hundred thousand and fifty. Eight million and seventy-four. Thirty-four million and twenty-five. One hundred million. Sixty-three thousand and four million, nine hundred and eight. Seventeen billion, eighty million and ninety-five. Eighty-nine thousand billion and eighty-five. Six hundred and twenty thousand trillion, seven million and seventy. Fifty thousand quadrillion, seventy-five trillion, one thousand million and eighty-six. Four hundred and seventy thousand and two quintillion, eighty-nine billion, four million and twenty. Seven hundred thousand and five sextillion. Eighty thousand octillion, ninety-five millions and fourteen. Eighty-nine thousand nonillions, sixty-five octillions, three hundred and fifty-nine thousand sextillions, seven trillions, one hundred and seventy-four thousand millions and seventy. Nine hundred thousand nonillions. Sixty-five thousand septillions. Nineteen sextillions. Five hundred quintillions. Four hundred and fifty-four thousand and ninety-nine trillions. Thirty-seven billions. Sixty-six thousand and fifty-four millions. Seven hundred and seven. Six hundred thousand nonillions.

So soon as the pupil is able to perform the exercises in Numeration, and Notation, with tolerable accuracy, and expedition, he should commence simple addition.

FUNDAMENTAL RULES.

THE fundamental rules of arithmetical calculations are ADDITION, and SUBTRACTION; which are subdivided into ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION.

These shall be treated of in their order, first in simple, and then in compound numbers.

SIMPLE ADDITION.

Simple Addition teaches how to collect several simple numbers into one sum.

Rule.—Write the numbers with the same places directly under each other, units under units, tens under tens, &c. and draw a line below. Add the figures in the units column, and place the right hand figure of the sum directly under it, carrying the other figure, or figures to the next column. Proceed in the same manner with every other column till you come to the last, under which write the whole sum it amounts to. The number thus obtained is the sum of the whole.

Proof.—Cut off the undermost number, and add the remaining numbers as before, then add this sum to the number cut off, the last sum will be the same as the first, when the work is right.

Example.—Add together 4243+2469+173+6549+699+99+7358.

	4243
	2469
	173
	6549
	699
	99
	7358
Sum =	21590
	14232
Proof =	21590

I here write the first number given, and under it, the other numbers in order, with the same places according to the numeration scale directly under each other. I then begin with the right hand column, and say 8 and 9 are 17, and 9 are 26, and 9 are 35, and 3 are 38, and 9 are 47, and 3 are 50; I write 0, the right hand figure of this number under the column added, and carry the 5 to next column, thus, 5 and 5 are 10, and 9 are 19, and 9 are 28, and 4 are 32, and 7 are 39, and 6 are 45, and 4 are 49; I write the 9 under the column added, and carry 4 to next column; thus, 4 and 3 are 7, and 6 are 13, and 5 are 18, and 1 are 19, and 4 are 23, and 2 are 25. I write 5 under the column added, and carry 2 to next column, thus, 2 and 7 are 9, and 6 are 15, and 2 are 17, and 4 are 21, which I write with the 1 under the column added, and the 2 on its left. I then cut off the undermost number 7358, and add up the remaining numbers as before, the sum is 14232, which being added to the line cut off, gives 21590, the same as the sum of the first addition. I therefore conclude that the work is right.

EXERCISES.

1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.
34	47	72	22	89	88	873	844
21	62	14	68	76	76	421	327
65	59	89	59	92	93	724	953
37	68	63	41	53	65	397	124

9th.	10th.	11th.	12th.	13th.	14th.
6824	4213	4321	3042	7541	7634
3241	5346	5692	4568	2932	2569
5632	1204	4368	9741	4320	8900
7463	6897	5213	4060	5142	7699
9821	8324	6108	2136	6163	8321

15th.	16th.	17th.	18th.	19th.	20th.
4324	8035	8725	7546	1431	8975
6390	4212	9798	2132	2754	4241
8276	7624	6341	5413	6893	5268
5943	3830	5420	7968	1234	7627
6254	7241	8224	9569	4213	9108
7032	6321	1409	2124	6761	2423

21st.	22d.	23d.	24th.	25th.	26th.
62134	48062	98796	12134	82854	424142
49032	17673	54213	37068	70103	897213
87639	21467	42416	45764	29798	625432
40608	98764	52459	89769	62431	897634
73241	52416	80740	21326	76243	323160
87654	47089	54369	42798	21632	825475
91232	65478	21423	44041	13218	421593

27th.	28th.	29th.	30th.	31st.	32d.
214562	853891	424678	424142	525682	556732
919875	724325	925468	939895	913927	879598
423648	398976	252732	242463	141682	654698
367214	254989	654298	242689	871487	418732
143128	694321	324293	873768	587963	526937
321989	978923	987938	549329	243158	989426
976548	485397	219287	213246	928976	549723
415948	212987	292841	424987	298957	524321

33d.	34th.	35th.	36th.	37th.	38th.
327985	321652	514123	829463	626358	672458
927691	937124	596927	754231	937692	728975
529625	876249	321432	432468	172182	486592
463259	732685	812459	327964	354921	498976
876832	426321	632638	283962	980546	213584
476821	434987	142734	474268	212938	968925
543958	624968	123298	212485	562894	864976
213246	939652	398765	989762	373987	234214
598947	848681	987243	452348	212843	367235
541935	184127	725986	283985	298724	676542

39th.	40th.	41st.	42d.	43d.
8342534	8346214	16241	62743	69012
28754	5124397	760398	208	58409
989342	693	4869373	57	236
3584	26458	6057892	8	1278
5143862	9732325	3241639	674	96234
8764239	674232	18070	45289	53
54362	2349789	795	4326	63760
543727	8942457	7489268	93617	91894
7614124	9873	862372	194	7368
3615	246254	62373	45968	79459
6734278	5691312	4657989	79839	9858
324368	1698714	8493146	2689	73634

44. Add 374, 8962, 4376, 97, 8329, 7854, 6376, 937, 421, 532.
45. Add 8763, 4989, 7247, 3836, 8984, 6523, 8972, 5431.
46. Add 974, 2532, 47, 6931, 8546, 7900, 521, 8986, 3425.
47. Add 80320, 7568, 213, 79241, 8089, 54963, 24907, 5806.
48. Add 62154, 321, 6742, 79894, 8280, 732, 9890, 76257, 5426, 34695, 423, 8762, 93957, 1047, 234, 97895.
49. Add 821, 6376, 21347, 90, 8674, 25473, 547, 4962, 87469, 5693, 259, 8789, 98, 725, 4791, 82543, 27682, 41327.
50. Add 3432168, 754239, 2142, 54987, 6243, 29894, 4513, 910837, 4246, 8245, 9324, 54562, 3432, 87632, 1444.
51. Add 2426, 329, 8762, 436, 98724, 5431, 8899, 989, 74871, 2324, 69897, 54958, 737465, 368964, 54369, 9897.
52. Add 3762, 8943, 2146, 9891, 6546, 8743, 2156, 3634, 89214, 3291, 5342, 635, 8971, 259, 83, 24367, 54236253.
53. Add 9990, 6784, 3742, 5163, 15426, 9194, 7542, 3674, 9542, 1736, 854, 29678, 4690, 7542, 9895, 4163.

54. Add 8705, 6582, 9766, 479, 385, 675, 4073, 258, 14695, 5416, 2681, 97215, 8762, 9395, 289, 4536, 876512.
55. Add 768, 2489, 7214, 5217, 62435, 607, 7215, 4729, 36316, 299, 9363, 4537, 46298, 7674, 92843.
56. Add 6542, 1769, 832, 4546, 3060, 725, 9876, 39754, 8269.
57. Add 968, 5427, 989, 700, 4621, 85, 2668, 7423, 98956, 2451, 76326, 7452, 666, 545, 372, 9800, 4256, 834316.
58. Add 865, 7324, 6973, 584, 98, 762, 3478, 9721, 54531.
59. Add 86854, 3275, 426, 9724, 7631, 896, 989, 762, 5478.
60. Add 754987, 54326, 54859, 4632, 58, 697, 2468, 972, 1687, 439, 58, 674, 5869, 7987, 434, 2673, 989, 546.
61. What is the sum of eighty-seven million and ninety-six. Forty-eight thousand million, nine hundred and fifty-seven. Seven hundred and sixty-eight thousand and eighty-four. Three million and sixty-seven thousand. Fourteen billion, ninety-nine million and eighty-six; and nine hundred and ninety-nine? Ans. 14048189837222.
62. What is the sum of seven hundred thousand and seventy-seven. Five million and fifty-nine thousand and eighty-three. Fifty-six thousand and ninety-one million; and one hundred and seventy-six thousand three hundred and ninety-seven? Ans. 56096935557.
63. A is indebted to B £359; to C £687; to D £189; to E £1342; to F £546; to G £746; to H £87; to K £862; to L £1697; to M £754; required A's debt.— Ans. 7269.
64. If the distance from Inverness to Banff be 71 miles; from Banff to Aberdeen 44 miles; from Aberdeen to Perth 81 miles; from Perth to Edinburgh 41 miles; from Edinburgh to Dumfries 72 miles; from Dumfries to Newton-Stewart 51 miles; and from Newton-Stewart to Portpatrick 35 miles;—How far is Inverness from Portpatrick, and how far is each of them from Edinburgh?

Ans.—From Inverness to Portpatrick 395, Edinburgh from Inverness 237, from Portpatrick, 158.
65. In what year will a man be three score and ten, who was born in 1790? Ans. 1860.
66. In what year will a man be 74, who was born in 1829? Ans. 1903.
67. A man who is now 37 years of age, was born in July 1789; in what month of what year will he be 84 years of age? Ans. In July 1873.

68. A gentleman left in his last will £7398 to his widow; £15680 to his old son; £6847 to each of his four younger sons; £4876 to each of his three daughters; £3750 to endow a school, and £675 to the poor.—How much did he leave in all? Ans. £69519.

SIMPLE SUBTRACTION.

Simple subtraction is the method of finding the difference between two simple numbers.

Rule.—Write the less number under the greater, with the same places directly under each other as in addition, and draw a line below. Begin on the right, and take each figure of the under number from the one directly above it, and place the difference right below. When any figure in the under number is greater than the one above it, borrow one to it from the next significant figure in the upper number, which suppose placed on its left, and take the under figure from the number thus formed; but if one or more ciphers intervene, between the figure you took the one from, and that to which it is taken, they become nines, and the first significant figure is diminished by one.

Proof.—Add the difference to the least number, and the sum will be equal to the greatest number, when the work is right. Or, subtract the difference from the greater number, and their difference will be equal to the least number when the work is right.

Example.—Take 4398321876 from 9300054817.

Greater	9300054817	minuend.
Less	4398321876	subtrahend.
Difference	4901732941	remainder.
Proof.	9300054817	by 1st method.
Proof	4398321876	by 2d. method.

Having placed the numbers under each other, I take 6 from 7 and 1 remains, which I write below the 6, then 7 from 1 I cannot, but borrow 1 from 8 to it, makes 11, and 7 from 11 leaves 4; then 8 from 7, I cannot, borrow 1 from 4 to it, makes 17, and 8 from 17 leaves 9; then 1 from 3 leaves 2; and 2 from 5 leaves 3; then 3 from 0, I cannot, borrow 1 from 3 to the 1st 0 on the left makes it 10, take 1 from it, leaves 9, and makes next 0 a 10, from which take 1 leaves 9, and makes the 1st 0 on the right 10, then 3 from 10 leaves 7; 8 from 9 leaves 1; 9 from

9 leaves 0; 3 from 2, I cannot, borrow 1 from 9 to it, makes 12, and 3 from 12 leaves 9, then 4 from 8 leaves 4.

I have given the proof by both methods. By the 1st method, the difference is added to the less number, which gives a sum equal to the greater; and by the 2d method, the difference is subtracted from the greater number, which leaves a remainder equal to the less number.

EXERCISES.

1st.	2d.	3d.	4th.	5th.
From 8736	From 7463	From 3215	From 9461	From 1679
Take 5314	Take 4251	Take 1203	Take 6350	Take 1456

6th.	7th.	8th.	9th.	10th.
63745	69796	47652	65472	41687
23851	57987	13987	16789	25819

11th.	12th.	13th.	14th.	15th.
375621	848965	451326	514237	643827
167694	398968	415983	214239	441928

16th.	17th.	18th.	19th.	20th.
685436	374219	729875	982143	263245
398175	127653	529876	871035	198794

21st.	22d.	23d.	24th.	25th.
4513642	6904326	6651674	9321367	4321436
2591783	5403987	3914865	6519876	2658209

26th.	27th.	28th.	29th.
98730054	62000352	23469765	47600041
68978075	46987549	12368976	29124567

30th.	31st.	32d.	33d.
45200000	65070601	70063000	98080842
41698973	26789543	19952992	63171753

34th.	35th.	36th.	37th.
111111111	987654321	432987689	270006415
11111112	87654322	356278940	179896724

	38th.	39th.	40th.	41st.
From	634253865	930000005	370005004	542300015
Take	379854326	876543219	276352068	436954623

42d.	43d.	44th.	45th.
613020054	876540397	162392453	754320060
463275946	487634298	156293625	213247123

46th.	47th.	48th.	49th.
100000000	100000080	370432000	690405060
27254687	279	247634594	254302987

50th.	51st.	52d.	53d.
710101012	790630012	615002317	394130001
372456109	549872134	297432128	218241162

54. From sixty-nine million, take seven thousand and eight. Ans. 68992992.
55. From eighty billion and eighty-five, take nine hundred and fifty-nine. Ans. 79,999999,999126.
56. From nine hundred thousand trillion, take a hundred thousand. Ans. 899999,999999,999999,900000.
57. From six nonillion, take nine trillion, five billion and fifty-five. Ans. 5,999999,999999,999999,999999,999999, 999990,999994,999999,999945.
58. From a hundred thousand septillions, take a thousand and forty-six. Ans. 99999,999999,999999,999999,999999, 999999,999999,998954.
59. Find the difference between ninety-nine million, and fifty trillion. Ans. 49,999999,999901,000000.
60. How much does a quintillion exceed nine hundred and ninety million? Ans. 999999,999999,999999,999001,000000.
61. Borrowed £1752, and paid back £1349, how much am I in debt? Ans. £403.
62. Lent my friend £875, and received at one time £195, at another time £356, and at another £172; how much is still due to me? Ans. £152.
63. Agricola's wall between Forth and Clyde was built in the year 85 of the Christian era; how old was it in 1829? Ans. 1744 years.
64. Glass was invented in England in the year 664, how many years is it since that period, this being 1829? Ans. 1165.

65. How long is it since decimal arithmetic was invented, which was in the year 1602, this being 1829? Ans. 227.
66. How old was the man who was born in 1745, and died in 1829? Ans. 84.
67. A has in goods and money £7629, but he owes to B £67; to C £98; to D £83; to E £179; to F £508; to G £104; and to H £110. K, has an estate worth £2656, besides L owes him £145; M £329; N £972; P £489; he has a ship worth £2500; Bank stock worth £500; household furniture worth £980; but he is indebted to Q £320; to R £650; and to S £573; what is the difference between their fortunes; and who has most?
Ans. K £548 more than A.
68. A was born in 1758, and B in 1800; how much is A older than B? Ans. 42 years.
69. Take one from ten thousand; and ten from a hundred thousand. Ans. 9999, and 999990.
70. Babel was built 2247 years, and Rome 753 years before the Christian era; how many years between these events; and in 1829 how long since each of them?
Ans. 1494 between, 4076 and 2582 since each.

SIMPLE MULTIPLICATION.

MULTIPLICATION is a short method of performing addition, when the same number is to be added, or repeated, any proposed number of times.

The number to be multiplied, is called the MULTIPLICAND; the number multiplied by, is called the MULTIPLIER; and the number arising from the multiplication is called the PRODUCT. The two numbers are also called FACTORS.

RULE.—Write the multiplier under the multiplicand, so, that like places may stand directly under each other; and draw a line below them. Begin at the right, and multiply the whole of the multiplicand, by each significant figure of the multiplier separately, from right to left; observing always to place the first figure of each product, directly under the figure you multiply by; and to the product of each subsequent place, add what you had to carry from the preceding product, as in addition. Add the several lines of products in the order in which they stand; their sum is the total product.

PROOF.—Multiply the multiplier by the multiplicand; and if the total product be the same as the former, the work is right.

EXAMPLE.—Multiply 765034 by 3049.

765034 Multiplicand.
3049 Multiplier.

6885306
3060136
2295102

2332588666 Product.

Having placed the numbers as in the example, I begin at the right, and say 9 times 4 are 36, write 6 under 9 and carry 3 to next product; 9 times 3 are 27 and 3 are 30, write 0 and carry 3; 9 times 0 and 3 are 3, write it down; 9 times 5 are 45, write 5 and carry 4; 9 times 6 are 54 and 4 are 58, write 8 and carry 5; 9 times 7 are 63 and 5 are 68, which write in full as in the example. Again multiplying by the 4 in tens place, I say 4 times 4 are 16, and write 6 under the 4 I am now multiplying by,—and carry 1 to next product as before; then 4 times 3 are 12 and 1 are 13, write 3 and carry 1; 4 times 0 and 1, write 1; 4 times 5 are 20, write 0 and carry 2; 4 times 6 are 24 and 2 are 26, write 6 and carry 2; 4 times 7 are 28 and 2 are 30, which write in full. Again 3 times 4 are 12, write 2 under the 3 and carry 1; 3 times 3 are 9 and 1 are 10, write 0 and carry 1; 3 times 0 are 0 and 1 are 1; 3 times 5 are 15, write 5 and carry 1; 3 times 6 are 18 and 1 are 19, write 9 and carry 1; 3 times 7 are 21 and 1 are 22, write in full. Add the different products in the order in which they stand, and we have 2332588666 the total product. This is proved by making the multiplicand the multiplier, and proceeding as before; and if the total product thus obtained be 2332588666 the work is presumed to be right.

EXERCISES.

1. Mult. 364 by 3.
2. Mult. 652 by 5.
3. Mult. 758 by 6.
4. Mult. 587 by 8.
5. Mult. 467 by 9.
6. Mult. 656 by 7.
7. Mult. 708 by 11.
8. Mult. 878 by 12.
9. Mult. 36732 by 34.
10. Mult. [illegible] by 356.
11. Mult. 360542 by 3405.
12. Mult. [illegible] by 6075.
13. Mult. [illegible] by 70604.
14. Mult. 656034 by 87[illegible].
15. Mult. 243241 by 37006.
16. Mult. 642063 by 87089.
17. 75436425 × 803004.
18. [illegible] × [illegible].
19. 87[illegible] × [illegible].
20. [illegible] × [illegible].
21. [illegible] × [illegible].
22. [illegible] × [illegible].
23. 8790554 × 897003.
24. 1342008 × 403050.

25. Multiply eighty-four thousand and ninety-five, by eight thousand and seventy-six. Ans. 679151220.
26. Multiply three millions four hundred thousand and twelve,

by eight hundred and fifty-three thousand and seventy-nine. Ans. 290046860001023б948.

27. How many days in 52 weeks, each week having seven days? Ans. 364.
28. How many days in 1829 years, each year having 365 days? Ans. 667585.
29. How many hours in 365 days, each day being twenty-four hours? Ans. 8760.
30. There are 52 Sabbaths in a year, how many are in 1829 years? Ans. 95108.

Contractions in Multiplication.

Contraction 1.—When the multiplier is 10, 100, 1000, &c.
Rule.—Annex the ciphers to the multiplicand, and the work is finished.

EXERCISES.

1. Mult. 6754 by 10.
2. Mult. 1426 by 100.
3. Mult. 7421 by 1000.
4. Mult. 76 by 10000.
5. Mult. 89 by 100000.
6. Mult. 54 by 1000000.

Contraction 2.

When there are ciphers on the right of one, or both factors.

Rule.—Proceed with the significant figures the same as if there were no ciphers present; then annex the ciphers to the product.

EXERCISES.

1. Mult. 3600 by 750
2. Mult. 5400 by 700
3. Mult. 605 by 860
4. Mult. 760 by 960
5. Mult. 807900 by 850600.
6. Mult. 640870 by 908700.
7. Mult. 463010 by 250200.
8. Mult. 754000 by 626450.

Contraction 3.

When the multiplier is the product of two, or more numbers not exceeding 12.

Rule.—Multiply the multiplicand by one of those numbers, and that product by another, and so on, the last product is the answer, or total product required.

EXERCISES.

1. Mult. 8964 by 24.
2. Mult. 3765 by 27.
3. Mult. 6243 by 36.
4. Mult. 8376 by 42.
5. Mult. 3426 by 56.
6. Mult. 4069 by 63.
7. Mult. 5735 by 81.
8. Mult. 4365 by 64.

9. Mult. 6057 by 72.
10. Mult. 7324 by 108.
11. Mult. 5603 by 77.
12. Mult. 5423 by 112.
13. Mult. 8976 by 84.
14. Mult. 9754 by 336.
15. Mult. 4967 by 96.
16. Mult. 4297 by 504.
17. Mult. 6734 by 224.
18. Mult. 3705 by 729.
19. Mult. 6051 by 152.
20. Mult. 3792 by 448.

CONTRACTION 4.

When a figure of the multiplier repeated any number of times under 13, gives any other figure, or number found in the multiplier.

RULE.—Multiply by that figure, then multiply the product by the number of times this figure must be repeated to give the other figure or number in the multiplier, observing to place the right hand figure of each product under its proper figure in the multiplier, as in the general rule.

EXAMPLE.—Multiply 76458 by 72936.

```
     76458
     72936
   -------
   688122
   2752488
  5504976
----------
5576540688
```

In looking over the multiplier we find that 9 multiplied by 4 gives 36, the two figures on the right; and that 36 multiplied by 2 gives 72, the two figures on the left; we therefore multiply first by nine, and that product by 4; and the product obtained after multiplying by 4, we multiply by two; and these three products added together give us the total product, much more concisely than by the general rule. The example shows the method of placing the products, and this being attended to, the figures in the multiplier may be taken in any order.

EXERCISES.

1. Mult. 85064 by 486.
2. Mult. 89087 by 819.
3. Mult. 57124 by 654.
4. Mult. 84321 by 824.
5. Mult. 63298 by 972.
6. Mult. 17865 by 426.
7. Mult. 7654324 by 84896.
8. Mult. 6203251 by 93756.
9. Mult. 1468024 by 26156.
10. Mult. 5639247 by 71684.
11. Mult. 8216423 by 8212168.
12. Mult. 1432876 by 255125.

CONTRACTION 5.

To multiply by any number between 12 and 20, in one line.

RULE.—Multiply by the figure in unit's place, and to the product of each figure, besides adding according to the general rule, add also the figure in the multiplicand immediately to the right of the one last multiplied; and to the left hand figure of the multiplicand add the last carriage.

EXAMPLE.—Multiply 4635287 by 15.

Here we multiply by 5 in unit's place; 5 times 7 are 35, 5 times 8 are 40+3+7 are 50; 5 times 2 are 10+5+8 are 23; 5 times 5 are 25+2+2 are 29; 5 times 3 are 15+2+5 are 22; 5 times 6 are 30+2+3 are 35; 5 times 4 are 20+3+6 are 29; 4 and 2 are 6.

$$\begin{array}{r} 4635287 \\ 15 \\ \hline 69529305 \end{array}$$

EXERCISES.

1. Mult. 7426 by 12.
2. Mult. 8543 by 14.
3. Mult. 4759 by 15.
4. Mult. 7698 by 16.
5. Mult. 187654 by 17.
6. Mult. 8734691 by 18.
7. Mult. 3424561 by 19.
8. Mult. 4236789 by 12.

PROMISCUOUS EXERCISES.

1. Mult. 874367 by 654.
2. Mult. 704325 by 462.
3. Mult. 843025 by 777.
4. Mult. 106342 by 184.
5. Mult. 989443 by 648.
6. Mult. 636309 by 100.
7. Mult. 816857 by 840.
8. Mult. 160600 by 400.
9. Mult. 747654 by 968.
10. Mult. 374768 by 108.
11. Mult. 474763 by 576.
12. Mult. 294821 by 563.
13. Mult. 7424654 by 87654.
14. Mult. 6300630 by 64816.
15. Mult. 7674325 by 14412.
16. Mult. 1321436 by 96468.
17. Mult. 7374625 by 10084.
18. Mult. 9809462 by 87635.
19. Mult. 8546451 by 12111.
20. Mult. 4546390 by 99194.
21. Mult. 6364912 by 56000.
22. Mult. 2345678 by 68095.
23. Mult. 5435343 by 70809.
24. Mult. 9999999 by 67654.
25. Mult. 674370956 by 384964812, in four lines of products.
26. Mult. 56234987 by 96896576, in four lines of products.
27. Mult. 76894060 by 48761043, and 636742 by 16.
28. Mult. 81687421 by 80040563, and 874632 by 17.
29. Mult. 24846329 by 67819312, and 406039 by 15.
30. Mult. 54320605 by 70060563, and 821264 by 18.
31. Mult. 62468047 by 16842624, and 763241 by 28.
32. Mult. 72724634 by 47687254, and 624965 by 35.
33. Mult. 93929875 by 65827293, and 634210 by 45.
34. Mult. 20430098 by 99887766 and 323423 by 10.
35. Mult. 91963387 by 45963252, and 543462 by 21.
36. Mult. 40243468 by 32345678, and 823216 by 63.

SIMPLE DIVISION.

Division is a compendious method of performing subtraction, or of finding how often one number is contained in another.

SIMPLE DIVISION

Is the method of finding how often one simple number is contained in another.

The number to be divided, or the containing number, is named the Dividend. The number divided by, or the contained number, is named the Divisor. The number of times the dividend contains the divisor, is named the Quotient. The number which is over after the dividend is exhausted, is named the Remainder.

General Rule.—Write the dividend with a curved line on the right and left of it, and write the divisor on the left.

Find how often the divisor is contained in the fewest possible figures on the left of the dividend; and write the figure expressing the number of times on the right, for a quotient.

Multiply the divisor by the quotient figure, and write the product as it arises, under the figures out of which it was taken. Draw a line under, and subtract it from the figures immediately above it; increase the remainder by annexing to it the next figure in the dividend.

Find how often the divisor is contained in the remainder thus increased; if it is not contained once, write a cipher in the quotient, and increase it further by next figure of the dividend; find how often the divisor is then contained in it, write the figure expressing the number of times in the quotient, and multiply the divisor by it, subtract the product from the increased remainder, this will give a new remainder, which increase as before; and proceed in the same manner until the dividend be exhausted; the quotient, and last remainder, if any, will be the answer.

Proof.—Multiply the quotient by the divisor, to their product add the remainder, this sum will be equal to the dividend when the work is right.

EXAMPLE.—Divide 235457 by 87.

Divisor.	Dividend.	Quotient.
87)	235457	(2706$\frac{35}{87}$
	174	
	614	
	609	
	557	
	522	
	35	

PROOF.
2706
87
18942
21648
235422
35
235457

I write the dividend with a curved line on the right and left of it, and write the divisor on the left. I see that the divisor, 87, is contained twice in 235, the three figures on the left of the dividend. I therefore write 2 in the quotient; and multiply the divisor by it, writing the product as it arises, under 235 out of which it was taken, placing the right hand figure of the product under the right hand place of the dividend which was taken in; draw a line under, and subtract, which gives us 61 of a remainder; to which annex 4, the next figure of the dividend, gives 614 for the increased remainder; in this the divisor goes 7 times, I therefore write 7 in the quotient, and multiply the divisor by it, and subtracting the product from the increased remainder, leaves 5 for a new remainder; this I increase by 5, the next figure in the dividend; which gives 55 for the new increased remainder, in this the divisor will not go once, I therefore write a cipher in the quotient, and further increase the remainder by annexing 7, the next figure in the dividend, which gives 557, in this the divisor goes 6 times, I write 6 in the quotient, and multiply the divisor by it, and subtract the product from 557 the increased remainder, which leaves 35; and the dividend being now exhausted, 35 is the last remainder, which I write in the quotient, with the divisor below it, in form of a fraction, thus $\frac{35}{87}$.

For Proof, multiply the quotient, 2706, by the divisor, 87, and add the remainder 35 to the product, the sum is the same as the dividend.

EXERCISES.

1. Divide 6247 by 5.
2. Divide 8975 by 9.
3. Divide 3467 by 11.
4. Divide 8842 by 13.
5. Divide 7987 by 15.
6. Divide 697654 by 21.
7. Divide 681213 by 29.
8. Divide 513243 by 34.
9. Divide 612132 by 37.
10. Divide 978943 by 46.
11. Divide 7698734 by 57.
12. Divide 8363424 by 59.
13. Divide 1231425 by 63.
14. Divide 2489721 by 69.
15. Divide 8725832 by 76.
16. Divide 4342489 by 87.
17. Divide 3498425 by 94.
18. Divide 8248329 by 112.
19. Divide 6362487 by 122.
20. Divide 8463243 by 208.
21. Divide 78734625 by 359.
22. Divide 67498725 by 486.

23. Divide [illegible] by [illegible].
24. Divide [illegible]4681 by [illegible].
25. Divide [illegible]248726 by [illegible].
26. Divide 62468754 by 856.
27. Divide [illegible] by [illegible].
28. Divide [illegible] by [illegible].
29. Divide [illegible] by [illegible].
30. Divide [illegible] by [illegible].

31. Divide thirty-five million, eight hundred and ninety-seven thousand one hundred and twenty-one, by five hundred and seven. Ans. 70803.
32. Divide twenty-three million, sixty-five thousand eight hundred and fifty-four, by six thousand three hundred and forty-two. Ans. 3637.
33. Divide one billion, four hundred and forty-nine thousand four hundred and fifty-two million, nine hundred and forty-three thousand nine hundred and twelve, by three million twelve thousand and thirteen. Ans. 481224.
34. If a nobleman's income be thirty thousand six hundred and sixty pounds sterling a-year; how much is that per day, allowing 365 days to the year? Ans. £84.
35. Suppose an army of 255794 men were to be drawn up in line of battle, 14 men deep, how many must be in each rank? Ans. 18271.
36. A party of soldiers, 425 miles from head quarters, have orders to be forward in 17 days; how many miles must they march each day? Ans. 25.
37. Divide a prize of £3564 among 54 sailors, giving the same sum to each. Ans. £66.
38. Divide £51113610 among 842 men, giving each an equal share. Ans. £60705.
39. Divide £12664278 equally among 4213 persons. Ans. £3006.
40. Divide £5146225 into 245 equal shares. Ans. 21005.

Contraction 1.

When the divisor is under 13.

Rule.—Multiply, and subtract, mentally, writing the quotient under the dividend.

Example.—Divide 539456 by 8.

$$8 \overline{)\,539456}$$
$$\quad 67432$$

Having placed the numbers as in the example, with a line under the dividend, I try how often 8 is contained in 53, and find it 6 times and 5 over, I write 6 under 3, and place the 5, or suppose it placed, on the left of next figure 9, making 59, in which 8 goes 7 times, and 3 over, I write 7 under 9, and the 3 which was over placed before next figure makes 34, in which 8 goes 4 times, and 2 over, I write 4, and the 2 placed before next figure makes 25, in which 8

goes 3 times and 1 over, write 3, and the 1 placed before next figure makes 16, in which 8 goes 2 times, and nothing over, write 2 under the right hand figure of the dividend, and the work is finished.

EXERCISES.

1. Divide 1962972 by 3.	6. Divide 24096360 by 8.
2. Divide 3061728 by 4.	7. Divide 87654366 by 9.
3. Divide 7987645 by 5.	8. Divide 41032104 by 2.
4. Divide 2188734 by 6.	9. Divide 50613211 by 11.
5. Divide 5844867 by 7.	10. Divide 37089804 by 12.

CONTRACTION 2.

To divide by 10. 100. 1000, &c.

RULE.—Cut off from the right of the dividend, so many figures as there are ciphers in the divisor. The figures cut off are the remainder, and those on the left the quotient.

EXAMPLE.—Divide 873657 by 10000.

$$87,\frac{3657}{10000}$$

I write down the dividend, and point off four places to the right, for the four ciphers in the divisor, under which I write the divisor in full, in form of a fraction for the remainder, and 87 on the right is the quotient.

EXERCISES.

1. Divide 8346 by 10.	5. Divide 71034216 by 100000.
2. Divide 7534 by 100.	6. Divide 4103246 by 1000000.
3. Divide 40325 by 1000.	7. Divide 31463412 by 10000000.
4. Divide 1608742 by 10000.	8. Divide 920012341 by 100000000

CONTRACTION 3.

When there are ciphers on the right of the divisor.

RULE.—Cut them off, and cut off as many figures from the right of the dividend, then divide. To the right of last remainder annex the figures cut off from the dividend for the true remainder.

EXAMPLE.—Divide 67843 by 3400.

```
34,00)678,43(19 3243/3400
      34
      ---
      338
      306
      ---
       32
       --
```

Here I cut off the two ciphers from the right of the divisor, and two figures from the right of the dividend; and then divide 678 by 34, by the general rule, which gives 19 in the quotient and 32 of a remainder, to which I annex the two figures cut off from the dividend, and write the whole of the divisor under it, as in the example.

EXERCISES.

1. Divide [illegible] by [illegible].
2. Divide 8240230 by 26800.
3. Divide 9873024 by [illegible].
4. Divide [illegible] by [illegible].
5. Divide [illegible] by [illegible].
6. Divide [illegible] by [illegible].
7. Divide 24137240 by [illegible].
8. Divide [illegible] by [illegible].
9. Divide [illegible] by [illegible].
10. Divide [illegible] by [illegible].

CONTRACTION 4.

When the divisor is the product of two or more numbers under 13.

RULE.—Divide by any one of those numbers, and the quotient thus arising by another, and so on; the last quotient is the answer. To find the true remainder. Multiply the last remainder by the preceding divisor, taking in its own remainder, if any, and this sum by next preceding divisor, taking in its remainder, and so on with all the preceding divisors and remainders, the last sum is the true remainder, under which write the whole divisor.

EXAMPLE.—Divide 687395 by 168.

4)687395	4 = Last remainder.
6)171848—3	6 = Preceding divisor.
7)28641—2	24 Their product.
4091—4	2 Remainder of division by 6.
	26
	4 Next preceding divisor.
$4091\frac{107}{168}$ Ans.	104
	3 Remainder of division by 4.
	107 True Remainder.

EXERCISES.

1. Divide 84679254 by 16.
2. Divide 12654987 by 18.
3. Divide 76549386 by 24.
4. Divide 20725685 by 32.
5. Divide 29807634 by 42.
6. Divide 78910234 by 72.
7. Divide 68424987 by 108.
8. Divide 68797685 by 132.
9. Divide 42436725 by 144.
10. Divide 62798542 by 112.
11. Divide 79620874 by 128.
12. Divide 97834245 by 192.
13. Divide [illegible] by [illegible].
14. Divide [illegible] by [illegible].
15. Divide 363005218 by 386.
16. Divide 980705004 by 432.
17. Divide 484625406 by 462.
18. Divide [illegible] by 504.
19. Divide 712108762 by [illegible].
20. Divide [illegible] by [illegible].
21. Divide [illegible] by [illegible].
22. Divide [illegible] by [illegible].
23. Divide 897200607 by [illegible].
24. Divide [illegible] by [illegible].

PROMISCUOUS EXERCISES.

1. Divide 32544 quarters of wheat, equally among 48 men. Ans. 678.
2. Divide £112 equally among 28 poor people. Ans. £4.
3. If the clothing, and furnishing of an army of 295684 men, cost £4730944; how much is that a man? Ans. £16.
4. Suppose the circumference of the globe in the latitude of Edinburgh be 24960 miles, and that the earth makes a complete revolution in 24 hours; at what rate per hour is that city carried from west to east? Ans. 1040 miles.
5. Suppose the earth's annual circuit round the sun be 509142880 miles, and that it completes a revolution in 365 days; at what rate per day are we carried by this motion? Ans. 1394912 miles.
6. Suppose there are one thousand million six thousand five hundred and sixty human beings on the earth, and that the average life of man is 36 years, of 365 days; how many die every day? Ans. 76104 daily.

REDUCTION.

REDUCTION is the method of changing money, weights, and measures from one denomination to another, without altering their value.

Reduction is generally divided into three cases. First, when the reduction is from a higher to a lower denomination; and is called reduction descending. Second, when the reduction is from a lower, to a higher denomination; and is called reduction ascending. Third, when it is required to know, how many of one denomination are contained in another, of which the higher denomination does not exactly contain the lower, and is called reduction ascending and descending.

CASE 1.

RULE.—Multiply the highest denomination, by the number of units in next inferior denomination which make one of it, adding to the product, the number in that inferior denomination. Multiply the denomination thus obtained, by the number in next inferior denomination which make one in it, adding to the product, the number in that denomination. Proceed thus, till you bring it to the denomination required.

To prove reduction, reverse the operation.

EXAMPLE.—Reduce £17 16s. 5½d. to farthings.

```
 £   s.  d.
 17  16  5½
 20
----
 356
  12
----
 4277
    4
-----------
4)17110=farthings.
12)4277 ½
2,0)35,6  5
 £17 16  5½—Proof.
```

I here multiply £17, the highest denomination, by 20, because 20 shillings make a £, and to the product I add the 16 shillings, which gives 356 shillings; I then multiply this sum by 12, because 12 pence make a shilling, and to the product I add the 5d. which gives 4277d.; I multiply this sum by 4, because 4 farthings make a penny, and to the product I add the 2 farthings, which gives 17110 farthings, which are the farthings in £17 16s. 5½d, and of the same value. For proof, I divide the farthings by 4, 12, and 20, which gives £17 16s. 5½d. the original sum.

EXERCISES.

1. Reduce £2, into shillings. Ans. 40s.
2. Reduce £5, into pence. Ans. 1200.
3. Reduce £7, into farthings. Ans. 6720.
4. Reduce £8 16s. 4d. into pence. Ans. 2116.
5. Reduce £12 15s. 6d. into sixpences. Ans. 511.
6. Reduce £7 18s. 7d. into halfpence. Ans. 3806.
7. Reduce 74 guineas into pence. Ans. 18648.
8. Reduce 18 shillings and 4¾d. into farthings. Ans. 883.
9. Reduce 84 sovgns. 16s. and 8d. into fourpences. Ans. 5090.
10. Reduce £157 11s. 10d. into twopences. Ans. 18911.
11. Reduce 754 half guineas into sixpences. Ans. 15834.
12. Reduce £734 16s. to sh. d. qr. Ans. 14696 sh. 176352d. 705408 qrs.
13. Reduce 7 lb. troy, into grains. Ans. 40320.
14. Reduce 4 lb. 7 oz. 15 dwts. 20 gr. into grains. Ans. 26780.
15. Reduce 20 lb. 6 oz. 3 dwts. into penny-weights. Ans. 4923.
16. In 3 cwt. 2 qrs. 14 lb. how many lbs.? Ans. 406.
17. In 15 ton 6 cwt. 3 qrs. 4 lb. 6 oz.; how many ounces? Ans. 549766.
18. In 2 cwt. 3 qrs. 23 lb. how many troy grains? Ans. 2317000.
19. How many scruples in 4lb. 6℥, 7ʒ ? Ans. 1317.
20. How many ℥s. ʒs. ℈s. and grains in 2lb. 6℥ ? Ans. 30℥, 240ʒ, 720℈, 14400 grs.
21. How many grains in 1lb. 6ʒ, 2℈, 8 grains? Ans. 6168.
22. Reduce 2 lasts, 6 sacks, 1 st. of wool into cloves. Ans. 1562.

23. Reduce 6 sacks wool into lbs. and troy grains.
Ans. 2184 lbs. 15288000 troy grains.
24. How many lbs. of straw in 36 loads? Ans. 46656.
25. How many lbs. of new hay, in 24 loads? Ans. 51840.
26. In 3 miles, 1 furlong, 2 poles, how many feet? Ans. 16533.
27. In 2 leag. 1 mile, 2 fur. how many yards? Ans. 12760.
28. In the earth's circumference, which is 360 degrees, and each degree $69\frac{1}{2}$ miles, how many imperial yards? Ans. 44035200.
29. Reduce 35 yds. 2 qrs. into inches. Ans. 1278.
30. Reduce 63 French ells, into nails. Ans. 1512.
31. Reduce 12 ells English, 4 qrs. 3 nails, to nails. Ans. [illegible]
32. How many square yards in 2 acres, 1 r. 4 p.? Ans. 11011.
33. How many square feet in 3 acres, 2 r. 20 p.? Ans. 157905.
34. In 1 hhd. 10 g. 1 qt. of wine; how many gills? Ans. 2344.
35. In 3 t. 1 p. 1 hhd. 42 gal. 2 qts. how many pts.? Ans. 7900.
36. In a puncheon of rum, how many gills? Ans. 2688.
37. Reduce 4 barrels, 6 gallons of ale, into quarts. Ans. 600.
38. Reduce 6 tons of ale, into pints. Ans. 10368.
39. Reduce 4 hds. 1 bar. 1 fir. 4 g. of ale, into gills. Ans. 8480.
40. In 3 qrs. 4 b. 2 p. 1 g. wheat; how many pints? Ans. 1832.
41. In 10 lasts, 1 w. 1 ch. 3 qrs. 1 coom of wheat; how many bushels? Ans. 900.
42. In 15 qrs. 2 bush. 3 pecks; how many gills? Ans. 31424.
43. How many pecks of coal, in 6 ch. 6 sac. 1 bu.? Ans. 940.
44. Reduce 365 days, 5 hours, 48 min. 48 sec. to seconds.
Ans. 31556928.
45. Reduce four-score years, each $365\frac{1}{4}$ days, into hours.
Ans. 701280.

Case 2.

Rule.—Divide the given denomination, by the number of it, which makes one of next higher denomination, the quotient is in that denomination; divide this quotient by the number of it, which makes one of next higher denomination; proceed thus till you obtain the required denomination. Remainders are invariably of the same denomination as the dividends to which they belonged.

Example.—How many £'s are in 14815 farthings?

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farthings.
4)14815
12)3703—3
2,0)30,8—7
£15 8 7¾
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I here divide farthings, the given denomination, by 4, because 4 farthings make 1 penny, and the quotient is pence, and the three which are over are 3 farthings; I then divide the last quotient by 12, because 12 pence make one shilling, and the quotient is in shillings, and the 7

which are over are 7d.; again, I divide the last quotient by 20, because 20 shillings make a £, and the quotient is £15 and 8 shillings over; therefore £15, together with the several remainders, is the answer, as in the example.

EXERCISES.

1. Reduce 700 shillings, into £s. Ans. £35.
2. Reduce 1040 pence, into £s. Ans. £4 6s. 8d.
3. Reduce 5402 farthings, into £s. Ans. £5 12s. 6½d.
4. Reduce 999 sixpences, into guineas. Ans. 23 guineas, 16s. 6d.
5. Reduce 7829 twopences, into £s. Ans. £65 4s. 10d.
6. Reduce 25880 sixpences, into half-crowns, and sovereigns. Ans. 5176 half-crowns—647 sovereigns.
7. Reduce 1687 crowns, into £s. Ans. £421 15s.
8. Reduce 75600 farthings, into half-guineas. Ans. 150.
9. In 70152 grs.; how many lbs. troy? Ans. 12 lbs. 2 oz. 3 dwts.
10. In 15568 dwts.; how many lbs.? Ans. 64 lbs. 10 oz. 8 dwts.
11. How many tons, in 6045216 drams? Ans. 10 t. 10 cwt. 3 qrs. 10 lb. 2 oz.
12. How many cwt. avoird., in 28098000 grains? Ans. 35 cwt. 3 qrs. 10 lb.
13. Reduce 43404 scruples, into lbs. Ans. 150 lbs. 8 ℥. 4 ʒ.
14. Reduce 11740 gr., to apothecaries ozs. Ans. 24 ℥. 3 ʒ. 2 ℈.
15. In 4393 lbs. of wool; how many tods? Ans. 156 tods, 1st. 1 cl. 4 lb.
16. In 2812 cloves of wool; how many lasts? Ans. 4 lasts 6 sacks, 1 tod.
17. How many loads of old hay in 44352 lbs? Ans. 22.
18. Reduce 4403520 inches, to Eng. miles. Ans. 69 m. 4 fur.
19. Reduce 2935680 feet, into degrees. Ans. 8 degrees.
20. In 1305 inches; how many imp. yds.? Ans. 36 yds. 1 qr.
21. In 912 nails; how many ells English? Ans. 45 ells 3 qrs.
22. How many acres, in 219699216 sq. inches? Ans. 35 a. 4 p.
23. How many acres, in 484000 sq. yards? Ans. 100.
24. Reduce 11452 gills wine, to hhds. Ans. 5 hhds. 42 gal. 3 qt. 1 pt.
25. Reduce 202692 pints of wine, to tons. Ans. 100 tons, 2 hhds. 10 galls. 2 qts.
26. In 12468 pints of ale; how many hhds? Ans. 28 hhds. 1 b. 1 f. 4 galls.
27. In 93992 qts. of ale; how many tons? Ans. 108 t. 1 bt. 1 hhd. 8 g.
28. How many qrs. of corn, in 59696 pints? Ans. 116 qrs. 4 bu. 3 pecks.

29. How many lasts of wheat, in 29024 pecks?
Ans. 90 lasts 1 ch. 3 qrs.

30. Reduce 20816 pecks of coal, into chaldrons.
Ans. 144 ch. 6 sa. 2 bu.

31. Supposing the year to consist of 365 days 6 hours, at what particular period of the Christian era had 960148815 minutes elapsed? Ans. 15 min. past 6 P.M. 7th July 1826.

32. Supposing the year to consist of 365 days, 5 hours, 48 minutes, 48 seconds; how old is the man who has lived 1136049408 seconds? Ans. 36 years.

33. In 9884160 inches; how many miles? Ans. 156.

Case III.

RULE.—Reduce the denomination given, to a denomination which is contained exactly in that required; and then reduce it to that denomination.

EXAMPLE.—Reduce 2368 guineas, into crowns.

```
   2368
     21
   ----
   2368
  4736
  -----
5)49728
  -----
Cr. 9945 3s.
```

I here multiply the given number by 21, the shillings in a guinea, which reduces them into shillings, I then divide by 5, because 5 shillings make a crown, this gives 9945 crowns, and 3 shillings over, as in the example.

EXERCISES.

1. Reduce 1240 guineas, to pounds. Ans. 1302.
2. Reduce £630, to guineas. Ans. 600.
3. In 765 half-guineas; how many half-crowns? Ans. 3213
4. In 625 guineas; how many crowns? Ans. 2625
5. In 6380 half-guin.; how many semi-sovereigns? Ans. 6699
6. How many dollars at 4s. 6d. will pay a debt of £884? Ans. 3928 dollars 4s.
7. How many avoirdup. lbs. are equal to 33600 troy lbs? Ans. 27648.
8. How many yards, in 6808 Flemish ells? Ans. 5106.
9. How many imp. yds., in 6342 French ells? Ans. 9513.
10. How many English ells, in 6840 yards? Ans. 5472.
11. How many scruples, in 1 cwt. avoirdupois? Ans. 39200.
12. Reduce 700 troy lbs. into avoirdupois lbs. Ans. 576.

PROMISCUOUS EXERCISES.

1. In £76 10s.; how many crowns, and sixpences?
Ans. 306 crs. 3060 sixps.

2. In £441; how many guin. and crs.? Ans. 420 guin. 1764 cr.
3. In 343 guineas; how many sixpences and farthings?
Ans. 14406 sixpences, 345744 farthings.
4. In 345744 qrs.; how many half-guineas? Ans. 686.
5. In 153600 qrs.; how many pounds? Ans. 160.
6. How many half-quart.; in 24 sacks of potatoes? Ans. 2304.
7. How many quarters, in 204800 gills of wheat? Ans. 100.
8. How many pecks of oats, in 654 lasts? Ans. 209280.
9. How many quart bottles will 100 tons of wine fill?
Ans. 700 gross.
10. A gentleman has in his cellars 15 tons of port, 12 tons of sherry, 17 tons of claret, and 6 tons 2 hhds. of other wines; how long will it serve his family at the rate of a dozen pint bottles per day? Ans. 23 years, 12 weeks, 5 days.
11. In 3 leagues; how many inches? Ans. 570240
12. How many deg. each 69½ miles, in 44035200 inc.? Ans. 10.
13. How many grs. in 10 tons. 4 cwt. avoird.? Ans. 159993600.
14. Reduce 2880000 grains, into troy lbs. Ans. 500.
15. Reduce 122320 fathoms, into miles. Ans. 139.
16. How many Flemish ells, in 356 French ells? Ans. 712.
17. How many nails, in 872 Flemish ells? Ans. 10464.
18. How many £s, in 6792 pounds Scots? Ans. 566.
19. In £624; how many pounds Scots? Ans. £7488.
20. In 11340 marks Scots; how many £s sterling? Ans. £630.
21. In 689631 placks Scots; how much sterling?
Ans. £957 16s. 5d.
22. How many avoirdup. lbs., in 484 Dutch lbs. each 22 oz.?
Ans. 665 lbs. 8 oz.
23. In 6846 crs.; how many dollars, each 4s. 6d? Ans. 7606 3s
24. In 620 Joannes; how many pounds? Ans. £1116.
25. In 576 moidores; how many guineas? Ans. 740 12s.
26. In 1928 Jacobi; how many pounds? Ans. £2410.
27. How much sterling money, in 21 bags, each containing 20 guineas? Ans. £441.
28. How many tea-spoons, each 12 dwts., in 14 lb. 6 oz. silver?
Ans. 290.
29. In 3 million qrs.; how many pounds and guineas?
Ans. £3125, 2976 guins. 4s.
30. In 80640 oz.; how many cwt.? Ans. 45.

COMPOUND ADDITION.

COMPOUND ADDITION, is the method of collecting numbers, of different denominations, into one sum.

RULE.—Write the numbers to be added, so, that the same places, and denominations may stand directly under each other, and draw a line below.

Add the denomination on the right; find how many of the next higher denomination that sum contains, which add to next column; and write the remainder under the column added. Proceed thus to the highest denomination, the sum of which, with the several remainders is the answer.

The PROOF is the same as in Simple Addition.

EXAMPLE.

£.	s.	d.	qrs.	
345	16	7	½	
429	12	10	¾	
147	17	4	¼	
632	15	9	¾	
32	10	11	¼	
529	19	6	¾	
2118	13	2	¼	Ans.
1588	13	7	½	
2118	13	2	¼	Proof.

The denomination of farthings is here the first on the right, and their sum is 13, and because 4 qrs. make 1 penny, I 13 ÷ 4 which gives 3 pence and ¼ remainder, write ¼ under the column of qrs, and add the 3 pence along with the column of pence, gives 50 pence, and because 12 pence make 1 shilling, I divide by 12, which gives 4 shillings, and 2 pence remainder, I write 2 under the column of pence and add 4 along with the column of shillings, which gives 93 shillings, and because 20 shillings make £1, I 93 ÷ 20, which gives £4, and a remainder of 13 shillings, which I write under the column of shillings, and add 4 to the next column, but next three columns being in the highest denomination are added according to the general rule for Simple Addition, which give £2118, and with the remainders, make the answer £2118 13s. 2¼d. To prove this we cut off the undermost number, and add all the remaining numbers as before, and by adding this second sum, to the number cut off, their sum is equal to the first, when the work is right.

EXERCISES.

1st.			2d.				3d.				4th.			
£	s.	d.	£	s.	d.	qr.	£	s.	d.	qr.	£	s.	d.	qr.
37	15	4	434	12	11	¾	246	13	3	¼	734	12	2	¾
87	13	6	197	17	10	¼	467	10	8	½	26	4	8	½
49	19	8	658	14	6	½	734	19	11	¾	7	17	7	¼
58	12	7	567	15	4	¼	827	17	5	½	86	15	4	½
12	16	5	131	10	3	¾	132	17	4	¼	174	19	11	¼

5th.				6th.				7th.				8th.			
£	s.	d.	qr.	£	s.	d.	qr.	£	s.	d.	qr.	£	s.	d.	qr.
179	19	11	¾	673	10	9	½	421	1	8		176	15	7	½
786	15	10	½	29	8	11		291	17	4	¼	69	12	8	¾
229	10	7	¼	8	17	3	¼	638	15	6	¾	826	16	11	
972	12	6	½	897	14	10		971	11	7		326	4	4	¼
347	19	8	¾	342	12	4		276	14	4	½	73	13	11	
625	17	7		2	2	2	¾	92	17	1		9	14	2	
349	12	6		78	18	11	½	54	4	6	¼	876	19	3	

9. Add £37 14s. 2d., £164 16s. 7½d., £98 12s. 11¾d., £3634 14s. 6½d., £981 1s. 10d., £63 16s. 4d., £5 4s. 2¼d., £24 10s. 6d., £47 4s. 8¾d. Ans. £5057 15s. 10¾d.
10. Add £167 17s. 4d., £981 2s. 2½d., £67 14s. 11d., £76 15s. 4¼d., £67 12s. 9d., £116 16s. 6d., £64 12s. 2¼d., £14 4s. 11¾d., £761 19s. 1½d., £41 14s. 7d.
Ans. £2360 9s. 11¼d.
11. Add £7 12s. 4½d., £3 0s. 4d., £21 7s., £915 5s. 0½d., £93, £86 2s. 6½d., £9 9s. 9d., £84 14s. 4d., £121, £106 7s., £31 0s. 6½d., £39 7s., £58, £374 4s. 8½d.
Ans. £1950 10s. 6¾d.
12. Add £372 4s. 6d., £82 1s. 7¾d., £908 12s. 6½d., £842 4s. 6d., £93 9s. 2½d., £4 7s. 6d., £12 6s. 8d., £781 0s. 4d., £87 7s. 6¾d., £151 11s. 11¾d., £24 10s., £143 3s. 6d., £2146 16s. 7½d., £4789 10s. 10d., £61423 7s. 11¾d.
Ans. £71862 17s. 4d.
13. A is indebted to B £432 0s. 6d.; to C £34 17s. 0½d.; to D £174 6s. 8d.; to E £1747 9s. 4d.; to F £4 2s.; to G £69 8s. 9d.; to H £147; to K £142; to L £769 5s. 4d.; to M £473 7s. 11½d.; to N £111 11s. 11¼d.; to O £87 5s.; to P £192 7s. 6¾d.; what is A's debt?
Ans. £4385 2s. 1d.
14. A bankrupt owes his landlord £346 16s. 8d.; his grocer £63 19s. 7½d.; his tailor £46 5s. 4¾d.; his baker £47 19s. 11d.; his flesher £73 13s. 2¼d.; his shoemaker £14 17s. 9d.; his servants £59 18s. 6d.; his banker £1487 16s. 10d.; his agent £134 17s. 4d.; and to sundry small creditors £15 10s. 8¾d.; what is his debt?
Ans. £2291 15s. 11¼d.
15. A merchant has in cash £3497 18s. 11½d.; in broad cloth £786 12s. 6d; in narrow do. £479 8s. 11½d.; in flannels £247 8s. 9d.; in linens £584 7s.; in silks £4287 4s. 7½d.; in Leghorn bonnets £223 6s. 8d.; in gentlemen's hats £132 3s. 11d.; in stockings £69 10s. 4d.;

in gloves £218 17s. 4d.; and in threads, tapes, buttons, &c. £698 19s. 8¼d; what is he worth? Ans. £11225 18s. 8¾d.

16. A builder paid for ground to build a house £746 12s. 8d.; for stones £543 10s.; for wood £659 8s. 9d.; for lime £97 16s. 6d.; for masons £327 7s. 11½d.; for carpenters £437 16s.; for slaters £230 0s. 6d.; for glaziers £274 6s. 8½d.; for smith's work £217 16s. 9½d.; for plasterers £179 17s. 6d.; for painters £187 15s. 4d.; and for other charges £87 14s. 9¾d.; what must he sell it for to clear £215 10s.? Ans. £4205 13s. 6¼d.

17. A banker upon examining his books, finds the following sums due by the bank; £673 16s.; £348 10s. 6d.; £1489 19s. 6d.; £31893 12s.; £132 17s. 8d.; £28 12s.; £91; £890 18s. 5d.; £1987 7s. 11¾d.; £57 16s.; £357 11s. 9d.; £8487 14s. 4d.; £982 10s. 3d.; £88 5s.; £322 2s. 8d.; and small sums amounting to £89 19s. 11¼d.; how much is due by the bank? Ans. £19922 14s.

18. A gentleman has an estate consisting of 14 farms, let at the following rent; 1st. £320 1s. 6d.; 2d. £434 10s. 8d.; 3d. £108 14s. 5½d.; 4th. £86 12s. 4d.; 5th. £1804 16s. 7¼d.; 6th. £112 10s.; 7th. £54 18s. 6d.; 8th. £36 4s. 10¼d.; 9th. £76 17s. 4d.; 10th. £181 11s. 8d.; 11th. £40 4s. 6d.; 12th. £762 11s. 9d.; 13th. £54 4s. 6d.; and 14th. £15 15s. 5d.; what was his income?

Ans. £4088 14s. 1d.

19. Add together the following sums; £67 17s. 6½d.; £12 10s. 8¼d.; £4 4s.; £123 17s. 9¾d.; £76 12s. 7¼d.; £1398 15s. 4½d.; £2486 9s. 9¾d.; £786 9s. 8½d.; £56 14s. 11¾d.; £7 19s. 8¼d.; £33 3s. 3¾d.

Ans. £5054 15s. 6¼d.

20. Add the following; £3. 3s. 4½d.; £2 1s. 6¼d.; £0 10s. 2¾d.; £0 6s. 3½d.; £1 19s. 3¾d.; £5 4s. 0½d.; £0 0s. 8½d.; 7¼d.; 9½d.; 11¾d.; 10½d.; £1 3s. 2½d.; £74 6s. 9¼d.; £19 9s. 7d.; £146 16s. 5d.; £1 8s. 3½d.

Ans. £256 13s. 5d.

Exercises in Troy Weight.

21st.				22d.				23d.			
lb.	oz.	dwt.	grs.	lb.	oz.	dwt.	grs.	lb.	oz.	dwt.	grs.
7	4	13	14	47	7	16	18	5	4	3	16
15	8	15	18	24	6	14	13	6	7	5	22
18	10	19	23	12	10	7	5	8	10	16	15
4	6	17	16	0	5	0	8	9	11	19	23
12	11	14	13	36	11	17	20	2	3	6	12
1	6	7	6	15	7	3	2	5	7	0	8

24th.

lb.	oz.	dwt.	grs.
16	7	3	14
14	4	15	6
8	9	17	18
14	10	12	13
2	6	7	8
19	9	9	9
3	10	11	15
4	10	5	9

25th.

oz.	dwt.	grs.
6	15	12
4	16	19
8	19	18
7	10	22
10	12	23
11	18	16
5	4	7
4	14	17

26th.

lb.	oz.	dwt.	grs.
147	7	16	17
489	11	17	18
976	10	12	20
708	6	15	16
237	7	16	21
76	5	4	23
727	10	19	4
453	3	7	12

27th.

lb.	oz.	dwt.	grs.
1576	6	5	3
2856	1	17	6
428	10	15	16
1747	7	18	19
4873	4	5	8
74	7	9	20
135	11	18	21

28th.

lb.	oz.	dwt.	grs.
45	7	6	3
85	5	15	16
47	10	18	19
30	4	6	11
69	9	19	8
87	10	12	14
99	11	19	23

29th.

lb.	oz.	dwt.	grs.
2118	6	15	12
489	10	12	13
1674	9	8	10
654	11	17	18
463	2	8	9
453	7	13	14
598	10	14	11

Exercises in Avoirdupois Weight.

30th.

Ton.	cwt.	qr.	lb.	oz.	dr.
42	16	3	17	8	13
19	10	1	24	10	7
76	19	2	22	14	15
36	15	1	8	4	3
9	6	0	27	15	12
99	13	3	0	8	1
52	16	1	12	12	10
16	7	2	8	9	13

31st.

Ton.	cwt.	qr.	lb.	oz.	dr.
12	16	1	15	5	10
15	10	3	16	10	12
14	13	0	27	13	15
7	9	2	16	11	4
48	17	1	0	0	11
10	15	3	19	12	6
25	4	1	8	7	5
31	16	2	14	14	13

32d.

Ton.	cwt.	qr.	lb.	oz.	dr.
7	4	1	3	6	7
15	16	3	14	5	15
2	7	0	8	7	12
9	19	1	27	5	4
17	14	3	7	14	13
42	12	2	9	9	8
32	15	1	17	14	11
8	7	3	6	5	14
11	16	1	15	7	6

33d.

Ton.	cwt.	qr.	lb.	oz.
6	7	1	13	6
4	12	2	17	4
19	19	3	19	15
14	14	1	14	14
9	9	2	9	9
12	13	3	17	13
21	15	1	18	10
14	10	3	4	6
17	16	2	11	12

34th.

Cwt.	qrs.	lb.	oz.	dr.
15	2	7	6	5
19	1	24	10	14
12	3	17	13	14
9	3	12	14	9
4	2	26	15	13
13	1	18	2	7
8	0	4	11	6
18	3	19	9	12
4	2	8	5	7
7	0	15	14	13

35th.

Cwt.	qrs.	lb.	oz.	dr.
16	1	14	3	2
15	3	7	14	13
13	2	12	7	9
7	1	26	13	11
10	3	16	15	14
8	2	10	12	8
19	3	20	7	6
6	1	17	10	12
18	2	16	11	15
6	3	7	6	9

36th.

Ton	cwt.	qr.	lb.
10	7	3	14
98	17	2	27
18	18	1	17
71	16	2	9
18	16	3	7
9	19	1	7
64	13	2	19
12	12	1	12
83	17	3	25
14	16	1	17

Exercises in Apothecaries' Weight.

37th.

lb.	℥	ʒ	℈	gr.
17	8	7	2	19
18	9	5	1	16
41	11	6	2	10
23	6	4	2	17
38	5	3	1	18
72	11	7	2	16
12	10	5	1	16
77	7	7	2	7
4	8	6	1	15

38th.

lb.	℥	ʒ	℈	gr.
22	10	3	1	7
27	7	1	2	18
14	11	7	2	14
17	7	6	1	5
35	9	2	2	12
15	5	4	2	16
18	4	7	1	15
29	9	6	2	17
18	4	5	2	13

39th.

lb.	℥	ʒ	℈	gr.
17	9	5	1	18
16	6	7	2	12
81	8	5	1	18
98	10	4	2	15
27	6	5	1	14
12	11	7	2	9
16	7	5	2	17
14	10	7	1	13
18	11	6	2	19

Exercises in Apothecaries' Fluid Measure.

40th.

Gal.	O	F℥	Fʒ	♏
4	3	12	6	50
9	7	15	7	35
12	5	14	5	59
24	4	12	6	21
18	5	4	2	13
15	3	9	4	15
25	2	13	7	19
16	6	5	4	32
14	7	15	7	18

41st.

Gal.	O	F℥	Fʒ	♏
17	6	5	4	11
6	5	13	5	24
9	7	14	6	25
15	5	13	7	15
12	6	15	5	19
8	4	5	2	3
5	3	8	5	16
25	5	15	4	39
17	7	10	6	54

42d.

Gal.	O	F℥	Fʒ	♏
5	3	10	5	40
6	7	15	4	37
8	5	13	6	12
9	7	14	4	58
7	6	12	7	17
12	1	10	5	24
9	7	15	4	14
15	3	14	7	29
8	7	6	5	58

Exercises in Wool Weight.

43d.

la.	sk.	wy.	td.	st.	cl.	lb.
17	11	1	5	1	1	6
13	10	1	6	1	0	5
12	9	0	4	0	1	4
14	11	1	5	1	1	2
16	7	1	4	1	1	6
3	8	1	6	1	1	5
15	7	1	3	1	1	3
27	6	1	4	1	1	6

44th.

la.	sk.	td.	lb.
15	10	6	20
17	11	5	24
15	5	4	27
23	3	3	12
8	9	2	14
19	8	5	18
15	5	4	24
27	10	6	13

45th.

la.	sk.	td.	st.	cl.
12	11	5	1	1
14	10	6	0	1
16	7	5	1	0
8	9	4	1	1
27	6	3	0	0
15	11	2	0	1
34	7	5	1	1
47	6	6	1	0

Exercises in Lineal Measure.

46th.

M.	f.	po.	yds.
75	7	39	5
58	6	23	4
89	5	16	2
89	6	18	4
37	3	34	1
19	5	13	3
16	6	17	4

47th.

Lea.	mi.	f.	po.	yds.	f.	in.
754	2	7	27	5	2	11
398	1	6	39	4	1	10
837	2	5	34	3	0	9
872	2	1	14	4	2	8
356	1	6	15	2	1	10
878	2	5	24	4	2	8
142	1	7	4	3	1	6

48th.

fur.	po.	yds.	f.	in.
18	39	4	2	10
17	16	5	1	9
37	17	3	2	8
19	25	1	2	5
12	13	0	1	11
21	29	4	0	7
14	16	5	2	6

Exercises in Cloth Measure.

49th.

Yds.	qrs.	nl.	in.
376	3	2	1
107	1	3	2
870	2	1	0
727	3	2	1
875	3	1	2
724	2	3	1
75	3	2	0
437	1	3	1

50th.

E. el.	qrs.	na.
746	4	3
621	3	2
105	2	1
859	3	3
247	4	2
978	2	3
727	6	0
423	1	2

51st.

Fl. el.	qrs.	na.
371	2	3
224	1	2
657	2	3
976	1	2
274	1	1
412	2	3
131	1	2
874	2	1

52d.

Fr.el.	qrs.	na.
716	5	2
217	4	3
825	3	1
271	5	3
879	3	2
321	4	1
794	5	3
241	3	2

Exercises in Land Measure.

53d.

Acres.	rds.	pls.	yds.
37654	3	21	25
16798	1	39	30
67281	3	25	14
68724	2	14	16
97213	3	23	25
10821	1	10	18

54th.

acres.	rds.	pls.
145	3	37
167	2	15
724	1	16
979	3	23
872	1	38
444	3	12

55th.

acres.	r.	pls.	yds.
57	3	30	20
19	2	14	15
67	3	17	16
37	2	25	24
76	3	17	18
94	2	35	27

Exercises in Wine Measure.

56th.

Ton.	p.	hhd.	g.	qt.	pt.	gill.
874	1	1	59	3	1	3
146	0	1	62	2	0	2
741	1	0	45	1	1	2
654	1	1	36	3	1	3
279	1	1	23	2	1	2
875	1	0	16	2	0	1
343	1	1	10	3	1	3

57th.

ton.	hhd.	gal.	pt.
324	3	56	7
342	2	62	6
594	3	15	5
247	1	24	1
935	3	32	6
172	1	14	4
198	3	43	7

58th.

ton.	hhd.	g.	pt.
17	3	13	4
37	2	42	5
28	1	58	7
7	3	15	4
89	2	61	3
54	3	17	1
19	1	29	5

59th.

Hhd.	gal.	pt.	gills.
17	43	2	6
28	13	3	7
64	19	2	5
72	53	1	3
25	36	3	1
77	47	2	1
16	15	1	6
97	39	3	5

60th.

Gal.	qs.	ps.	gs.
43	2	1	3
37	1	0	2
85	3	1	3
88	1	1	1
25	3	1	3
67	2	1	2
32	3	0	1
63	2	1	3

61st.

Tons.	hd.	gal.
43	3	34
15	1	42
22	3	56
35	2	62
74	1	29
57	3	15
15	2	12
19	1	10

Exercises in Ale and Beer Measure.

62d.

Bt.	hhd.	b.	kil.	fir.	gal.	qt.	pt.	gill.
324	1	1	1	1	8	3	1	3
237	1	1	1	1	7	2	1	2
576	0	1	0	0	5	3	0	3
421	1	1	1	1	8	2	1	1
671	1	1	1	0	6	1	1	3
167	1	0	0	1	4	2	1	0
475	0	1	1	0	3	2	0	2

63d.

Hhd.	bar.	f.	gal.
364	0	3	8
216	1	2	7
814	1	3	4
673	1	2	5
432	1	0	6
224	1	3	7
895	1	2	4

Exercises in Dry Stricken Measure.

64th.

La.	w.	qr.	bu.	pe.	gal.
342	1	4	7	3	1
237	1	3	5	2	1
716	1	2	4	3	1
374	1	3	6	2	1
112	0	4	7	1	0
642	1	2	3	2	1
421	1	2	5	1	1

65th.

Qrs.	bu.	gal.	qts.	pts.
234	7	7	3	1
146	5	4	2	1
347	6	5	1	1
242	5	6	3	1
672	4	7	2	0
498	7	6	3	1
897	4	5	1	1

66th.

La.	qr.	bu.	p.
47	9	7	3
67	8	5	2
98	7	6	6
48	4	4	2
72	9	3	5
89	8	7	4
54	5	6	6

Exercises in Dry Heaped Measure.

68th.

Ch.	Sa.	Bu.	P.
347	11	2	3
467	10	1	2
637	9	2	3
476	8	1	1
685	4	2	2
746	8	2	3
124	7	1	2
416	3	0	1

69th.

Ch.	Sa.	Bu.	P.
489	10	2	3
987	11	2	1
463	4	1	2
874	9	2	0
467	8	2	3
625	4	2	1
542	11	1	2
476	5	2	3

70th.

Sc.	Ch.	Sa.	B.
324	20	11	2
834	13	2	1
549	17	5	2
897	12	10	0
465	9	8	2
546	15	4	1
872	14	5	2
193	20	11	1

71st.

Sc.	Ch.	Sac.
15	14	10
26	16	7
89	10	11
35	17	9
98	20	7
14	13	11
18	9	7
13	16	10

Exercises in Time Measure.

72d.

Year.	M.	W.	D.	H.	M.	S.
3654	11	3	6	23	59	59
4732	6	2	3	21	35	45
3231	7	3	5	14	16	19
5432	8	1	4	16	27	12
1813	4	2	6	5	18	19
4762	5	2	1	18	45	54
6768	10	3	5	15	13	33
6476	9	3	4	22	56	24

73d.

Y.	M.	W.	D.
46	10	3	6
59	11	2	4
87	6	2	5
18	10	2	3
46	8	3	5
59	11	1	6
95	7	2	4
39	9	3	5

74th.

D.	H.	M.	S.
34	7	52	35
23	21	59	49
87	16	42	23
98	12	56	45
67	23	42	36
95	18	14	16
74	21	36	42
97	18	39	57

Exercises in Astronomy.

75th.

Sign	D.	M.	S.
10	29	59	59
11	27	35	43
16	15	27	34
18	19	17	16
42	12	33	25
14	16	19	43
82	25	54	37
19	27	33	46

76th.

Great C.	S.	D.	M.	S.
237	11	29	59	59
437	5	14	16	12
987	9	25	14	34
542	7	18	16	10
143	6	25	36	49
187	7	16	13	10
576	10	17	49	36
184	11	25	38	42

77th.

D.	M.	S.
234	46	37
427	18	26
498	37	16
234	39	48
756	45	54
879	58	35
421	19	23
426	37	34

78. A goldsmith bought at one time 18 lb. 7 oz. 16 dwt 14 gr. of gold; at another time 5 lb. 7 oz. 3 gr; at another time 24 lb. 16 dwt.; at another time 12 lb. 6 oz. 20 gr.; at another time 10 lb. 15 dwt.; at another time 14 lb. 4 oz. 17 dwt. 16 gr.; at another time 21 lb. 6 oz. 22 gr.; and lastly, 6 oz. 15 dwt. 17 gr.; how much did he buy in all? Ans. 107 lb. 4 oz. 2 dwt. 20 gr.

79. A grocer bought 37 cwt. 3 qr. 10 lb. 7 oz. of sugar; 2 cwt. 3 qr. 7 lb. 5 oz. of tea; 34 cwt. 1 qr. 24 lb. of cheese; 12 cwt. 19 lb. of soap; 17 cwt. 3 qr. 17 lb. 5 oz. of tobacco: 2 cwt. 2 qr. 17 lb. 14 oz. of candles; 23 cwt. 2 qr. 27 lb. 6 oz. of butter; 37 cwt. 3 qr. 27 lb. 6 oz. 14 dr. of salt; 3 cwt. 2 qr. 16 lb. of honey; for how much weight had he to pay carriage? Ans. 173 cwt. 26 lb. 11 oz. 14 dr.

80. An Apothecary mixed together seven different ingredients, the weight of the first was 2 lb. 10 oz. 6 dr. 2 sc. 10 gr; the second, 3 lb. 10 oz. 1 sc.; the third, 11 oz. 2 sc. 16 gr; the fourth, 5 lb. 6 oz. 4 dr. 2 sc. 17 gr.; the fifth, 1 lb. 2 sc. 18 gr.; the sixth, 9 oz. 6 dr. 12 gr.; and the seventh, 5 lb. 10 oz. 5 dr. 1 sc. 15 gr.; how much was in the whole mixture? Ans. 20 lb. 11 oz. 1 dr. 2 sc. 8 gr.

81. An Apothecary mixed together seven different liquids, of the first, 2 gals. 7 O, 15 F℥, 7 Fʒ, 59 ♏; the second, 3 gals. 12 F℥, 30 ♏; the third, 4 O, 13 F℥, 6 Fʒ, 10 ♏; the fourth, 4 gal. 3 O, 12 F℥, 6 Fʒ; the fifth, 7 O, 9 F℥, 5 Fʒ; and the sixth, 5 gal. 3 O, 4 F℥, 3 Fʒ, 40 ♏; how much was of the mixture?

Ans. 16 gal. 6 O. 4 ℥, 4 ʒ, 43 ♏.

82. A woolstapler bought from A, 6 la. 8 sk. 4 t. 5 lb.; from B, 12 la. 7 sk. 5 t. 1 s. 6 lb.; from C, 10 sk. 1 w. 3 t. 1 s. 1 cl. 5 lb.; from D, 7 la. 8 sk. 1 w. 6 t. 1 cl.; from E, 1 w. 6 t. 1 s. 1 cl. 3 lb.; from F, 16 la. 6 t. 5 lb.; from G, 4 la. 11 sk. 3 t. 1 s. 5 lb.; from H, 10 sk. 1 w. 6 t. 1 cl. 4 lb.; and from K, 7 la. 9 sk. 1 w. 4 t. 1 s. 1 cl. 6 lb.; how much had he in all?

Ans. 57 la. 9 sk. 2 t. 1 st. 4 lb.

83. A pedestrian walked ten days as follows, viz. 1st day 43 m. 6 f. 30 p. 4 yd; 2nd day 52 m. 7 f. 25 p. 4 yd. 2 f. 10 in; 3d day 48 m. 20 p. 2 yd. 6 in.; 4th day 53 m. 6 f. 19 p.; 5th day 57 m. 12 p. 3 yd. 2 f. 10 in.; 6th day 38 m. 4 f. 5 yd. 6 in.; 7th day 45 m. 4 p. 2 yd.; 8th day 52 m. 5 f. 2 yd. 1 foot; 9th day 43 m. 28. p. 5 yd. 9 in.; and 10th day 47 m. 3 f. 37 p. 2 yd. 1 ft.; required the distance travelled.

Ans. 482 m. 3 f. 20 p. 4 yd. 1 f. 11 in.

84. A merchant bought 476 yd. 3 qr. 2 n. of broad cloth; 357 yd. 1 qr. 3 n. 2 in. narrow; 547 yd. 2 n. 1 in. flannel; 4268 yd. 3 qr. 2 n. 2 in. linen; 437 yd. 3 qr. 2 n. shalloon; 1259 yd. 2 qr. 3 n. 2 in. printed cotton; 356 yd. 1 qr. 3 n. black silk; 245 yd. 3 n. 2 in. flowered silk; 137 yd. 2 qr. 3 n. French cambric; 355 yd. 3 q. 1 n. 2 in.

Scotch cambric, and 1674 yd. 1 qr. 2 n. of other goods; how many yards had he in all?

Ans. 10144 yd. 2 qr. 2 n. 2 in.

85. A farmer has in pasture 157 ac. 3 r. 29 p. 25 yds. 8 f.; under oats, 16 ac. 2 r. 10 p.; under wheat, 75 ac. 1 r. 35 p. 17 yd. 6 f.; under barley, 23 ac. 3 r. 14 p. 7 y.; under rye, 13 ac. 37 p. 28 yd.; under beans, 12 ac. 3 r. 15 yd.; under turnips, 37 ac. 3 r. 29 p. 30 yd.; under potatoes, 27 ac. 1 r. 3 p.; under fallow, 41 ac. 2 r. 19 p. 18 yd.; under hay 66 ac. 3 r. 25 yd. and occupied by houses and fences, 12 ac. 3 r. 14 p. 19 yds. 7 f.; for how much has he to pay rent? Ans. 486 ac. 36 p. 4¼ yd. 3 f.

86. A wine merchant brought into his cellars 6 t. 1 p. 1 hhd. 54 gal. 3 qt. 1 pt. port; 2 t. 3 hhd. 36 gal. 3 qt. Burgundy; 1 t. 2 hhd. 56 gal. 1 pt. sherry; 1 p. 59 gal. 3 qt. claret; 5 t. 3 hhd. 25 gal. 2 qt. Madeira; 2 hhd. 54 gal. 2 qt. 1 pt. champagne; 1 p. 2 hhd. 15 gal. 3 qt. hock; 3 t. 1 p. 1 hhd. 52 gal. 3 qt. brandy; 3 t. 1 hhd. 56 gal. 1 qt. rum; 3 hhd. 39 gal. 3 qt. 1 pt. hollands; 4 t. 1 p. 1 hhd. 45 gals. 2 qt. 1 pt. 3 gills whisky; how much had he in all? Ans. 33 tons 56 gal. 3 qt. 1 pt. 3 gills.

87. A brewer sold ale as follows, to A 12 bt. 2 b. 3 fir. 7 gal. 2 qts.; to B 15 bt. 1 hhd. 1 b. 1 kil. 1 fir.; to C 23 bt. 3 b. 7 g.; to E 19 bt. 3 k. 15 g. 3 qt. 1 pt.; to F 37 bt. 1 b. 3 f. 7 gal.; to G 3 hdd. 2 k. 12 gal.; to H 27 bt. 1 hhd. 1 b. 5 gal. 3 qt.; to K 37 bt. 2 b. 8 gal.; to L 2 bt. 1 hdd. 1 b. 1 f. 6 gal. 3 qt. 1 pt.; how much did he sell?

Ans. 180 bt. 1 hhd. 1 b. 1 f. 6 gal.

88. A cornchandler bought of wheat 246 lasts 1 w. 4 qrs. 7 b. 3 p.; barley 157 lasts, 8 qr. 5 b.; oats 379 la. 1 w. 4 qr.; rye 98 la. 1 w. 4 qr. 5 b.; beans 234 la. 3 qr. 5 b.; peas 19 la. 5 qr. 6 b.; for how much had he to pay carriage?

Ans. 1137 lasts 1 w. 1 qr. 4 b. 3 p.

89. A coal merchant, in the month of January sold 735 chal. 10 sk. 2 b.; in February 697 chal. 8 sk. 1 b. 3 p.; in March 587 chal. 11 sk. 2 b.; in April 578 chal. 5 sk. 1 b.; in May 498 chal. 9 sk. 1 b. 3 p.; in June 452 chal. 3 sk. 2 bu.; in July 385 chal. 10 sk. 3 p.; in August 287 chal. 7 sk. 2 b.; in September 247 chal. 3 sk. 3 p.; in October 574 chal. 10 sk. 2 b.; in November 825 chal. 11 sk. 1 b. 2 p.; and in December 879 chal. 1 sk. 2 b. 3 p.; how many did he sell in the year? Ans. 6751 ch. 11 sk. 1 b. 2 p.

90. An old man being asked his age, replied, I was 5 years, 3 m. 2 w. 15 h. when I first went to school, where I remained 7 y. 9 m. 3 w. 6 d. 18 h.; after this I was idle for

3 m. 1 w. 5 d. 6 h. and was then bound an apprentice for 7 years; after my apprenticeship was finished, I remained in the same shop as a journeyman for 1 y. 6 m. 3 w. 4 d. 18 h.; I then served three other masters, with the first I was 5 y. 4 m. 1 w. 5 d., the second 3 y. 11 m. 2 w. 2 d. 12 h., the third 7 y. 6 m. 3 w. 6 d. 2 h.; I then travelled for 11 m. 2 w. 3 d.; I then commenced business for myself, which I carried on for 37 y. 10 m. 3 w. 5 d. 6 h.; I have now been retired from business 15 y. 7 m. 3 d. 23 h.; how old was he? Ans. 93 years 4 m. 2 w. 1 d. 4 h.

91. A gentleman was 25 y. 6 m. 3 d. old when his oldest son was born, and after 3 y. 1 m. 3 d. his second son was born; 1 y. 11 m. 3 w. 5 d. after this his oldest daughter was born; 2 y. 7 m. 6 d. after this, his third son was born; 3 y. 2 w. 5 d. after this, his second daughter was born, and when she was 4 y. 3 m. 3 d. old, his young son was born, who was 21 y. 9 m. 2 w. 3 d. old when his father died, how old was the father? Ans. 62 y. 3 m. 3 w.

COMPOUND SUBTRACTION.

Compound Subtraction, is the method of finding the difference between numbers, consisting of several denominations.

Rule.—Write the less number, below the greater, and the same places and denominations directly under each other, and draw a line below them.

Begin at the right hand, and take each denomination in the under number, from that above it, and write the remainder below.

But if any under denomination be greater than that above it, add to the upper number the value of a unit taken from next higher denomination, and subtract the under from the sum; proceed thus through all the denominations.

Proof; same as in simple addition.

Example.—From £37 14s. 6¾d. take £26 9s. 4½d.; and from £57 16s. 8½d. take £38 18s. 9¾d.

1st.	£	s.	d.	qrs.
From	37	14	6	3
Take	26	9	4	2
Difference	11	5	2	1
Proof	37	14	6	3

2d.	£	s.	d.	qrs.
From	57	16	8	2
Take	38	18	9	3
Difference	18	17	10	3
Proof	57	16	8	2

The 1st example is simple, for each denomination of the under number can be taken from that above it. Beginning with farthings I say 2 from 3 and 1 remains; 4 from 6 and 2 remain; 9 from 14 and 5 remain; 6 from 7 and 1 remains; 2 from 3 and 1 remains; proof 1 and 2 are 3, 2 and 4 are 6, 5 and 9 are 14, 1 and 6 are 7, 1 and 2 are 3. 2d Example, 3 from 2 I cannot, I therefore add to it a unit taken from pence which is equal to 4 farthings, 4 and 2 are 6—3=3, and as 1 was taken from 8 it is now 7, 9 from 7, I cannot, I therefore take a unit or 1 from shillings which is equal to 12 pence, and 12+7=19—9=10, and 16 was diminished by 1, I therefore say 18 from 15, I cannot, I take a unit from pounds, which is equal to twenty shillings, and 20+15=35—18=17, then 8 from 6, I cannot, but 8 from 16 and 8 remain, 3 from 4 and 1 remains; which leaves the difference as in the example.

EXERCISES.

	1st. £	s.	d.	qrs.	2d. £	s.	d.	qrs.	3d. £	s.	d.	qrs.
From	769	15	4	3	589	17	9	2	489	19	11	3
Take	357	12	3	2	468	15	7	1	357	16	10	2

4th. £	s.	d.	qrs.	5th. £	s.	d.	qrs.	6th. £	s.	d.	qrs.
7854	15	8	1	5432	10	9	2	6753	12	4	3
6797	17	9	2	4624	9	9	3	2749	19	6	2

7th. £	s.	d.	qrs.	8th. £	s.	d.	qrs.	9th. £	s.	d.	qrs.
4987	1	0	3	5952	13	7	1	6742	0	9	2
2978	12	6	3	4267	12	8	2	3569	7	11	3

10th. £	s.	d.	qrs.	11th. £	s.	d.	qrs.	12th. £	s.	d.	qrs.
2369	12	7	0	1763	13	7	2	8765	17	9	1
989	19	4	2	1457	19	9	0	8675	18	1	3

13th.				14th.				15th.			
£	s.	d.	qrs.	£	s.	d.	qrs.	£	s.	d.	qrs.
1798	14	0	2	1810	0	7	0	6245	10	10	1
979	14	8	3	1547	12	8	2	5896	12	0	3

16th.				17th.				18th.			
£	s.	d.	qrs.	£	s.	d.	qrs.	£	s.	d.	qrs.
76953	16	5	0	7689	12	0	1	18974	1	8	2
37876	17	10	2	4626	14	9	2	12987	19	10	1

19th.				20th.				21st.			
£	s.	d.	qrs.	£	s.	d.	qrs.	£	s.	d.	qrs.
76	18	10	2	687	19	9	1	7894	14	7	3
49	18	10	3	597	19	10	2	6598	13	10	2

22d.				23d.				24th.			
£	s.	d.	qrs.	£	s.	d.	qrs.	£	s.	d.	qrs.
7876	1	5	2	8764	12	2	1	1000	0	0	0
2887	19	11	3	4989	10	7	3	999	19	11	3

25. A was indebted to B £259 10s. but has paid him £99 18s. 6d.; how much remains due? Ans. £159 11s. 6d.
26. C borrowed £875 13s. 4½d. from D, and afterwards paid him £599 19s. 6¾d.; how much is still due? Ans. £275 13s. 9¾d.
27. A merchant has capital to the value of £3897 9s. 2¼d., and has debts due by him, to the amount of £989 19s. 10¾d.; what is he worth? Ans. £2907 9s. 3½d.
28. A bankrupt owes his creditors £13787 17s. 9¼d., and his estate is worth only £6989 18s. 11½d.; what do they lose by him? Ans. £6797 18s. 10¼d.

Exercises in Troy Weight.

29th.				30th.				31st.			
lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
35	10	15	18	98	6	7	16	675	5	14	20
29	11	13	12	56	8	14	15	286	9	18	23

32d.				33d.				34th.			
lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
65	10	12	10	543	1	19	5	632	0	4	3
498	9	18	20	246	9	18	12	281	7	16	9

35th.

lb.	oz.	dwt.	gr.
54	0	3	4
6	5	7	18

36th.

lb.	oz.	dwt.	gr.
67	0	0	0
1	8	16	17

37th.

lb.	oz.	dwt.	gr.
100	0	0	0
34	4	3	18

Exercises in Avoirdupois Weight.

38th.

T.	c.	q.	lb.	oz.	dr.
35	16	2	16	10	12
27	18	3	12	13	9

39th.

T.	c.	q.	lb.	oz.	dr.
67	3	0	20	8	12
47	16	3	27	14	15

40th.

Cwt.	q.	lb.	oz.	d.
17	2	15	6	7
9	3	14	12	8

41st.

T.	cwt.	q.	lb.	oz.	dr.
15	0	3	27	6	0
5	16	3	27	10	9

42d.

T.	cwt.	q.	lb.	oz.	dr.
47	0	0	0	0	4
17	16	1	17	5	9

43d.

Cwt.	q.	lb.	oz.	d.
175	2	18	6	14
137	3	19	7	15

44th.

T.	c.	q.	lb.	oz.	dr.
168	0	2	0	7	0
89	0	3	20	0	12

45th.

T.	cwt.	q.	lb.	oz.	dr.
47	2	1	3	0	5
38	15	3	14	10	6

46th.

C.	q.	lb.	oz.	dr.
15	0	0	0	7
1	0	26	7	10

Exercises in Apothecaries Weight.

47th.

lb.	℥	ʒ	℈	gr.
69	6	2	1	14
34	5	4	2	7

48th.

lb.	℥	ʒ	℈	gr.
74	10	6	2	16
47	10	7	2	18

49th.

lb.	℥	ʒ	℈	gr.
25	0	2	0	4
15	7	6	2	16

50th.

lb.	℥	ʒ	℈	gr.
79	10	6	0	17
37	11	7	1	16

51st.

lb.	℥	ʒ	℈	gr.
81	2	0	0	0
47	6	3	1	19

52d.

lb.	℥	ʒ	℈	gr.
175	7	1	2	0
99	9	7	2	17

Exercises in Apothecaries Fluid Measure.

53d.

G.	O.	F℥	Fʒ	♏
67	2	10	4	47
29	7	12	6	58

54th.

G.	O.	F℥	Fʒ	♏
17	0	5	6	10
6	4	12	7	24

55th.

Gal.	O.	F℥	Fʒ	♏
374	2	4	5	17
256	6	12	6	59

Promiscuous Exercises in Compound Subtraction.

109. A merchant is indebted to B £274 6s. 4½d.; to C £789 5s. 10d.; to D £509 10s. 4¾d.; and to E £679 19s. 4¼d. he has in houses £876 6s.; in goods £516 7s. 2½d.; in recoverable debts £307 16s. 1¼d.; in bank stock £1203 3s. 7¾d.; in bills £2347 12s. 10¼d.; and in landed property £16047 12s. 4¾d.; what is he worth?

Ans. £19045 16s. 3¼d.

110. A jeweller received from a nobleman the following articles of old plate, six dozen table spoons, weighing 14 lb. 3 oz. 1 dwt. 16 gr.; four dozen and a half tea-spoons, weighing 4lb. 2 oz. 4 dwt. 15 gr.; five dozen and four dessert spoons, weighing 6 lb. 7 oz. 5 dwt. 8 gr.; and sundry other articles, weighing 20 lb. 10 oz. 15 dwt. 17 gr.; which he wishes made into spoons to the weight of 19 lb. 10 oz. 16 gr.; forks 12 lb. 10 oz. 10 dwt. 3 gr.; candlesticks 10 lb. 4 oz. 6 dwt.; cups 4 lb. 7 dwt. 12 gr.; tea pot, &c. 6 lb. 7 oz. 16 gr.; and sundry other articles 12 lb. 7 oz. 6 dwt. 4 gr.; how much additional silver will the jeweller require to complete the order, supposing none of the old were lost in the working?

Ans. 20 lb. 4 oz. 3 dwt. 19 gr.

111. A grocer bought 2 ton 3 cwt. 1 qr. 16 lb. of sugar in the month of January, and sold 1 t. 16 cwt. 3 qr. 20 lb.; in February he bought 3 t. 14 lb. 6 oz.; and sold 2 t. 19 cwt. 27 lb.; in March he bought 4 ton 16 cwt. 16 lb.; and sold 4 t. 17 cwt. 3 qr. 17 lb. 10 oz. how much remained unsold?

Ans. 5 cwt. 2 qr. 9 lb. 12 oz.

112. An apothecary had a composition of several ingredients, weighing 2 lb. 3 ℥. 7 ʒ. 16 gr.; of which he sold to A 2 ℥. 1 ℈. 6 gr.; to B 6 ℥. 4 ʒ. 14 gr.; to C 8 ℥. 2 ℈.; and to D 1 ℥ 5 ʒ 17 gr.; how much is unsold?

Ans. 9 ℥. 4 ʒ. 1 ℈. 19 gr.

113. A manufacturer bought four lots of wool, the first weighed 4 la. 6 sk. 2 t. 6 lb.; the second 7 la. 5 t. 1 s. 3 lb.; the third 5 la. 10 sk. 1 st. 9 lb.; and the fourth 6 la. 11 sk. 2 t. 1 s. 6 lb. Of this he made into broad cloth 7 la. 10 sk. 4 t. 7 lb; into flannel 8 la. 6 td. 1 st.; into blankets 5 la. 7 sk. 6 lb.; and into carpets 2 la. 4 sk. 2 td. 9 lb.; how much has he yet on hand?

Ans. 5 sk. 4 t. 1 st. 2 lb.

114. A gentleman on express, posted 116 m. 6 f. 7 p. 3 yd. the first day: 127 m. 20 p. 3 y. the second; and 99 m. 7 f. 36 p. 4 yd. the third; and returning by the same road,

he travelled 67 m. 4 f. 16 p. 2 yd. the first day; 87 m. 6 f. 17 p. the second; and 79 m. 7 f. 29 p. 4 yd. the third; how far was he from the place he started?

Ans. 108 m. 4 f. 1 p. 4 yds.

115. A draper bought three pieces of broad cloth, the 1st. measured 64 yds. 2 qr. 3 n.; the 2d. 57 yds. 2 n.; the 3d. 47 yds. 3 qr. 1 n.; and the 4th. 39 yds. 3 qr. 2 n. He sold to A 4 yds. 3 qr.; to B 8 yds. 1 qr. 2 n.; to C 23 yds. 3 n.; to D 36 yds. 3 qr. 2 n.; to E 51 yds. 1 qr. 3 n.; and used for his own family 19 yds. 3 qr. 3 n.; how much remains? Ans. 64 yds. 3 qr. 3 n.

116. A gentleman has an estate consisting of 7397 a. 3 r. 2 p. within the ring fence; and divided into ten farms, as follows; first 374 a. 3 r. 1 p.; second 279 a. 2 r. 3 p.; third 610 a. 1 p.; fourth 587 a. 2 r.; fifth 826 a. 1 r. 2 p.; sixth 479 a. 2 r.; seventh 853 a. 3 r. 2 p.; eight 974 a. 3 r.; ninth 284 a. 2 p.; and tenth 323 a. 2 r. 2 p.; policy and gardens 389 a. 0 r. 13 p.; the remainder is a lake; how much does it cover? Ans. 1415 a. 16 p.

117. A spirit dealer bought a hogshead of whisky, of which he has sold the following quantities, to A 3 g. 1 qt. 1 p. 2 gill.; to B 6 g. 3 qt. 2 gill.; to C 7 g. 1 pt. 3 gill.; to D 10 g. 3 qt. 1 p.; to E; 14 g. 1 qt.; to F 12 g. 3 qt. 1 p. 2 gill.; to G 1 g. 1 qt. 3 gill.; but the crane being left open by mistake, the remainder ran out; how much was lost by this accident? Ans. 6 g. 1 p.

118. A brewer, in the month of October, brewed 43 bt. 1 h. 1 f. strong ale, and 3 bt. 1 h. 2 k. of table beer; in November 51 bt. 8 gal. strong ale, and 4 bt. 1 b. 1 fir. table beer; in December 57 bt. 2 b. 2 fir. strong ale, and 5 bt. 1 h. 1 b. table beer; in January 65 b. 1 h. 1 b. 1 k. 7 g. strong ale, and 6 bt. 2 b. 3 fir. table beer; in February 57 bt. 1 h. 2 k. strong ale, and 5 bt. 2 b. 2 fir. table beer. In March 49 bt. 1 h. 1 b. 1 f. 8 g. strong ale, and 18 bt. 1 b. 1 k. 1 f. table beer. In October he sold 37 bt. 3 b. strong ale, and 2 bt. 1 b. table beer; in November 39 bt. 1 h. 2 k. strong ale, and 3 bt. 1 b. 3 fir. table beer; in January 44 bt. 1 h. 1 b. 1 fir. strong ale, and 4 bt. 1 b. 1 k. table beer; in February 29 bt. 1 b. 1 fir. strong ale, and 3 bt. 1 b. 1 k. 8 gal. table beer; in March 42 bt. 1 b. strong ale, and 7 bt. 1 k. 1 fir. table beer; how much of each sort, had he on hand at the end of March?

Ans. Ale 131 bt. 1 h. 1 b. 8 g. Beer 24 bt. 1 k. 1 g.

119. A corn chandler bought the following lots of wheat; from A 4 la. 1 qr. 7 bu., and paid 3 la. 1 wy. 4 qr.; from B 7 la. 1 qr. 6 bu., and paid 2 la. 1 wy. 2 qr. 4 bu.; from C 16 la.

4 qr. 7 bu. and paid 12 la. 2 qr. 4 bu.; from D 27 la. 1 wy. 3 qr. 5 bu., and paid 17 la. 3 qr. 2 bu.; from E 35 la. 5 bu. and paid 22 la. 1 wy. 3 qr. 7 bu.; for how much is he debtor? Ans. 31 la. 1 wy. 1 qr. 5 bu.

120. I wish to know how many coals are in that fold, into which, in the month of January, were put 647 ch. 6 sa. 2 bu., and sold out 597 ch. 10 sa. 1 bu.; in February put in 749 ch. 1 sa. 1 bu., and sold out 634 ch. 11 sa. 2 bu.; in March put in 842 ch. 9 sa., and sold out 785 ch. 2 bu.; and in April put in 698 ch. 6 sa., and sold out 768 ch. 9 sa. 2 bu. Ans. 151 ch. 2 sa. 2 bu.

121. A man was born on the 13th of March 1749, and died on the 31st of July 1825; how old was he? Ans. 76 yrs. 4 m. 18 d.

122. A gentleman was born at ½ past 10 A. M. on the 21st of November 1753, and his son ¾ past 7 P. M. on the 30th of April 1788; how much older is the father than his son? Ans. 34 yrs. 5 m. 1 w. 2 d. 9 h. 15 m.

123. A gentleman was born 4th March 1765, married 17th Novr. 1789, and died 23d August 1825; and his lady was born 9th January 1772; how old was the man on his marriage day, and what was the lady's age on the day of her husband's death?
Ans. Man's age 24 y. 8 m. 13 d. Lady's age 53 y. 7 m. 14 d.

124. If 3 Sign 15 deg. 3 m. 45 sec. be taken from one great circle; how much remains? Ans. 8 Sig. 14 dg. 56 m. 15 s.

COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION is the method of finding what any number, consisting of several denominations, will amount to, when repeated a given number of times.

RULE.—Write the multiplier under the lowest denomination of the multiplicand, which multiply by it; find by division, how many units of next higher denomination the product contains, write the remainder under it, and add the quotient to the product of next higher denomination; proceed thus with every denomination.

EXAMPLE.—Multiply £2 16s. 8½d. by 7.

£	s.	d.
£2	16	8½
		7
£19	16	11¼

I here place the multiplier 7, under the farthings of the multiplicand, and say 7 times 2 are 14 qrs.; the 4's in 14 are three times and 2 qrs. over, I write the ½ under the qrs.; then seven times 8 are 56 and 3, the last quotient added make 59,

the 12's in 59 are 4 times and 11, write 11 under pence; then 7 times 16 are 112 and 4 the last quotient added are 116, the 20's in 116 are 5 times and 16 over, write 16 under shillings; then 7 times 2 are 14 and 5 the last quotient added are £19, which write in full under £s, and the work is finished.

Exercises.

1. Multiply £17 4s. 6d. by 2. Ans. £34 9s.
2. Multiply £27 14s. 8½d. by 3. Ans. £83 4s. 1½d.
3. Multiply £37 17s. 9¾d. by 4. Ans. £151 11s. 3d.
4. Multiply £59 12s. 2¼d. by 5. Ans. £298 0s. 11¼d.
5. Multiply £84 4s. 6½d. by 6. Ans. £505 7s. 3d.
6. Multiply £97 19s. 10¾d. by 7. Ans. £685 19s. 3¼d.
7. Multiply £56 6s. 11½d. by 8. Ans. £450 15s. 8d.
8. Multiply £37 14s. 9½d. by 9. Ans. £339 13 1½d.
9. Multiply 15 lb. 6 oz. 10 dwt. 12 gr. by 5. Ans. 77lb. 8 oz. 12 dwt. 12 gr.
10. Multiply 6 t. 12 cwt. 3 qr. 14 lb. 6 oz. 9 dr. by 7. Ans. 46 t. 10 cwt. 16 lb. 13 oz. 15 dr.
11. Multiply 16 lb. 8 ℥, 6 ʒ, 2 ℈, 15 gr. by 9. Ans. 150 lb. 7 ℥, 6 ʒ, 15 gr.
12. Multiply 12 g. 6 O. 12 F ℥, 7 F ʒ, 20 ♏ by 6. Ans. 77 gall. 13 F ℥, 4 F ʒ.
13. Multiply 42 la. 9 sk. 1 w. 6 t. 1 s. 1 cl. 5 lb. by 8. Ans. 342 la. 8 sk. 3 t. 1 cl. 5 lb.
14. Multiply 54 m. 6 f. 20 p. 3 yd. 2 f. 9 in. by 7. Ans. 383 m. 5 f. 24 p. 5 yd. 1 f. 3 in.
15. Mult. 27 yd. 3 qr. 2 n. 1 in. by 5. Ans. 139 yd. 2 qr. ½ in.
16. Multiply 97 a. 3 r. 20 p. 15 yd. 6 f. by 9. Ans. 880 a. 3 ro. 24 p. 20 yd.
17. Multiply 16 t. 2 h. 35 g. 3 q. 1 p. by 12. Ans. 199 t. 2 h. 52 g. 2 qts.
18. Multiply 18 bt. 2 br. 1 k. 1 f. 7 ga. by 6. Ans. 113 bt. 2 br. 1 k. 6 ga.
19. Multiply 63 la. 1 w. 4 q. 6 b. 3 p. 1 ga. by 11. Ans. 703 la. 1 wy. 3 qr. 3 b. 2 p. 1 ga.
20. Mult. 98 ch. 10 sa. 2 bu. 3 p. by 8. Ans. 791 ch. 3 sa. 1 b.
21. Multiply 57 year 8 m. 2 w. 6 d. 12 h. 10 m. by 12. Ans. 692 y. 8 m. 3 w. 1 d. 2h.
22. Mult. 15 deg. 45 m. 48 sec. by 8. Ans. 126 deg. 6 m. 24 s.

Case 2d. When the multiplier is composite, and the product of numbers under 13.

Rule.—Multiply successively by these numbers, in any order.

EXAMPLE.—Multiply £6 9s. 8¼d. by 42.

£6	9	8¼
		6
38	18	1½
		7
£272	6	10½

Here the multiplier is a composite number, of which the factors are 6 and 7, I therefore multiply by 6, which gives £38 18s. 1½d., and this product by 7, gives £272 6s. 10½d.; which is the same as if £6 9s. 8¼d. had been multiplied at once by 42.

EXERCISES.

1. Multiply £16 15s. 5½d. by 14. Ans. £234 16s. 5d.
2. Multiply £167 1s. 9¼d. by 16. Ans. £2673 8s. 4d.
3. Multiply £12 19s. 11¾d. by 18. Ans. £233 19s. 7½d.
4. Multiply £18 18s. 8½d. by 21. Ans. £397 12s. 10½d.
5. Multiply £86 12s. 2¼d. by 27. Ans. £2338 9s. 0¾d.
6. Multiply £65 0s. 6¾d. by 32. Ans. £2080 18s.
7. Multiply 16 lb. 7 oz. 12 dwt. 8 gr. by 35.
   Ans. 582 lb. 2 oz. 11 dwt. 16 gr.
8. Multiply 29 ton, 15 cwt. 3 qr. 16 lb. by 42.
   Ans. 1251 t. 7 cwt. 2 qr.
9. Multiply 36 lb. 10 ℥, 6 ʒ, 1 ℈, 15 gr. by 48.
   Ans. 1771 lb. 3 ℥, 4 ʒ.
10. Multiply 17 la. 8 sk. 10 t. 1 s. 8 lb. by 56.
    Ans. 993 la. 2 sk. 6 td.
11. Multiply 45 m. 6 f. 20 p. 4 y. 2 f. by 72.
    Ans. 3298 m. 5 f. 21 p. 1 f. 6 in.
12. Multiply 126 yd. 3 qr. 2 n. 2 in. by 96.
    Ans. 12185 yd. 1 qr. 1 n. ¾ in.
13. Multiply 18 a. 2 r. 5 p. 20 yds. 6 f. by 120.
    Ans. 2224 a. 1 r. 1 p. 29¾ yd.
14. Multiply 27 t. 3 h. 36 gal. 2 q. 1 p. by 144.
    Ans. 4016 t. 3 h. 45 gal.
15. Multiply 29 bt. 2 b. 3 f. 8 gal. by 180. Ans. 5576 bt. 1 b.
16. Multiply 46 la. 1 w. 2 qr. 3 b. 2 p. by 200.
    Ans. 9348 la. 1 w. 2 qr. 4 b.
17. Multiply 69 ch. 8 s. 1 b. 2 p. by 240. Ans. 16730 ch.
18. Multiply 18 y. 6 m. 2 w. 3 d. 12 h. by 512.
    Ans. 9498 y. 8 m.
19. Multiply 79° 15′ 25″ 45‴ by 432. Ans. 34239° 5′ 24″

CASE 3d. When the Multiplier is not a composite number.

RULE.—Take the composite number nearest to it, whether greater or less, and multiply by its parts as before; then add to this product, as many times the multiplicand, as the number taken, is less than the given multiplier, but subtract when the number taken, is greater than the multiplier.

EXAMPLES.—Multiply £3 12s. 6½d. by 17.

1st method.				
3	12	6	½	
			4	
14	10	2		
		4		
58	0	8		
3	12	6	½	add.
£61	13	2	½	Ans.
2d method.				
3	12	6	½	
			6	
21	15	3		
		3		
65	5	9		
3	12	6	½	subtr.
£61	13	2	½	Ans.

Because 17 is not a composite number, or is not the product of any two numbers; by the first method, I take 16 as the composite number nearest to, and less than 17, and multiply as in last case, by its factors, and then add once the multiplicand or £3 12s. 6½d. to the product, because 16 wanted 1 of the given multiplier 17.

By the second method, I take 18 as the composite number nearest to the multiplier, and greater, and 3 and 6 as its factors, and proceed with them as before; but from the product, I subtract once the multiplicand, because 18 is one greater than the given multiplier 17. The result is the same by both methods.

EXERCISES.

1. Multiply £16 17s. 4½d. by 19. Ans. £320 10s. 1½d.
2. Multiply £25 12s. 8¾d. by 23. Ans. £589 12s. 9¼d.
3. Multiply £46 18s. 9¼d. by 29. Ans. £1361 4s. 4¼d.
4. Multiply £12 12s. 10½d. by 37. Ans. £467 16s. 4½d.
5. Multiply 16 lb. 6 oz. 15 dwt. 4 gr. by 43.
Ans. 712 lb. 2 oz. 12 dwt. 4 gr.
6. Multiply 15 cwt. 3 qr. 20 lb. 6 oz. by 58.
Ans. 924 cwt. 5 lb. 12 oz.
7. Multiply 21 lb. 6 ℥, 3 ʒ, 2 ℈, 12 gr. by 67.
Ans. 1443 lb. 2 ℥, 3 ʒ, 4 gr.
8. Multiply 45 yd. 3 qr. 2 n. by 76. Ans. 3486 yds. 2 qrs.
9. Multiply 79 a. 3 r. 14 p. by 87. Ans. 6945 a. 3 r. 18. p.
10. Multiply 16 m. 6 f. 14 p. 3 yd. 2 f. by 94.
Ans. 1578 m. 6 f. 18. p. 3 yd. 2 f.
11. Multiply 24 la. 6 qr. 7 b. 3 p. 1 gal. by 106.
Ans. 2618 la. 2 b. 3. p.
12. Multiply 18 y. 3 m. 2 w. 5 d. 4 h. by 115.
Ans. 2105 years, 3 m. 2 w. 6 d. 4 h.

CASE 4th.—When the Multiplier is large.

RULE.—Multiply the given price or quantity successively by 10, so often, wanting one, as there are places in the multiplier; then, multiply the last product by the left hand figure of the multiplier, the preceding product by next highest figure,

and so on with all the preceding products, and figures in order, writing the products under each other; their sum is the answer.

Example.—What is the value of 3243, at £6 4s. 3½d. each.

£6	4	3½	× 3
		10	
62	2	11	× 4
		10	
621	9	2	× 2
		10	
6214	11	8	× 3
18643	15	0	
1242	18	4	
248	11	8	
18	12	10½	
£20153	17	10½	Ans.

Because the multiplier consists of 4 places, I multiply the price of one, or £6 4s. 3½d. three times successively by 10; I then multiply the last product £6214 11s. 8d. by 3, the left hand figure of the multiplier, and the preceding product £621 9s. 2d. by 2, the next highest figure of the multiplier; the next preceding product £62 2s. 11d. by 4, the next highest figure of the multiplier; then the price of one, by the 3 in units place of the multiplier; and write the products under each other, as in the example; their sum is £20153 17s. 10½d.; the value of 3243 at £6 4s. 3½d.

Exercises.

1. What cost 349 yds. muslin, at 1s. 6½d. a-yard? Ans. £26 18s. 0½d.
2. What cost 598 yds. cloth, at £1 4s. 6½d. a-yd.? Ans. £733 15s. 11d.
3. What cost 3165 lb. tea, at 7/6¼ a-lb.? Ans. £1190 3s. 5¼d.
4. What cost 1284 cwt. sugar, at £3 19s. 4d. a cwt.? Ans. £5093 4s.
5. What cost 6053 bullocks, at £12 15s. 6d. a-head? Ans. £77327 1s. 6d.
6. What cost 569 score sheep, at £1 7s. 8½d. a-head? Ans. 15766 0s. 10d.
7. What cost 59 ton butter, at 1/4½d. a-lb.? Ans. £9086.
8. Multiply 16 lb. 7 oz. 15 dwt. 6 gr. by 16834. Ans. 420350 lb. 2 oz. 17 dwt. 12 gr.
9. Multiply 12 lb. 10 oz. 12 dr. by 43235. Ans. 4891 cwt. 2 qr. 20 lb. 8 oz. 4 dr.
10. In 65004 bags, each 2 cwt. 1 qr. 6 lb. 3 oz.; how many cwt.? Ans. 149850 cwt. 20 lb. 4 oz.
11. In 240106 parcels of land, each 1 a. 2 ro. 10 p.; how many acres? Ans. 375165 a. 2 r. 20 p.
12. Multiply 3 m. 4 f. 20 p. 4 yd., by 60280402. Ans. 214707694 m. 3 f. 8 p. 4 yd.

Case 5th.—The following method is often considered the simplest, although it sometimes lengthens the operation.

Rule.—Reduce the multiplicand to the lowest denomination mentioned in it, which multiply by the given multiplier, and then reduce it to the denomination required.

Example.—Multiply £4 6s. 6½d. by 323.

```
      £4  6s. 6½d.
       20
      ---
       86
       12
     ----
     1038
        4
     ----
     4154
      323
    -----
    12462
    8308
  12462
 --------
4)1341742
 --------
12)335435 ½
 -------
2,0)2795,2 11
Ans. £1397 12s. 11½d.
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In this example I reduce the £4 6s. 6½d. into farthings, which gives 4154 qrs. and then multiply by the given multiplier 323, gives 1341742 qrs., and these divided by 4, 12 and 20 give £1397 12s. 11½d. as in the example.

Exercises.

1. Multiply £17 14s. 6d., by 75. Ans. £1329 7s. 6d.
2. Multiply £4 15s. 7½d., by 249. Ans. £1190 10s. 7½d.
3. What cost 237 lb. coffee, at 2/4¼ a-lb.? Ans. £27 17s. 11¼d.
4. If the inhabitants of Edinburgh, and Leith require £1640 12s. 6d. to supply them in bread, for one day; how much do they spend in a year for bread alone? Ans. £598828 2s. 6d.
5. If the inhabitants of Edinburgh, and Leith consume 4 ton 15 cwt. 2 qr. 18 lb. of butter each day, at 1/1½ per lb.; how much money do they spend annually on this article? Ans. £219971 16s. 3d.
6. In 96 pieces of silk, each containing 37 yd. 2 qr. 3 n.; how many yards, and what does it cost at 6/4¼ a-yd.? Ans. 3618 yds., cost, £1149 9s. 4½d.
7. What is the rent of an estate containing 138 farms, and each farm on an average consisting of 173 ac. 2 r.,—and let one with another at 13/10½d. per acre? Ans. £16610 9s. 1½d.

Promiscuous Exercises.

1. What is the product of £7 15s. 6½d. multiplied by eleven? Ans. £85 10s. 11½d.
2. What is the value of 27 yds. linen, at 4/7¼ a-yd. Ans. £6 4s. 3¾d.

3. What cost 69 lb. tea, at 8/9½. a-lb.? Ans. £30 6s. 7½d.

4. What is the value of 16968 yds. carpeting, at 4/6½d. a-yd.? Ans. £3853 3s.

5. How many pounds of silver, in nine ingots, each 12 lb. 7 oz. 2 dwt. 3 gr.? Ans. 113 lb. 3 oz. 19 dwt. 3 gr.

6. In 3474 hhds. sugar, each 15 cwt. 2 qrs. 17 lb.; how many ton? Ans. 2718 ton 14 cwt. 1 qr. 6 lb.

7. An apothecary received 24 chests of medicines, each containing 18 parcels, and each parcel 3 lb. 7 ℥, 4 ʒ, 2 ℈, 10 gr.; how many pounds had he in all? Ans 1569 lb. 9 ℥.

8. A woolstapler bought 72 lots of wool, each weighing 15 la. 10 sk. 1 w. 5 td. 1 st. 8 lb.; how much had he in all? Ans. 1145 lasts 8 sk. 1 st. 2 lb.

9. If the distance between Edinburgh and Dumfries be 71 m. 7 fur. 30 p. 5 yds.; how many miles does the mail coach, between these two places run in a year, supposing it to go, and return every day? Ans. 52539 m. 2 f. 3 p. 3½ yds.

10. A manufacturer makes ready for the market, 64 pieces of broadcloth every month; how many yds. does he manufacture in a year if each piece contain 57 yds. 3 qrs. 2 n.? Ans. 44448.

11. How many acres are there in an estate which is divided into 32 farms, and each farm into 14 inclosures, each inclosure containing 10 ac. 3 ro. 8 p.? Ans. 4838 ac. 1 r. 24 p.

12. If a wine merchant sell 2 hhd. 24 gal. 3 qts. of wine every lawful day; how much does he sell in 15 years? Ans. 11234 hhd. 29 gal. 1 qt.

13. If a brewer make 18 hhd. 2 kil. 1 fir. 6 gal. ale in a week; how much does he make in seven years at that rate? Ans. 6914 hhd. 2 kil. 3 gal.

14. If there are a million of inhabitants in London, and if each individual consume 1 qr. 3 bu. 2 p. of wheat annually; how many quarters do they consume in seven years? Answer 10062500 quarters.

15. If a coal pit discharge 25 ch. 8 sk. 2 bu. coal every lawful day; how many do seven such pits discharge in four years? Ans. 225429 ch. 6 sk. 2 bus.

16. How many seconds from the birth of Christ, to Christmas 1829, reckoning solar years? Ans. 57717621312.

17. How many minutes are there, in the circumference of 56 great circles? Ans. 1209600.

18. How much sterling is there in that coffer, in which are sixteen drawers, and in each drawer seven divisions, and in each division three guineas, three sovereigns, three crowns, three shillings, three sixpences, three pence, and three farthings? Ans. £799 15s.

19. If the inhabitants of Edinburgh and Leith are 150000, and spend at an average 3/4½d. per day; how much do they spend in ten years? Ans. £92390625.

20. If a bankrupt owes his creditors, £37320, and pays them 8/4½d. a £; how much does he pay them in all? Ans. £15627 18s.

21. If one bushel of wheat weigh 2 qrs. 7 lb.; how many tons will 64 qrs. weigh? Ans. 14 ton 8 cwt.

22. An operative manufacturer earns 3/7½d. every work day; how much does he earn in 5 years, supposing he lose 40 days by bad health? Ans. £276 8s. 1½d.

23. A certain sum of money yields £1 3s. 6d. interest a-day; how much is that a-year? Ans. £428 17s. 6d.

COMPOUND DIVISION.

COMPOUND DIVISION is the method of dividing a compound number, into any proposed number of equal parts; or of finding how often any proposed part is contained in a compound number.

RULE.—Write the divisor on the left of the dividend, and divide the highest denomination by it; multiply the remainder by the number of units of next inferior denomination which make one of that denomination the remainder is in, taking in the corresponding denomination of the dividend; divide this number, and proceed thus with every remainder, and denomination to the lowest.—Proof by compound Multiplication.

EXAMPLE.—Divide £46 18s. 1½d. by 15.

1st.

	£	s.	d.	£	s.	d.	
15)	46	18	1½(	3	2	6½	Ans.
	45					5	
	1			15	12	8½	
	20					3	
	38	Proof		£46	18	1½	
	30						
	8						
	12						
	97						
	90						
	7						
	4						
	30						
	30						

I here divide £46, the highest denomination in the dividend, by 15, which gives £3 in the quotient, and £1 over, I multiply the remainder £1 by 20 the shillings in a £ and take in the 18 shillings in the dividend, the sum is 38 shillings, which divided by 15 gives 2 shs. in the quotient, and 8 shs. of a remainder, I multiply the 8 by 12 the pence in a shilling and take in the 1 pen-

2d. Method.

		£	s.	d.
15	5	46	18	1½
	3	9	7	7½
Ans.		£ 3	2	6½

ny in the dividend to the product, which gives 97 pence, this divided by 15, gives 6d. in the quotient, and 7d. over; I multiply the 7d. by 4 the farthings in a penny and take in the 2 qrs. in the dividend, the sum is 30 which divided by 15 gives 2 qrs. in the quotient. The complete quotient is £3 2s. 6½d., the 15th part of £46 18s. 1½d. The second method is performed by contraction 4th in Simple Division, reducing remainders mentally.

Exercises.

1. Divide £37 5s. 3¾d., by 3. Ans. £12 8s. 5¼d.
2. Divide £27 19s. 11d., by 4. Ans. £6 19s. 11¾d.
3. Divide £76 8s. 6½d., by 5. Ans. £15 5s. 8½d.
4. Divide £50 5s. 10½d. by 6. Ans. £8 7s. 7¾d.
5. Divide £629 2s. 7¾d., by 7. Ans. £89 17s. 6¼d.
6. Divide £453 19s., by 8. Ans. £56 14s. 10½d.
7. Divide £716 16s. 9¾d., by 9. Ans. £79 12s. 11¾d.
8. Divide £1541 11s. 10½d., by 10. Ans. £154 3s. 2¼d.
9. Divide £2233 12s. 1¾d., by 11. Ans. £203 1s. 1¼d.
10. Divide £3997 19s. 9d., by 12. Ans. £333 3s. 3¾d.
11. Divide 46 lb. 1 oz. 20 gr., by 13. Ans. 3 lb. 6 oz. 10 dwt. 20 gr.
12. Divide 245 lb. 7 oz. 7 dwt., by 14. Ans. 17 lb. 6 oz. 10 dwt. 12 gr.
13. Divide 99 cwt. 1 qr. 21 lb. 8 oz., by 15. Ans. 6 cwt. 2 qr. 14 lb. 8 oz.
14. Divide 252 ton. 15 cwt. 2 qr. 23 lb., by 16. Ans. 15 ton. 15 cwt. 3 qr. 25 lb. 15 oz.
15. Divide 264 lb. 3 ℥, 5 ʒ, 4 gr., by 17. Ans. 15 lb. 6 ℥, 4 ʒ, 1 ℈, 12 gr.
16. Divide 682 lb. 1 ℥, 2 ʒ, 1 ℈, 10 gr. by 18. Ans. 37 lb. 10 ℥, 5 ʒ, 2 ℈, 15 gr.
17. Divide 264 la. 1 sk. 1 w. 4 td. 1 st., by 21. Ans. 12 la. 6 sk. 1 w. 5 td. 1 st. 8 lb.
18. Divide 414 miles, 36 p. 3 yds. 2 feet, by 25. Ans. 16 miles, 4 f. 20 p. 3 yds. 2 feet.
19. Divide 1298 miles, 6 f. 3 p. 3½ yds., by 36. Ans. 36 miles, 24 p. 3 yds.
20. Divide 6737 yds. 2 qr. 1 n., by 43. Ans. 156 yds. 2 qr. 3 n.
21. Divide 18000 yds. 2 qr., by 56. Ans. 321 yds. 1 qr. 3 n.

22. Divide 3916 acres, 3 r. 8 p., by 68. Ans. 57 a. 2 r. 16 p.
23. Divide 2518 acres, 7 p. 18¼ yds., by 72.
Ans. 34 acres, 3 r. 35 p. 20 yds.
24. Divide 4836 tons, 20 gall. 1 pt., by 85.
Ans. 56 tons, 3 hhd. 36 gall. 2 qts. 1 pt.
25. Divide 8465 hhd. 7 gal. 3 qt. 1 p. 3 gill., by 97.
Ans. 87 hhd. 16 gal. 3 qt. 1 pt. 3 gill.
26. Divide 429 bt. 1 h. 2 k. 1 f. 2 gal. 1 qt., by 108.
Ans. 3 bt. 1 h. 2 k. 1 f. 6 gal. 3 qt. 1 p. 2 gill.
27. Divide 8126 hhds. 1 f. 3 gal., by 224.
Ans. 36 hhds. 1 f. 6 gall.
28. Divide 14800 la. 1 w. 4 qr. 3 b. 1 gal. 1 qt., by 315.
Ans. 46 la. 1 w. 4 qrs. 6 b. 3 p. 1 gal. 3 qt.
29. Divide 45997 la. 2 qr., by 576. Ans. 79 la. 8 qr. 4 b. 2 p.
30. Divide 116400 ch. coal, by 864. Ans. 134 ch. 8 sk. 2 b.
31. Divide 150072 ch. 9 sk. coal, by 729. Ans. 205 ch. 10 sk. 1 b.
32. Divide 24596 years 1 w. 5 d., by 324.
Ans. 75 y. 10 m. 3 w. 6 d.
33. Divide 1186 years 10 m. 3 w. 6 d. 12 h., by 81.
Ans. 14 y. 7. m. 3 w. 2 d. 12 h.

Case 2.

When the divisor consists of several denominations.

RULE.—Reduce both it, and the dividend, to the same denomination, then divide by simple division.

EXAMPLE.—If one yard of cloth cost 14/6½; how many yards may be bought for £69 16s.?

```
 s. d.
 14 6½,  £69 16s.
 12        20
 ---      ----
 174      1396
   2        12
 ---     -----
 349     16752
             2
        ------
   349)33504(96 yds. Ans.
        3141
        ----
         2094
         2094
         ====
```

I here reduce the divisor 14/6½ into halfpence, and also the dividend into halfpence, and then divide; which gives 96 for quotient, the number of yards that £96 16s. will purchase.

EXERCISES.

1. How many yds. of muslin may be bought for £2 5s. at 1s. 8d. per yd.? Ans. 27.

2. If 1 lb. of tea cost 7s. 6½d.; how many lbs. may be bought for £23 15s. 1½d.? Ans. 63 lb.
3. Into how many inclosures of 14 ac. 2 r. each may a farm be divided, which contains 406 acres? Ans. 28.
4. If 1 ac. 3 r. graze one bullock; how many bullocks may be grazed on 315 acres? Ans. 180 bullocks.
5. If a suit of clothes can be made from 3 yd. 2 qr. of cloth; how many can be made from 115½ yards? Ans. 33 suit.
6. If a pedestrian walk 37 m. 3 f. 10 p. in one day; in how many days would he walk 1346 m. 5 fur.? Ans. 36 days.

Case 3.

When a fraction is annexed to the divisor.

Rule.—Multiply both the divisor, and dividend, by the under number of the fraction, adding its upper number to the product of the divisor, then divide.

Example.—Divide £150 3s. 4d. by $9\frac{5}{7}$.

```
  9 5/7   £150  3  4
  7              7   £   s.  d.
  --
  68    )1051  3  4(15   9   2
Divisor   68
         ---
         371
         340
         ---
          31
          20
         ---
         623
         612
         ---
          11
          12
         ---
         136
         136
         ===
```

I here multiply 9, the whole number of the divisor, by 7 the under number of the fraction, and to the product add 5, the upper number of the fraction, which gives 68 for a divisor. I then multiply £150 3s. 4d. the dividend also by 7, which gives £1051 3s. 4d. for a new dividend, which divide by the 68, as in the example.

Exercises.

1. Divide £42 18s. 4d. by 6¼. Ans. £6 17s. 4d.
2. If 6⅔ yds. silk, cost £2 17s. 1d.; what is it a yard? Ans. 8/6¾.
3. What is a gallon of brandy worth, when 9⅞ gal. cost £16 0s. 11¼d.? Ans. £1 12s. 6d.
4. What will 1 oz. of silver cost, when $15\frac{6}{25}$ oz. cost £7 10s. 9¾d.? Ans. 9/10¾.
5. If $18\frac{7}{16}$ galls. wine be bought for £19 19s. 5¾d.; what is the price of a gallon? Ans. £1 1s. 8d.

6. Bought 35$\frac{15}{32}$ qrs. wheat for £114 13s. 7¾d.; what did one quarter cost? Ans. £3 4s. 8d.

Case 4.

When it is required to divide any sum of money, or any quantity of goods into an equal number of unequal shares,

Rule.—Add together one share of each value, or quantity; reduce the sum to the lowest denomination mentioned in it, for a divisor, then reduce the whole sum or quantity to be divided into the same denomination, for a dividend, and divide; the quotient is the equal number of shares.

Example.—A quantity of silver worth £2835 is to be coined into an equal number of crowns, half-crowns, shillings, and sixpences; how many should be of each?

	s.	d.
1 crown	= 5	0
1 half cr.	= 2	6
1 shilling	= 1	0
1 sixpence	= 0	6
Sum	= 9	0

£
2835
20
9)56700
Ans = 6300 of each kind.

I here add together a crown, half a crown, a shilling, and a sixpence, the sum of which is 9 shillings, the divisor; I then reduce the £2835 into shillings and divide them by 9, which gives 6300 the number of each that should be coined out of the silver.

Exercises.

1. A manufacturer requires £202 16s. a-week to pay the wages of his operatives, the men have 16/6, the women 8/2, the children 3/6 per week; there are an equal number of men, women, and children employed; how many are there of each? Ans. 144.
2. A gentleman has a farm containing 265 acres, 2 r. 20 p. which he wishes to stock with an equal number of horse, bullocks, and sheep; now, if a horse requires 1 acre 20 poles, a bullock 3 roods, and a sheep 1 rood; how many of each sort of cattle will he require to stock his farm? Ans. 125.
3. A grocer has 3 hhds. of sugar weighing together 26 cwt. 12 lb. which he orders to be made up in an equal number of parcels of 14 lb., 7 lb., 1 lb., ½ lb., ¼ lb., 1 oz., ½ oz.; how many should he have of each? Ans. 128.
4. An apothecary gave 7 lb. of calomel to make into pills of 1, 2, 3, 4, and 5 grains, and to have an equal number of each; how many dozen had he of each kind? Ans. 224 doz.
5. A wine merchant is desirous of drawing off a pipe of wine

into an equal number of quart, pint, and half pint bottles; how many dozen will he require of each? Ans. 24 dozen.

6. A candlemaker has a ton of tallow, which he wishes to make into an equal number of long and short fours, long and short sixes, and short eights per lb.; how many dozen should he have of each sort, supposing 9 lb. of the tallow to be lost in the process? Ans. 194 dozen.

7. A plumber has 90 tons of lead, which he wishes to make into pipes of 9 lb., 12 lb., 18 lb., and 24 lb. per yard, and to have an equal number of yards of each size; how many yards should he have of each? Ans. 3200 yds.

CONTRACTION 1.

When the Divisor is under 13.

RULE.—Divide as directed in contraction 1st. Simple Division, reducing the remainders mentally.

EXAMPLE.—Divide £56 13s. 5¼d., by 9.

9)56 13 5¼ I here divide £56 by 9, which gives £6,
Ans. £6 5 11¼ and £2 over, which reduced into shillings and the 13 added gives 53s., and divided by 9 gives 5s., and 8s. over, which reduced into pence, and the 5d. added gives 101d. and divided by 9 gives 11d., and 2d. over, which reduced into farthings and the ¼ qr. added gives 9 qrs., and divided by 9 gives ¼ as in the example.

EXERCISES.

1. If 3 lb. of sugar cost 2/10½ what is it a-lb.? Ans 11½d.
2. What is the price of 1 lb. tea, when 4 lb. cost £1 11s.? Ans. 7s. 9d.
3. Bought 5 yds. muslin for 9/8¾; what was it a yd.? Ans. 1/11¾.
4. Sold 6000 quills for £10 1s. 6d.; what were they a thousand? Ans. £1 13s. 7d.
5. Bought 7 reams paper for £8 4s. 6d.; what was it a ream? Ans. £1 3s. 6d.
6. What is the price of a yd. of broad cloth of which 9 yds. cost £11 9s. 10½d.? Ans. £1 5s. 6½d.
7. If 11 lb. of beef cost 6s. 10½d.; what is it a lb? Ans. 7½d.
8. What will 1 oz. of gold cost, when 12 oz. are worth £59 15s. 6d.? Ans. £4 19s. 7½d.
9. If 9 pieces of cloth measure 302 yds. 2 qrs. 2 nails, what is the length of one piece? Ans. 33 yds. 2 qr. 2 n.
10. What is the weight of 1 hhd. of sugar, when 11 hhds. weigh 83 cwt. 1 qr. 4 lb.? Ans. 7 cwt. 2 qr. 8 lb.

11. What is the weight of an ingot of silver, when 7 ingots weigh 87 lb. 7 oz. 2 dwt. 4 gr.? Ans. 12 lb. 6 oz. 3 dwt. 4 gr.
12. Divide 187 a. 2 r. into 12 equal shares. Ans. 15 a. 2 r. 20 p.

Contraction 2.

When the divisor is the product of two or more numbers under 13.

Rule.—Divide successively by these numbers, reducing the remainders mentally.

Example.—Divide 396 cwt. 1 qr. 7 lb. by 112.

		cwt.	qr.	lb.	
112 =	4	396	1	7	
	4	99	0	8	12
	7	24	3	2	3
		3	2	4	5

I first reduce the divisor 112 into its factors 4, 4, 7. I then divide 396 cwt. 1 qr. 7 lb by 4, which gives 99 cwt. 0 qrs. 8 lb. 12 oz.; and this quotient divided by 4 gives 24 cwt. 3 qr. 2 lb. 3 oz.; and this quotient again divided by 7, the other factor, gives 3 cwt. 2 qr. 4 lb. 5 oz. as in the example.

EXERCISES.

1. Divide £91 12s. 8d. equally among 16 men. Ans. £5 14s. 6½d.
2. Divide £147 19s. 6d. into 18 equal shares. Ans. £8 4s. 5d.
3. If two dozen silver forks weigh 6 lb. 3 oz.; what is the weight of one of them? Ans. 3 oz. 2 dwt. 12 gr.
4. Paid £44 2s. for 36 weeks board; what was that a-week? Ans. £1 4s. 6d.
5. The dinner bill for a company of 48 gentlemen amounted to £61 10s.; how much was that a-piece? Ans. £1 5s. 7½d.
6. If a prize of £3243 16s. be divided equally among 56 sailors; what is the share of each? Ans. £57 18s. 6d.
7. If 948 yards of cloth be sufficient to clothe 96 soldiers; how much is that a man? Ans. 9 yds. 3 qrs. 2 n.
8. If 19 ton 13 cwt. 16 lb. of sugar be divided into 256 equal parcels; how much is there in each? Ans. 1 cwt. 2 qr. 4 lb.

Contraction 3.

When the divisor is 10, 100, 1000, &c.

Rule.—Cut off from the right of the highest denomination of the dividend as many places as there are ciphers in the divisor, the figures on the left are the quotient, and those cut off are the remainder which reduce, and cut off as before.

EXAMPLE.—Divide £497 14s. 2d. into 100 equal shares.

£4,97 14s. 2d.	I here cut off the 97 from the £s and the 4 on the left is the quotient in £s, I then reduce the £97 cut off into shillings taking in the 14sh. from which I cut off as before, and so on with every remainder. The several numbers cut off, placed in order, complete the Ans., thus £4 19s. 6½d.
20	
19,54	
12	
6,50	
4	
2,00	

EXERCISES.

1. What is the quotient of £193 6s. 3d. divided by 10? Ans. £19 6s. 7½d.
2. If a farm of 135 acres 3 r. 20 p. be divided into 10 equal inclosures; how much is in each? Ans. 13 a. 2 r. 14 p.
3. Divide 2196 galls. 3 qts. 1 pint of wine into 100 equal shares. Ans. 21 galls. 3 qt. 1 pt. 3 gills.
4. What is the quotient of 17687 yds. 2 qrs. divided by 1000? Ans. 17 yds. 2 qrs. 3 n.
5. If 10000 soldiers require 86875 yards for clothing, how much is that for each? Ans. 8 yds. 2 qrs. 3 n.
6. If the inhabitants of London be one million, and spend daily £627083 6s. 8d., how much is that a piece? Ans. 12/6½.

PROMISCUOUS EXERCISES ON THE SEVERAL RULES.

1. If 7 yards of silk cost £2 19s. 6d.; what is it per yard? Ans. 8/6.
2. If 56 lb. of butter, give £2 2s.; what is the price of a lb.? Ans. 9d.
3. Bought 75 lb. of tea, for £29 4s. 4½d.; what was it per lb? Ans. 7s. 9½d.
4. If 1 acre 1 rd. of pasturage feed a bullock; how many will a farm of 640 acres graze? Ans. 512.
5. If a yard of ribbon cost 1/2; how many yards can I buy for £8 9s. 2d.? Ans. 145 yds.
6. If 7¾ lb. of tea cost £3 19s. 11¼d. what is it per lb.? Ans. 10/6½.
7. Paid £107 8s. 9d. for 95½ yds. broad cloth; what did it cost per yard? Ans. £1 2s. 6d.
8. If there are 4187½ stones of butcher meat each 16 lb. consumed in Edinburgh and Leith in a day, at the rate of 8 oz. for every individual; how many inhabitants are there in these two places? Ans. 134000.
9. Bought 2 cwt. of Stilton cheese for £16 6s. 8d.; what did it cost per lb.? Ans. 1/5½.

10. If the value of tea annually imported into Great Britain, and Ireland, be six million nine hundred and seventy-six thousand and forty-six pounds sterling, at an average of five shillings and six pence per lb.; how many hundred-weights are imported? Ans. 226495.

11. Of all the fixed stars Syrius or the dogstar is supposed to be the nearest to this earth, the computed distance of which is two billion and two hundred thousand million of miles. Now supposing the world to be created four thousand and four years before the Christian era, and that a messenger had been dispatched from Syrius the first moment of time, moving without intermission, at the rate of sixteen miles an hour, at what particular period of the Christian era would this messenger arrive at the earth, if the year consisted of three hundred and sixty-five and a fourth days? Ans. 10 P. M., 20th June 15681600.

12. A cheese monger wishes to lay out £1056 on cheese in the following manner, viz. on Parmisan at 2/ a lb., Stilton at 1/3, double Gloucester 8½d., Wiltshire 7¼d., Dunlop 6½d., and Dutch Goudie 4¾d., and to have an equal quantity of each kind; what weight should he have of each? Ans. 34 cwt. 1 qr. 4 lb.

13. If 100 horse be bought for £2483 6s. 8d.; what is their average price? Ans. £24 16s. 8d.

14. Bought 15 cwt. 2 qrs. 14 lb. of palm soap, for £105 14s. 7d.; what did it cost per pound? Ans. 1/2½.

15. If a stone of beef be bought for 7/8; how many stones can I buy for £128 16s.? Ans. 336 stones.

16. If the clothing of 1000 sailors cost £2525; how much is that for one? Ans. £2 10s. 6d.

17. If 1 hhd. rum cost £58 5s. 6d.; what is it per gallon? Ans. 18s. 6d.

18. A gentleman laid out on silver plate £1145, at an average of £5 14s. 6d. per lb.; how many lbs. had he, and what did it cost per oz.? Ans. 200 lb., and cost 9s. 6½d. per oz.

19. A silversmith has 79 lb. 12 dwt. of silver, which he wishes to make into an equal number of tea spoons of 12 dwt. 8 gr., table spoons of 2 oz. 15 dwt. 6 gr., dessert spoons of 1 oz. 6 gr., punch ladles of 1 oz 2 dwt. 12 gr., and forks of 2 oz. 8 dwt. 16 gr.; how many should he have of each sort? Ans. 124.

20. Sold 3 cwt. 2 qrs. 16 lb. of soap, for £11, 18s., what was it per lb.? Ans. 7d.

21. Bought a ton of salt for £9 6s. 8d., what was it a lb.? Ans. 1d.

22. The flash of a minute gun on board a ship in distress is

seen 33 seconds before the report is heard on shore; how far is the ship from land, supposing sound to travel 1142 feet per second? Ans. 7 miles 1 f. 4. p.

23. Paid 6¾d. for a quart of ale, how many barrels will £243 purchase at that rate? Ans. 60.

24. A corn chandler laid out on wheat £7616, at the rate of 8/6 per bush. how many lasts did he purchase? Ans. 224.

25. A bankrupt owes his creditors £302400, and pays them 2/6 a £; how much does he pay in all? Ans. £37800.

26. How much coffee may be bought for £44 19s., at 2/7 a lb? Ans. 3 cwt. 12 lb.

27. If a manufacturer earns 3/7½ a day; in how many days can he earn £276 8s. 1½d. at the same rate? Ans. 1525.

28. How far does a mail coach run in an hour, at the rate of 78894 miles in a year, of 365 days 6 h.? Ans. 9 miles.

29. A merchant realized a fortune of £56940 at the rate of £4 6s. 8d. per day; how long was he in business, supposing the year to be 365 days? Ans. 36 years.

30. Sold 75⅜ acres of land for £3040 2s. 6d.; how much was that per acre? Ans. £40 6s. 8d.

PRACTICE.

Practice is a compendious method of calculating the value of any quantity of goods.

Case 1.

To find the value of any number of articles when the price of one is given.

Rule.—Multiply the given price by the number of articles, the product is the value of the proposed quantity.

Example.—Calculate the value of 36 yds. of broad cloth at £0 15s. 6½d.

	15	6½
		6
4	13	3
		6
£27	19	6 Ans.

This operation is performed by Rule 2d. in Compound Multiplication.

Exercises.

Calculate the value of

1. 3 lb. sugar at /11½. Ans. 2/10½.
2. 5 lb. tobacco at 3/9¾. Ans. 19/0¾.
3. 7 lb. cheese at /9½. Ans. 5/6½.
4. 9 lb. tea at 7/4. Ans. £3 6s.
5. 10 lb. beef at /7½. Ans. 6/3.
6. 14 yds. at 1/2½. Ans. 16/11.
7. 16 ells at 2/1½. Ans. £1 14s.
8. 27 lb. at 1/1. Ans. £1 9s. 3d.
9. 34 gall. at 9/6. Ans. £16 3s.
10. 56 doz. at £2 1s. 6d. Ans. £116 4s.
11. 15 cwt. at £2 19s. 8d. Ans. £44 15s.
12. 17 gall at 16/8. Ans. £14 3s. 4d.
13. 112 lb. at 4/3. Ans. £23 16s.
14. 63 gall. at £1 1s. 6d. Ans. £67 14s. 6d.
15. 30 cwt. at £1 12s. 8d. Ans. £49.
16. 336 lb. at 2/6. Ans. £42.
17. 19 barrels at £2 12s. 7d. Ans. £49 19s. 1d.
18. 105 yds. at 9/8½. Ans. £50 19s. 4½d.
19. 3875 doz. at 1/1½. Ans. £217 19s. 4½d.
20. 26424 gall. at £1 8s. 6d. Ans. £37654 4s.

Case 2.

To find the value of one, when the value of any given number of articles is known.

Rule.—Divide the whole price by the given number of articles according to the rules for Compound Division, the quotient is the price of one.

Example.—Calculate the value of 1, when 54 cost £74 5s.

$$54 = \left\{\begin{array}{l} 6)74 \quad 5 \\ 9)\underline{12 \quad 7 \quad 6} \end{array}\right.$$
$$\text{Ans. } £1 \quad 7 \quad 6$$

This operation is performed by the second contraction in Compound Division.

Calculate the value of one, when

1. 6 yards cost £0 15s. 3d. Ans. £0 2s. 6½d.
2. 21 acres cost £31 11s. 9d. Ans. £1 10s. 1d.
3. 56 lb. cost £3 7s. 8d. Ans. 1/2½
4. 29 qrs. wheat cost £81 13s. 8d. Ans. £2 16s. 4d.
5. 10 lb. tea cost £6 5s. 5d. Ans. 12/6½
6. 15½ lb. cost £1 3s. 10¾d. Ans. 1/6½
7. 8 cwt. butter cost £44 16s. Ans. £5 12s.
8. 100 thousand quills cost £152 10s. Ans. £1 10s. 6d.
9. 48 yds. cambric cost £5 14s. Ans. 2/4½
10. 36 sheep cost £45 12s. Ans. £1 5s. 4d.

11. 85 bullocks cost £1099 2s. 6d.	Ans. £12 16s. 6d.
12. 224 lb. soap cost £7.	Ans. /7½.
13. 67 stone flour cost £12 19s. 7½d.	Ans. 3/10½.
14. 120 miles posting cost £10 15s.	Ans. 1/9½
15. 72 firkins butter cost £163 16s.	Ans. £2 5s. 6d.

Before commencing case third, the pupil should commit to memory the following table of aliquot Parts.

Of sixpence.	Of a shilling.	Of a £.		Of a cwt.	Of an oz. troy.
3d. = ½	6d. = ½	10s. = ½	1s. = $\frac{1}{20}$	2 qrs. = ½	10 dwts. = ½
2 = ⅓	4 = ⅓	6/8 = ⅓	/8 = $\frac{1}{30}$	1 = ¼	6 16 gr = ⅓
1½ = ¼	3 = ¼	5 = ¼	/6 = $\frac{1}{40}$	16 lb. = $\frac{1}{7}$	5 = ¼
1 = ⅙	2 = ⅙	4 = ⅕	/4 = $\frac{1}{60}$	14 = ⅛	4 = ⅕
¾ = ⅛	1½ = ⅛	3/4 = ⅙	/3 = $\frac{1}{80}$	Of a qr.	3 8 = ⅙
½ = $\frac{1}{12}$	1 = $\frac{1}{12}$	2/6 = ⅛	/2 = $\frac{1}{120}$	7 = ¼	2 12 = ⅛
		2 = $\frac{1}{10}$		4 = $\frac{1}{7}$	2 = $\frac{1}{10}$
				3½ = ⅛	

Case 3.

When the price is an aliquot part of a penny, shilling or pound.

Rule.—Divide the number of articles by that part, the quotient is the answer in the denomination of which the price was an aliquot part. If it is in pence or shillings reduce it to £s.

Example.—6781 yards at ¼, 4d., and 4 shillings.

1st.	2d.	3d.
qr.	d.	sh.
1 | ¼ | 6871	4 | ⅓ | 6871	4 | ⅕ | 6871
12 | 1717 ¾	2,0 | 229,0 4	£ | 1374 4s.
2,0 | 14,3 1	£ | 114 10s. 4d.	Ans.
£7 3s. 1¾d.	Ans.	
Ans.		

1st. Ex.; because ¼ is the ¼th part of a penny, I therefore take the ¼th part of the number of articles, and the quotient represents the pence which they are worth, and dividing by 12 and 20 reduces them to £s, or £7 3s. 1¾d.

2d. Ex.—Because 4d. is the ⅓d. part of a shilling, I take the ⅓d part of the number of articles and the quotient is their price in shillings, which divided by 20 gives £s. or £114 10s. 4d.

3d. Ex.—Because 4 shillings is the ⅕th part of a £, I take

the $\frac{1}{5}$th part of the number of articles, and the quotient is the answer in £s. or £1374 4s.

EXERCISES.

CALCULATE THE VALUE OF

	Answer. £	s.	d.		Answer. £	s.	d.
1. 679 yds. at ¼.	0	14	1¾	10. 7586 .. at 1/8.	632	3	4.
2. 6987 lb. at ¼.	7	5	6¾	11. 4635 .. at 2/.	463	10	0.
3. 8324 oz. at ½.	17	6	10	12. 6489 .. at 2/6.	811	2	6.
4. 2123 .. at 1½d.	13	5	4½	13. 1897 .. at 3/4.	316	3	4.
5. 4732 .. at 2d.	39	8	8	14. 9879 .. at 4/.	1975	16	0.
6. 6210 .. at 3d.	77	12	6	15. 2987 gal. at 5/.	746	15	0.
7. 2123 yds. at 4d.	35	7	8	16. 8796 .. at 6/8.	2932	0	0.
8. 1067 .. at 6d.	26	13	6	17. 4345 .. at 10/.	2172	10	0.
9. 5654 .. at 1/	282	14	0				

CASE 4.

When the price is less than a shilling, and not the aliquot part of a shilling.

RULE.—Take an aliquot part of a shilling which is greater than half the given price, divide the remainder of the price either into aliquot parts of a shilling or of parts already taken, and the sum of the quotients thus arising is the answer in shillings; which reduce to £s.

EXAMPLE.—Calculate the value of 7634 yards at 7½d.

1st. Method.

6d.	½	7634 at	7½d.		
1½d.	¼	3817			
		954	3		
	2,0	477,1	3		
		238	11	3	Ans.

2d. Method.

4d.	⅓	7634	at	7½d.
3d.	¼	2544	8	
½	$\frac{1}{6}$	1908	6	
		318	1	
	2,0	477,1	3	
	£	238	11	3 Ans.

1st Method.—The aliquot parts here, are 6d. the ½ of a shilling and 1½d. the ¼ of 6d. The 2d part or ¼ is taken out of the quotient of the given number of articles divided by the first part, being an aliquot part of the first, and not of a shilling, and the sum of the quotients is the answer in shillings, which reduce to £s.

2d Method.—The parts here are 4d. the ⅓ and 3d. the ¼ of a shilling and ½ the $\frac{1}{6}$ of 3d. The first two being the aliquot parts of a shilling are both taken out of the given number of articles, and the last being a part of 3d. is taken out of the

quotient arising from the division by that part, and the sum of these quotients is the answer in shillings, which reduce to £s.

Exercises.

Calculate the value of

1.	3689 yds.	at /1¾.	per yard.	Ans. £26 17s. 11¾d.
2.	6421 yds.	at /2¼.	——	Ans. £60 3s. 11¼d.
3.	4897 yds.	at /2½.	——	Ans. £51 0s. 2½d.
4.	7632 yds.	at /2¾.	——	Ans. £87 9s. 0d.
5.	4763 yds.	at /3¼.	——	Ans. £64 9s. 11¾d.
6.	3249 yds.	at /3½.	——	Ans. £47 7s. 7½d.
7.	5657 yds.	at /3¾.	——	Ans. £88 7s. 9¾d.
8.	9872 yds.	at /4¼.	——	Ans. £147 16s. 4d.
9.	6340 yds.	at /4½.	——	Ans. £118 17s. 6d.
10.	1072 yds.	at /4¾.	——	Ans. £21 4s. 4d.
11.	2904 yds.	at /5.	——	Ans. £60 10s. 0d.
12.	3986 yds.	at /5¼.	——	Ans. £87 3s. 10½d.
13.	9909 yds.	at /5½.	——	Ans. £227 1s. 7½d.
14.	7698 yds.	at /5¾.	——	Ans. £184 8s. 7½d.
15.	2790 yds.	at /6¼.	——	Ans. £72 13s. 1½d.
16.	8786 yds.	at /6½.	——	Ans. £237 19s. 1d.
17.	4400 yds.	at /6¾.	——	Ans. £123 15s. 0d.
18.	7890 yds.	at /7.	——	Ans. £230 2s. 6d.
19.	2324 yds.	at /7¼.	——	Ans. £70 4s. 1d.
20.	1980 yds.	at /7½.	——	Ans. £61 17s. 6d.
21.	6893 yds.	at /7¾.	——	Ans. £222 11s. 8¾d.
22.	9291 yds.	at /8.	——	Ans. £309 14s. 0d.
23.	2000 yds.	at /8¼.	——	Ans. £68 15s 0d.
24.	3333 yds.	at /8½.	——	Ans. £118 0s. 10½d.
25.	4444 yds.	at /8¾.	——	Ans. £162 5s. 0d.
26.	7676 yds.	at /9d.	——	Ans. £287 17s. 0d.
27.	8584 yds.	at /9¼.	——	Ans. £330 16s. 10d.
28.	9897 yds.	at /9½.	——	Ans. £391 15s. 1½d.
29.	1345 yds.	at /9¾.	——	Ans. £54 12s. 9¾d.
30.	8480 yds.	at /10.	——	Ans. £353 6s. 8d.
31.	1667 lb.	at /10¼.	per lb.	Ans. £71 3s. 10¾d.
32.	6114 lb.	at /10½.	——	Ans. £267 9s. 9d.
33.	7227 lb.	at /10¾.	——	Ans. £323 14s. 2¼d.
34.	6262 lb.	at /11.	——	Ans. £287 0s. 2d.
35.	7672 lb.	at /11¼.	——	Ans. £359 12s. 6d.
36.	4235 lb.	at /11½.	——	Ans. £202 18s. 6½d.
37.	2339 lb.	at /11¾.	——	Ans. £114 10s. 3¼d.

CASE 5.

When the given price is above one shilling, and less than two shillings, and not the aliquot part of a £.

RULE.—Take the part, or parts with so much of the price as exceeds a shilling. by last case, and let the given number of articles stand for shillings, the sum of these and the several quotients is the answer in shillings, which reduce to £s.

EXAMPLE —Calculate the value of 3762 lb. at 1/3¾.

3d.	¼	3762 at 1s. 3¾d.
		940 6
¾	¼	235 1½
	2,0	493,7 7½
		Ans. £246 17 7½

The parts in this example are 3d. the ¼ of a shilling, and ¾ the ¼ of 3d., which gives the value by last case at 3¾d., and adding these quotients to the given number of articles considered as shillings, gives the value of the whole quantity at 1s. 3¾d. according to the rule.

EXERCISES.

CALCULATE THE VALUE OF

1. 6397 yds. at 1/1.	per yard.	Ans. £346 10s. 1d.
2. 7321 yds. at 1/1½.	——	Ans. £411 16s. 1½d.
3. 4216 yds. at 1/2.	——	Ans. £245 18s. 8d.
4. 3769 yds. at 1/2¾.	——	Ans. £231 12s. 8¾d.
5. 5728 yds. at 1/3.	——	Ans. £358.
6. 7472 yds. at 1/3½.	——	Ans. £482 11s. 4d.
7. 9872 yds. at 1/4½.	——	Ans. £678 14s.
8. 7374 yds. at 1/5¾.	——	Ans. £545 7s. 4½d.
9. 6231 yds. at 1/6.	——	Ans. £467 6s. 6d.
10. 1398 yds. at 1/6¾.	——	Ans. £109 4s. 4½d.
11. 2893 yds. at 1/7½.	——	Ans. £235 1s. 1½d.
12. 6429 yds. at 1/7¾.	——	Ans. £529 1s. 0¾d.
13. 6287 yds. at 1/8½.	——	Ans. £537 0s. 3½d.
14. 7768 yds. at 1/8¾.	——	Ans. £671 12s. 2d.
15. 6976 yds. at 1/9.	——	Ans. £610 8s.
16. 4246 yds. at 1/9½.	——	Ans. £380 7s. 5d.
17. 4679 yds. at 1/10.	——	Ans. £428 18s. 2d.
18. 9896 yds. at 1/10½.	——	Ans. £927 15s. 0d.
19. 7063 yds. at 1/10¾.	——	Ans. £669 10s. 3¼d.
20. 6286 yds. at 1/11.	——	Ans. £602 8s. 2d.
21. 5575 yds. at 1/11¼.	——	Ans. £540 1s. 6¾d.
22. 4849 yds. at 1/11½.	——	Ans. £474 15s. 11½d.
23. 6298 yds. at 1/11¾.	——	Ans. £623 4s. 9½d.
24. 7998 yds. at 1/11¾.	——	Ans. £791 9s. 4½d.

CASE 6.

When the price is in shillings.

RULE.—Multiply by half the number of shillings, double the right hand figure of first product for shillings, those on the left are £s, and if there is an odd shilling, add $\frac{1}{20}$ of the number of articles to the former product. Or multiply the number of articles by the shillings, and divide by 20, as in Case 1st.

EXAMPLES.—Calculate the value of 6274 at 6/ and at 15/.

6274 at 6s.		1/ | $\frac{1}{20}$ | 6274 at 15s.	
3		7	
£1882	4s.	4391	16
Ans.		$\frac{1}{20}$=313	14
		£4705	10 Ans.

In the first example I multiply by 3, half the number of shillings, and double 2, the right hand figure of first product for shillings, and add the one which was to carry to next product, and all on the left of the 4 shillings are £s as in the example.

In the 2d example multiply by 7, half the greatest even number of shillings, which gives the value at 14 shillings as in last example, I then add $\frac{1}{20}$ of the number of articles, which is the value at 1 shilling, the sum is the value at 15/.

EXERCISES.

CALCULATE THE VALUE OF

	Answer. £	s.		Answer. £	s.
1. 2362 yds. at 2/ =	236	4	13. 3040 yds. at 14/=	2128	0
2. 3843 yds. at 3/ =	576	9	14. 2324 yds. at 15/=	1743	0
3. 5458 yds. at 4/ =	1091	12	15. 6743 yds. at 16/=	5394	8
4. 7636 yds. at 5/ =	1909	0	16. 3100 yds. at 17/=	2635	0
5. 6324 yds. at 6/ =	1897	4	17. 8723 yds. at 18/=	7850	14
6. 3724 yds. at 7/ =	1303	8	18. 4200 yds. at 19/=	3990	0
7. 4269 yds. at 8/ =	1707	12	19. 6870 yds. at 22/=	7557	0
8. 7646 yds. at 9/ =	3440	14	20. 1003 yds. at 24/=	1203	12
9. 6657 yds. at 10/=	3328	10	21. 4846 yds. at 25/=	6057	10
10. 5873 yds. at 11/=	3230	3	22. 2806 yds. at 26/=	3647	16
11. 8762 yds. at 12/=	5257	4	23. 1068 yds. at 28/=	1495	4
12. 9400 yds. at 13/=	6110	0	24. 3765 yds. at 30/=	5647	10

CASE 7.

When the price is in shillings and lower denominations.

RULE.—Multiply the number of articles by the shillings, and take parts for the rest of the price, by Case 4th. which add to the first product, the sum is the answer in shillings.

Or, divide the price so, that the first may be an aliquot part of a £, and the rest either aliquot parts of a £ or of a part already taken, the sum of the quotients is the answer in £s.

EXAMPLE.—Calculate the value of 3643 yds. at 9/8 per yd.

```
   1st. method.              2d. method.             3d. method.
6d.|½| 3643               5/|¼|3643               1/8|1/12|3643
   | |    9               4/|⅕|  910 15                   4 by case 6th.
   | |-----                /8|⅙|  728 12                1457  4
/8 |⅛|32787                      121  8  8             303 11  8
2d.|⅓| 1821  6            Ans £1760 15  8             £1760 15  8
        607  2
2,0)3521,5  8
  £1760 15  8 Ans.
```

1st. Method. I here multiply by 9 shillings and take parts for the 8d. by case 4th., and the quotients added to the former product is the answer in shillings, which reduce to £s.

2d. Method. I here divide the price into 5 shillings the $\frac{1}{4}$, 4 shillings the $\frac{1}{5}$ of a £, and 8d. the $\frac{1}{6}$ of 4 shillings, the sum of the quotients by these parts is the answer in £s.

3d. Method. I here multiply by 4, half the greatest even number of shillings by case 6th., and take 1/8 the $\frac{1}{12}$ of a £, and the product and quotient added together give the answer in £s.

EXERCISES.

CALCULATE THE VALUE OF

1. 7691 gals. at 2/6¾. per. gal.		Ans. £985 8s. 2¼d.
2. 6768 gals. at 3/4½.	——	Ans. £1142 2s. 0d.
3. 8789 gals. at 4/9¾.	——	Ans. £2114 17s. 0¾d.
4. 7222 gals. at 5/6½.	——	Ans. £2001 1s. 11d.
5. 9887 gals. at 6/10.	——	Ans. £3378 1s. 2d.
6. 4242 gals. at 7/9¾.	——	Ans. £1657 0s. 7½d.
7. 8439 gals. at 8/4.	——	Ans. £3516 5s. 0d.
8. 3675 gals. at 8/9½.	——	Ans. £1615 9s. 4½d.
9. 7234 gals. at 9/6¾.	——	Ans. £3458 15s. 1½d.
10. 1070 gals. at 9/9¾.	——	Ans. £524 19s. 4½d.
11. 6582 gals. at 10/10.	——	Ans. £3565 5s. 0d.
12. 7998 gals. at 11/8.	——	Ans. £4665 10s. 0d.
13. 2027 gals. at 12/9½.	——	Ans. £1296 8s. 8½d.
14. 7491 gals. at 13/4.	——	Ans. £4994 0s. 0d.
15. 5435 gals. at 13/9½.	——	Ans. £3747 17s. 8½d.
16. 4834 gals. at 14/7½.	——	Ans. £3534 17s. 3d.
17. 9009 gals. at 15/10.	——	Ans. £7132 2s. 6d.
18. 2983 gals. at 16/8.	——	Ans. £2485 16s. 8d.
19. 4235 gals. at 17/11½.	——	Ans. £3802 13s. 6½d.

20. 7374 gals. at 18/6. Ans. £6820 19s. 0d.
21. 2234 gals. at 18/11¾. Ans. £2119 19s. 5½d.
22. 3668 gals. at 19s/6¾. Ans. £3587 15s. 3d.
23. 5407 gals. at 19/10¼. Ans. £5367 11s. 5¾d.
24. 6374 gals. at 19/11¾. Ans. £6367 7s. 2½d.

Case 8.

When the price is in £s, and lower denominations.

Rule.—Multiply the number of articles by the £s, and take parts for the lower denominations by the preceding rules, the sum of the products, and quotients, is the answer in £s.

Or, reduce the £s and shillings into shillings; multiply the number of articles by them, take parts for the pence, &c. the sum of the products, and quotients is the answer in shillings, which reduce to £s.

Example.—Calculate the value of 6324 lb. at £3 12s. 6d.

£. 1st method.	s. 2d method.	£. 3d method.
10/ | ½ | 6324	/6 | ½ | 6324	/6 | $\frac{1}{40}$ | 6324 × 6 = ½ of 12/ by
3	72	3 Case 6th.
2/6 | ¼ | 18972	12648	18972=value at £3.
3162	44268	3794..8=value at 12/.
790..10	3162	158..2=value at /6.
Ans. £22924 10	2,0)45849,	Ans. £22924 10=value at
	Ans. £22924 10	£3 12 6.

1st Method. I here multiply the given number of articles by £3, and for 12/6 I take 10/ the ½ of a £. and 2/6 the ¼ of 10/, the sum of the product by 3, and the quotients by ½ and ¼ is the answer in £s.

2d Method. I here multiply the given number of articles by 12, the shillings in £3 12s., and take 6d. the ½ of a shilling, the sum of the products, and quotient, is the answer in shillings, which reduce to £s.

3d Method. I here multiply by the £3, and by 6 the ½ of 12s. by Case 6th, and then take 6d. the $\frac{1}{40}$ of a £, the sum of the products, and quotient is the answer in £s.

EXERCISES.

Calculate the value of

1. 2632 cwt. at £2 15s. 10d. per cwt. Ans. £7347 13s. 4d.
2. 3426 cwt. at £3 5s. 6¾d. per do. Ans. £11230 17s. 1½d.
3. 4121 cwt. at £3 16s. 8d. per cwt. Ans. £15797 3s. 4d.

CALCULATE THE VALUE OF

4. 1204 cwt. at	£4 10s. 11½d.	per cwt.	Ans. £5475 13s. 10d.
5. 5425 ——	£4 13s. 4d.	——	Ans. £25316 13s. 4d.
6. 6468 ——	£5 17s. 6d.	——	Ans. £37999 10s. 0d.
7. 8721 ——	£6 8s. 4d.	——	Ans. £55959 15s. 0d.
8. 7298 ——	£6 18s. 6¾d.	——	Ans. £50561 9s. 1½d.
9. 9116 ——	£7 10s. 11½d.	——	Ans. £68806 16s. 2d.
10. 4499 ——	£7 15s. 6¾d.	——	Ans. £34993 15s. 8¼d.
11. 6397 ——	£8 14s. 10½.	——	Ans. £55933 15s. 4½d.
12. 7439 ——	£8 19s. 0d.	——	Ans. £66579 1s. 0d.
13. 4362 ——	£9 9s. 9¾d.	——	Ans. £41398 2s. 1½d.
14. 2632 ——	£10 10s. 11½d.	——	Ans. £27762 2s. 4d.
15. 6065 ——	£11 17s. 7½d.	——	Ans. £72059 15s. 7½d.
16. 4243 ——	£12 18s. 9½d.	——	Ans. £54902 13s. 0½d.
17. 2229 ——	£13 13s. 4d.	——	Ans. £30463 0s. 0d.
18. 9873 ——	£14 14s. 4½d.	——	Ans. £145318 4s. 4½d.

CASE 9.

When the given price is greater or less than 20 shillings by any aliquot part of a £.

RULE.—Consider the quantity £s., divide it by that part; add the quotient to the quantity if the price is greater, but subtract if less than a £.

When the price is greater or less than 1/ by any aliquot part of 1/, we proceed as above, by considering the quantity shillings, and dividing the result by 20.

EXAMPLES.—Calculate the value of 3675 yds. at 16/8, 26/8, 9d., 16d.

1st Example.

```
3/4 | 1/6 | 3675  at 16/8.
    |     |  612 10 subt.
          £3062 10 Ans.
```

2d Example.

```
6/8 | 1/3 | 3675  at 26/8.
    |     | 1225  add.
          £4900 Ans.
```

3d Example.

```
/3 | 1/4 | 3675  at 9d.
         |  918   9 subt.
     2,0 | 275,6  3
         £137 16  3 Ans.
```

4th Example.

```
/4 | 1/3 | 3675 at 16d.
         | 1225 add.
     2,0 | 490,0
         £245 Ans.
```

1st. Example.—Because 16/8 is less than a £ by 3/4 or ⅙ of a £, I consider the quantity £s, ⅙ of which is £612 10s. and this quotient subtracted from the dividend £3675 leaves £3062 10s. the answer.

2d Example.—Because 26/8 is greater than a £ by 6/8 or ⅓ of a £, I add ⅓ of the quantity considered as £s, to itself, which gives £4900 the answer.

3d Example.—Because 9d. is less than a shilling by 3d or ¼ of a shilling, I subtract ¼ of the quantity from itself, which leaves 2756s. 3d. and divided by 20 gives £137 16s. 3d. the answer.

4th Example.—Because 16d. is greater than a shilling by 4d. or ⅓ of a shilling, I add ⅓ of the quantity to itself, and divide the sum 4900 by 20 gives £245 the answer.

EXERCISES.

CALCULATE THE VALUE OF

1.	6734 yds. at	13/4	per yard.	Ans. £4489 6s. 8d.
2.	5436 ——	15/.	——	Ans. £4077.
3.	3892 ——	16/.	——	Ans. £3113 12s.
4.	2798 ——	16/8.	——	Ans. £2331 13s. 4d.
5.	7966 ——	17/6.	——	Ans. £6970 5s.
6.	1076 ——	18/.	——	Ans. £968 8s.
7.	8500 ——	19/.	——	Ans. £8075.
8.	9006 ——	£1 1s.	——	Ans. £9456 6s.
9.	3424 ——	£1 2s.	——	Ans. £3766 8s.
10.	6248 ——	£1 3s. 4d.	——	Ans. £7289 6s. 8d.
11.	4649 ——	£1 4s.	——	Ans. £5578 16s.
12.	6243 ——	£1 5s.	——	Ans. £7803 15s.
13.	7246 ——	£1 6s. 8d.	——	Ans. £9661 6s. 8d.
14.	3998 ——	£1 10s.	——	Ans. £5997.
15.	5243 ——	8d.	——	Ans. £174 15s. 4d.
16.	2134 ——	9d.	——	Ans. £80 0s. 6d.
17.	7962 ——	10d.	——	Ans. £331 15s.
18.	7549 ——	10½d.	——	Ans. £424 12s. 7½d.
19.	9824 ——	1/2.	——	Ans. £330 5s. 4½d.
20.	8656 ——	1/3.	——	Ans. £573 1s. 4d.
21.	1069 ——	1/4.	——	Ans. £71 5s. 4d.
22.	5976 ——	1/6.	——	Ans. £448 4s.

CASE 10.

When there is a fraction in the price for which parts cannot be easily taken.

RULE.—Multiply the price by the under number of the fraction, adding in the upper number; take parts with the sum, and divide the result, by the under number of the fraction; for the answer.

EXAMPLE.—Calculate the value of 6373 yards at 1/8$\frac{4}{5}$.

1/8 $\frac{4}{5}$	6/8	$\frac{1}{3}$	6373		
5	1/4	$\frac{1}{5}$	2124	6	8
8s. 8d.	/8	$\frac{1}{2}$	424	17	4
			212	8	8
		5)	2761	12	8
		Ans. £	552	6	6 $\frac{1}{4}$ $\frac{3}{5}$

I here multiply 1/8 by 5 the under number of the fraction, and add 4 the upper number, gives 8/8, with which I take 6/8, the $\frac{1}{3}$ of a £, 1/4 the $\frac{1}{5}$ of 6/8, and 8d. the $\frac{1}{2}$ of 1/4, the sum of the quotients by these parts is £2761 12s. 8d., which divided by 5 the under number of the fraction gives £552 6s. 6$\frac{1}{4}$d. $\frac{3}{5}$, the answer.

EXERCISES.

CALCULATE THE VALUE OF

1. 3746 gals. at 3/6$\frac{2}{3}$ per gal. Ans. £665 19s. 1$\frac{1}{4}$d. $\frac{1}{3}$.
2. 4573 gals. at 4/5$\frac{3}{5}$ per gal. Ans. £1021 6s. 0$\frac{3}{4}$d. $\frac{1}{5}$.
3. 6258 gals. at 6/4$\frac{4}{7}$ per gal. Ans, £1996 12s.
4. 8972 gals. at 7/8$\frac{3}{5}$ per gal. Ans. £3460 0s. 8$\frac{1}{4}$d. $\frac{2}{5}$.
5. 2673 gals. at 8/3$\frac{4}{11}$ per gal. Ans. £1106 13s. 3d.
6. 5306 gals. at 8/9$\frac{6}{13}$ per gal. Ans. £2331 11s. 6$\frac{3}{4}$d. $\frac{8}{13}$.
7. 2045 gals. at 9/3$\frac{4}{17}$. per gal. Ans. £947 16s. 4$\frac{3}{17}$d.
8. 6648 gals. at 10/6$\frac{7}{15}$ per gal. Ans. £3500 8s. 1$\frac{1}{4}$d. $\frac{1}{15}$.

CASE II.

When there is a fraction in the quantity.

RULE.—Calculate for the whole number as before, to the result add the same part of the price which the fraction is of unity, the sum is the answer :—

Or thus—Multiply the quantity by the under number of the fraction, to the product add the upper number, find the value of this new quantity, which divide by the under number of the fraction for the answer.

EXAMPLE.—Calculate the value of 6246$\frac{5}{8}$ lb. at 6/3 per lb.

1st Method.

5/	$\frac{1}{4}$	6246			
1/3	$\frac{1}{4}$	1561	10		
		390	7	6	
$\frac{5}{8}$ lb. =			5	2$\frac{1}{2}$	
		£1952	2	8$\frac{1}{2}$	Ans.

$\frac{4}{8}$	$\frac{1}{2}$	6	3
$\frac{1}{8}$	$\frac{1}{4}$	3	1$\frac{1}{2}$
		2	1
$\frac{5}{8}$ =		5	2$\frac{1}{2}$

2d Method.

		6246$\frac{5}{8}$			
		6			
5/	$\frac{1}{4}$	37481			
1/3	$\frac{1}{4}$	9370	5		
		2342	11	3	
	6)	11712	16	3	
		£1952	2	8$\frac{1}{2}$	Ans.

1st Method. I first calculate the value of 6246 lb. at 6/3, then the value of the ⅝ lb, and these two values added together give the Answer. But instead of taking parts with the fraction as in the example, its value is often calculated more concisely, thus; Multiply the price by the upper number of the fraction, and divide the product by the under number, the quotient is its value.

2d Method. I here multiply the given quantity by the under number of the fraction, adding the upper to the product, I then calculate the value of this sum as if it were the given quantity, and divide the result by the under number of the fraction for the answer.

EXERCISES.

CALCULATE THE VALUE OF

1. 6324 ⅓ lb. at	4/7½	per lb.	Ans. £1462 10s. 0½d.
2. 4236 ⅖ ——	5/3¾	——	Ans. £1125 5s. 10½d.
3. 5287 ⅙ ——	6/9	——	Ans. £1784 8s. 4½d.
4. 7895 3/7 ——	6/6¾	——	Ans. £2590 13s. 9d.
5. 8682 ⅝ ——	7/6	——	Ans. £3255 19s. 8¼d.
6. 9367 4/9 ——	10/8¼	——	Ans. £5005 14s. 6¾d.
7. 2435 7/10 ——	11/10½	——	Ans. £1446 3s. 11¼d.
8. 3684 5/11 ——	12/10	——	Ans. £2364 3s. 10d.
9. 4786 5/12 ——	13/6	——	Ans. £3230 16s. 7½d.
10. 1989 4/13 ——	14/4½	——	Ans. £1427 14s. 10½d.

CASE 12.

When the quantity is in several denominations, and the price of the highest given.

RULE.—Multiply the price by the highest, and take parts of it for the lower denominations, the sum of the products and quotients is the answer.

EXAMPLE.—Calculate the value of 35 lb. 4 oz. 8 drs. at £2 4s. 6d. per lb.

		£	s.	d.	
oz. 4	¼	2	4	6	
				5	
		11	2	6	
				7	
drs		77	17	6	= value of 35 lb.
8	⅛	0	11	1½	= —— 4 oz.
		0	1	4 ½ ¾	= — 8 drs.
		£78	10	0 ¼ ¾	Ans.

I here multiply the price of 1 lb. by 35 the number of lbs., and as 4 oz. is the ¼ of a lb., I take ¼ of the price of a lb. for its value, again because 8 drs. is the ⅛ of 4 oz., I take ⅛ of the last quotient for its value, and the sum of the product and quotients is the answer as in the example.

EXERCISES.

CALCULATE THE VALUE OF

1. 3 lb. 6 oz. 8 dwt. at £3 4s. 6d. per lb. Ans. £11 7s. $10\frac{3}{4}$d. $\frac{1}{6}$.
2. 6 lb. 8 oz. 12 dwt. at £4 7s. $8\frac{1}{2}$d. per lb.
Ans. £29 9s. $1\frac{1}{4}$d. $\frac{1}{8}$ — $\frac{1}{4}$.
3. 16 lb. 6 oz. 12 drs. at £1 16s. 7d. per lb.
Ans. £30 0s. $9\frac{3}{4}$d. $\frac{11}{16}$ — $\frac{13}{16}$.
4. 28 lb. 2 oz. 8 drs. at £2 9s. 10d. per lib.
Ans. £70 3s. $1\frac{1}{4}$d. $\frac{3}{8}$.
5. 33 cwt. 3 qr. 7 lb. at £6 7s. 8d. per cwt.
Ans. £215 16s. $8\frac{3}{4}$d.
6. 57 cwt. 1 qr. 21 lb. at £5 4s. 8d. per cwt.
Ans. £299 7s. $10\frac{1}{2}$d. $\frac{1}{4}$ — $\frac{3}{4}$.
7. 48 cwt. 2 qr. 14 lb. at £7 6s. per cwt. Ans. £354 19s. 3d.
8. 65 cwt. 1 qr. 4 lb. at 18s. 5d. per cwt. Ans. £60 2s. $4\frac{3}{4}$d. $\frac{4}{7}$.
9. 36 cwt. 0 qr. 14 lb. at £1 13s. 6d. per cwt.
Ans £60 10s. $2\frac{1}{4}$d.
10. 42 cwt 3 qr. 7 lb. at £3 6s. 8d. per cwt. Ans. £142 14s. 2d.
11. 19 ton 15 cwt. 2 qr. at £2 6s. 3d. per ton.
Ans £45 14s. $7\frac{3}{4}$d. $\frac{1}{8}$.
12. 12 ton 12 cwt. 2 qr. at £16 16s. 4d. per ton.
Ans. £212 6s. $2\frac{1}{2}$d — 1 qr.
13. 6 ton 18 cwt. 2. qr. at £9 18s. 4d. per ton.
Ans. £68 13s $5\frac{1}{2}$d.
14. 9 ton 4 cwt. 2 qr. 8 lb. at £4 3s. 9d. per ton.
Ans. £38 12s. $10\frac{1}{2}$d. $\frac{5}{7}$.
15. 16 acres 2 r. 10 p. at £1 12s. 4d. per acre.
Ans. £26 15s. $6\frac{1}{4}$d.
16. 27 acres 3 r. 20 p. at £2 4s. 6d. per acre.
Ans. £62 0s. $5\frac{1}{4}$d.
17. 216 acres 1 r. 12 p. at £3 14s. 10d. per acre.
Ans. £809 8s. $3\frac{3}{4}$d. $\frac{3}{8}$ — $\frac{7}{8}$
18. 56 qrs. 6 bus. 3 pecks, at £2 13s. 4d. per quarter.
Ans. £151 11s. 8d.
19. 16 qrs. 3 bu. 2 p. at £3 2s. 3d, per quarter.
Ans. £51 3s. $2\frac{3}{4}$d. $\frac{1}{4}$ — $\frac{5}{4}$.
20. 14 yds. 3 qr. 2 n. at £1 6s. 7d. per yd.
Ans. £19 15s. $5\frac{3}{4}$d. $\frac{1}{8}$.
21. 63 yds. 1 qr. 3 n. at £0 18s. 8d. per yd. Ans. £59 4s. 2d.
22. 84 gal. 3 qt. 1 pt. at £1 4s. 9d. per gal.
Ans. £105 0s. $7\frac{3}{4}$d. $\frac{1}{2}$.
23. 26 gal. 2 qt. at £1 10s. 2d. per gal. Ans. £39 19s. 5d.
24. 36 ton 2 hhd. 21 gal. at £46 15s. 9d. per ton.
Ans. £1711 12s. $10\frac{1}{2}$d.

25. 42 ton 3 hhd. 28 gal. at £63 7s. 8d. per ton.
Ans. £2716 13s. 7$\frac{0}{4}$d. $\frac{8}{9}$.

26. 16 miles 3 f. 18 p. at £12 16s. 10d. per mile.
Ans. £211 0s. 1$\frac{0}{4}$d. $\frac{9}{20}$.

COMMERCIAL ALLOWANCES, OR TARE AND TRET,

Are certain deductions made from goods which have been weighed in the CHEST, CASK, BAG, or whatever contains them.

GROSS WEIGHT, is the weight of any goods together with that which contains them.

DRAFT is an allowance of so much per chest, bag, &c. that the weight may hold out when sold by retail, and when allowed is first deducted from the whole weight.

TARE is the weight of whatever contains the goods, and is deducted after the draft, and first when no draft is allowed.

TRET is an allowance sometimes given of 4 lb. per 104 or $\frac{1}{26}$ on goods subject to waste, and is deducted after the tare.

CLOFF is an allowance which is also sometimes given to retailers, of 2 lb. per 3 cwt. or $\frac{1}{168}$ for the turn of the scale, and is deducted after the tret.

Neat weight is what remains after making all the stipulated deductions.

RULE.—1st. When draft is allowed, subtract it from the gross weight.

2d. When the whole tare is given, without farther allowances, take it from the gross, the remainder is the neat weight.

3d. When the tare is at so much per box, cask, &c. without farther allowances, multiply the tare by the number of boxes, &c. the product is the whole tare, which subtracted from the gross, leaves the neat weight.

4. When the tare is at so much per cent., or per cwt., take of it aliquot parts of 100, or of 112, with which divide the gross weight, the sum of the quotients is the tare, and subtracted from the gross, leaves the neat weight.

5. When both tare and tret are allowed; find the tare as before, and subtract it from the gross leaves the tare suttle, which divide by 26 the quotient is the tret, and subtracted from the tare suttle, leaves the neat weight.

6. When tare, tret, and cloff are allowed, deduct tare and

tret as before, divide the remainder, or tret suttle by 168, the quotient is the cloff, which subtracted from the tret suttle leaves the neat weight.

Examples.

1. What is the neat weight of 36 casks of madder weighing gross 63 cwt. 2 qrs. 14 lb., draft 4 lb. per cask, tare on the whole 3 cwt. 0 qr. 24 lb.?

	63	2	14	= gross
	1	1	4	= draft
	62	1	10	
	3	0	24	= tare
Ans.	59	0	14	= neat

The draft is found by multiplying the 4 lb. by 36 the number of casks and dividing the product by 28 and 4, which I subtract, and then from the remainder I subtract the tare, which leaves the neat weight.

2. What is the neat weight of 56 cwt. 3 qrs. gross, tare 20 lb. per cwt., allowing also the usual tret and cloff?

lb.		cwt.	qr.	lb.	
16 =	$\frac{1}{7}$	56	3	0	= gross.
4 =	$\frac{1}{4}$	8	0	12	
		2	0	3	
		10	0	15	= Tare.
4 =	$\frac{1}{26}$	46	2	13	= Tare suttle.
		1	3	5	= Tret.
2 =	$\frac{1}{168}$	44	3	8	= Tret suttle.
			1	2	= Cloff.
	Ans.	44	2	6	Neat weight.

I here take parts with the tare, viz. 16 lb. the $\frac{1}{7}$th of a cwt. and 4 lb. the $\frac{1}{4}$th of 16 lb. I then take the same parts of the gross that the tare is of a cwt. the sum of the quotients is the tare, which subtracted from the gross leaves the tare suttle. Then as the tret is $\frac{1}{26}$, I divide the tare suttle by 26, the quotient is the tret, which subtracted from the tare suttle, leaves the tret suttle. Again I divide the tret suttle by 168, the quotient is the cloff, which subtracted from the tret suttle leaves the neat weight.

Exercises.

1. What is the neat weight of 15 bags of cotton wool, weighing gross 64 cwt. 3 qrs. 19 lb., draft 1 lb. per bag, tare on the whole 2 qrs. 18 lb? Ans. 64 cwt. 14 lb.
2. What is the neat weight of 144 casks madder, each weighing 1 cwt. 2 qr. 8 lb. gross, draft 4 lb. and tare 15 lb. per cask? Ans. 201 cwt. 3 qrs. 12 lb.
3. What is the neat weight of 42 barrels of pot-ash, each weighing gross 1 cwt. 3 qrs. 14 lb.; tare 10 lb. per cent.? Ans. 70 cwt. 3 qrs. 14 lb.

4. What is the neat weight of 49 hhds. tobacco, each weighing gross 5 cwt. 2 qrs. 14 lb.; tare 24 lb. per cwt.?
Ans. 216 cwt. 2 qr. 7 lb.

5. What is the neat weight of 35 hhds. sugar weighing gross 216 cwt. 1 qr. 14 lb.; tare 16 lb. per cwt. also deducting the usual tret? Ans. 178 cwt. 1 qr. $9\frac{1}{13}$ lb.

6. What is the neat weight of 67 cwt. 2 qr. 12 lb. gross; tare 25 lb. per cent., deducting also the usual tret and cloff? Ans. 48 cwt. 1 qr. 24 lb.

7. In 28 casks of prunes each 2 cwt. 1 qr. 14 lb. gross, what is the neat weight, deducting 14 lb. per cwt. tare, and the usual cloff? Ans. 57 cwt. 3 qrs. 10 lb.

8. In 16 casks of currants weighing gross 105 cwt. 3 qrs. 16 lb.; tare 15 lb. per cent. tret and cloff as usual, how much neat? Ans. 86 cwt. 0 qr. 3 lb.

9. Calculate the neat weight of 62 chests of Souchong tea, weighing gross 73 cwt. 3 qrs. 20 lb.; draft 1 lb. per chest, and tare 21 lb. per cwt. Ans. 59 cwt. 2 qrs. 13 lb.

10. Calculate the neat weight of 8 bags of cotton wool, each weighing gross 2 cwt. 3 qrs. 6 lb., draft 1 lb. per bag, and tare 5 lb. per cent. Ans. 21 cwt. 0 qr. 27 lb.

11. Calculate the neat weight, and value of 5 barrels of rice, weighing gross 14 cwt. 26 lb., at $3\frac{3}{4}$d. per lb. neat; allowing draft 1 lb. per barrel, tare on the whole 2 cwt. 3 qrs. 19 lb. and tret and cloff as usual?
Ans. 10 cwt. 3 qrs. 2 lb., value £18 16s. $10\frac{1}{2}$d.

12. What is the neat weight of 15 barrels of anchovies, each weighing gross 36 lb.; tare 8 lb. per. cwt.?
Ans. 4 cwt. 1 qr. $25\frac{5}{7}$ lb.

13. What is the neat weight, and value of 6 barrels indigo weighing gross 22 cwt. 1 qr. 16 lb.; tare 12 lb. per cent?
Ans. 19 cwt. 2 qr. 23 lb.

14. What is the neat weight, and value of 10 barrels molasses, each weighing gross 12 cwt. 1 qr. 8 lb.; tare 14 lb. per cwt.; allowing also the usual tret and cloff, at $3\frac{1}{2}$d. per lb. neat? Ans. 103 cwt. 6 lb., value £168 6s. 5d.

15. What is the neat weight, and value of 23 cwt. 2 qr. 16 lb. gross weight of oil; tare $12\frac{1}{2}$ lb. per cent; at 6/8 per gallon of 9 lb. neat to the gallon?
Ans. 20 cwt. 2 qr. 21 lb., value £85 16s. $3\frac{1}{4}$d. $\frac{2}{9}$.

16. What is the tare on 34 cwt. 1 qr. 4 lb. at 21 lb. per cwt.?
Ans. 6 cwt. 1 qr. 20 lb.

17. What is the tret on 14 cwt. 1 qr. 16 lb.? Ans. 2 qrs. 6 lb.

18. What is the cloff on 52 cwt. 2 qrs.? Ans. 1 qr. 7 lb.

SIMPLE PROPORTION,

OR

THE RULE OF THREE.

SIMPLE PROPORTION, OR THE RULE OF THREE is the method of finding a fourth proportional to three given numbers.

RULE FOR STATING THE GIVEN TERMS.

Consider which of the given terms is of the same kind as that which the question demands, and write it down.

Then consider from the nature of the question whether the answer must be greater or less than this term. If greater, place the least of the remaining terms first on the left, and the greatest in the middle; but if less, place the greatest on the left, and the least in the middle with two points (:) between the 1st, and 2d on the left and (::) points between the 2d and 3d.

Reduce the two first terms to the same denomination, and the third term to the lowest denomination mentioned in it.

RULE FOR WORKING.

Multiply together the second and third terms thus reduced, and divide their product by the first term, the quotient is the answer in that denomination which the third term was reduced to, and if there is a remainder it is also in the same denomination as the third term, and may be reduced to the next lower and then divided, and so on, till it is brought to the lowest denomination.

PROOF.—Make the answer of the former stating the third term of the proportion, and the 1st of the former the 2d of this, and the 2d the first, and the answer will be the same as the third term of the first proportion, when the work is right. Or thus, Multiply the answer by the 1st term, and if this product is the same as that of the 3d term multiplied by the 2d, the work is right.

EXAMPLE.—What must I pay for 9 yds. 3 qrs. of broad cloth, of which 7 yds. cost £9 13s. ?

```
yds.  yd. qr.      £. s.
As 7  :  9  3  ::  9 13
   4     4         20
  28    39        193
                   39
                 1737
                  579    2,0
              28)7527(26,8
                  56   £13 8 9¾ 3/7
                 192           Ans.
                 168
                  247
                  224
                   23
                   12
                 )276
                  252
                   24
                    4
                  )96
                   84
                   12
                   -- ÷4=3/7
                   28
```

I here see that the question demands money, or the price of 9 yds. and 3 qrs. I therefore write £9 13s. the money given in the question for the third term of the proportion, with :: points before it. I then say, if 7 yds. cost £9 13s. will 9 yds. cost more or less, and because 9 yds. will cost more than 7 yds. and consequently, the answer must be more than £9 13s. I therefore write 7 yds. the least of the two remaining terms 1st on the left with : points after it, and the 9 yds. 3 qrs. in the middle. The numbers are then arranged in proportion, according to the rule as represented in the example. This done, I reduce the 1st and 2d terms to qrs. of a yard, being the lowest denomination mentioned in either of them, and the 3d term to shillings, being the lowest denomination mentioned in it. I then multiply 193 the 3d term by 39, the 2d term, and divide the product 7527 by 28 the first term, which gives 268 in the quotient, and 23 of a remainder, and because the third term was reduced to shillings, both the quotient and remainder are in shillings. I therefore divide the quotient by 20, which gives £13 8s. ; I then multiply the remainder by 12 and divide the product by the former divisor, gives 9d. and 24 pence of a remainder, which multiplied by 4 and divided, gives ¾ and 12/28 of a remainder, and this divided by 4 gives 3/7, therefore the answer is £13 8s. 9¾d. 3/7 as in the example.

1. If 9 lb. of sugar cost 6s.; what must I pay for 36 lb. at the same rate? Ans. £1 4s.
2. If 36 lb. of sugar cost £1 4s.; what must I pay for 9 lb. at the same rate? Ans. 6 sh.
3. If £1 4s. buy 36 lb. of sugar; how many lbs. will 6 shs. buy at the same rate? Ans. 9 lb.
4. If 9 lb. of sugar cost 6 sh.; how many lbs. will £1 4s. purchase at the same rate? Ans. 36 lb.
5. What must I pay for 96 yds. of cambric, when I am charged 13/6 for 12 yds.? Ans. £5 8s.
6. How much will 2 cwt. 3 qrs. of soap cost, when 14 lb. is 8 sh. and 2d.? Ans. £8 19s. 8d.
7. Paid £18 3s. for 8 quarters 2 bushels of wheat; how much will £660 purchase at the same rate? Ans. 300 qrs.
8. How much silver plate may be bought for £675 16s. 8d., when £3 11s. 3d. is paid for 7 oz. 10 dwts? Ans. 118 lb. 6 oz. 16 dwt 3 $\frac{7}{15}$ grs.
9. If nineteen pounds four shillings buy four ounces of gold; how much will one thousand pounds purchase at the same rate? Ans. 17 lb. 4 oz. 6 dwt. 16 gr.
10. An apothecary bought six bottles of East India castor oil, each weighing 110 lb. avoirdupois gross, tare 4 lb. per bottle, and sold it out at 3½d. per ounce apothecaries weight, how much money did it bring him? Ans. £135 5s. 2½d.
11. If I pay 6 sh. 4½d. for 8½ lb. of cheese; how much will £56 16s. purchase? Ans. 13 cwt. 2 qr. $2\frac{2}{3}$ lb.
12. If 7 lb. of butter cost 7/10½, what is the price of 3 cwt. 2 qr? Ans. £22 1s.
13. What must be paid for 72 yds. of satin, 8 yds. of which cost £3 8s. 3d.? Ans. £30 14s. 3d.
14. How far will fifty pounds defray the expense of travelling in a post chaise, at the rate of sixteen shillings and four pence halfpenny for nine and a half miles? Ans. 580 miles 1 f. $8\frac{112}{131}$ p.
15. A wool stapler paid six shillings and four pence for a clove of wool; what will fourteen lasts cost him at that rate? Ans. £2766 8s.
16. If seven yds. two qrs. one nail of cloth is manufactured from 9¼ lb. of wool, how much cloth of the same kind can be made from four lasts eight sacks one wey of wool of the same quality? Ans. 16814 yds. $1\frac{19}{37}$ n.
17. A gentleman ordered his steward to purchase 56 loads of new hay at 7½d. per stone of 14 lb.; how much money will pay this order? Ans. £270.

18. A silk mercer bought 378 French ells of silk, and sold it at £1 12s. for three yards; how much did it produce?
Ans. £302 8s.

19. If 36 square feet of land cost $5\frac{1}{4}$d.; how much money will purchase 12 acres 2 roods at the same rate?
Ans. £330 17s. $2\frac{1}{4}$d.

20. Bought a ton of port wine, old measure, for £226 16s. how must it be sold per imperial gallon, which is equal to $1\frac{1}{5}$ old wine gallons, in order to gain £21 6s. 6d. on the whole? Ans. £1 3s. $7\frac{1}{2}$d. $\frac{2}{7}$

21. If 7 gallons 2 pints of whisky cost £3 18s. 9d.; what will a puncheon cost? Ans. £45 12s. $4\frac{3}{4}$.d. $\frac{25}{29}$.

22. What will $3\frac{1}{2}$ tons of ale cost at 5/8 per 2 gals.?
Ans. £107 2s.

23. How much wheat will a thousand pounds purchase at the rate of £1 15s. for six bushels? Ans. 428 qrs. 4 bu. $2\frac{2}{7}$ p.

24. If two pecks of potatoes cost 1/5; what money will purchase a chaldron? Ans. £5 2s.

25. If 16 men cut a trench in 40 days; how many men can do the same in 8 days? Ans. 80 men.

26. A farmer engaged 40 men to cut his crop in 18 days, but before commencing he finds it necessary to have it all cut in 8 days; how many additional men will he require?
Ans. 50.

27. What must I pay for the use of £875 10s. for a year at the rate of £5 per hundred pounds? Ans. £43 15s. 6d.

28. Bought eight pieces of holland, each 24 ells Flemish, at 5/6 per imperial yard; how much did it cost me?
Ans. £39 12s.

29. How many English miles are equal to one hundred Irish miles, the Irish pole being seven yds. Ans. $127\frac{3}{11}$ miles.

30. How many quarters of wheat at £1 19s. are equal in value to 656 bolls of potatoes at 8/8. Ans. 145 qrs. $6\frac{2}{9}$ bu.

31. If six engineers can finish two steam engines in four weeks; how long will two engineers require to do the same?
Ans. 12 weeks.

32. How long must I lend my friend £65 to requite his kindness for allowing me the use of £200 for 7 months?
Ans 21 m. 2 w. $1\frac{1}{13}$ d.

33. What quantity of sugar at 7d. per lb. is equal in value to 96 lb. of tea at 6/6 per lb. Ans. 9 cwt. 2 qrs. $5\frac{5}{7}$ lb.

34. How much Stilton cheese at 1/6 per lb. must I give for four hundred weight of double Gloucester cheese at $9\frac{1}{2}$d. per lb.? Ans. 2 cwt. $12\frac{4}{9}$ lh.

35. How many yards of carpeting 3 qrs. wide, and of crumb

cloth 5 qrs. wide respectively, will cover a room 36 feet long and 30 feet broad?

Ans. 160 yds. of carpeting, 96 yds. of crumb cloth.

36. How many yards of paper 18 inches wide will cover a wall 72 feet long, and 10 feet high? Ans. 160 yds.

37. What is my commission on £1364 10s., at £2½ per £100? Ans. £34 2s. 3d.

38. Calculate the insurance on £3629 15s., at 4/8 per hundred pounds. Ans. £8 9s. 4¼d. [illegible]

39. Calculate the insurance on £1068, at 6 guineas per £100. £67 5s. 8$\frac{4}{25}$d.

40. How many yards of serge 3 quarters wide will line coats for 6000 soldiers, each coat containing 2½ yds. of cloth 5 qrs. wide? Ans. 25000 yards.

41. Part of the wall of a garrison which was built by 30 men in 8 days was broken down, and must be repaired in 2 days; how many men can accomplish the work in that time? Ans. 120 men.

42. If a dress-maker require 10½ yards of silk 2 qrs. 2 n. wide to make a dress; how much print 5 qrs. wide will make a dress in the same style? Ans. 5¼ yds.

43. If a tailor require 4½ yards of cloth 6 qrs. wide to make a suit of clothes; how many yards of 3 qrs. 2 n. wide will make the same? Ans. 7$\frac{5}{7}$ yds.

44. A merchant failed and was found to be owing in all £6376 15s. 6d., his effects are worth only £898 16s. 10d.; how much can he pay per £, and what should B receive to whom he owes £674 18s.?

Ans. 2s. 9¾d. [illegible] per £., B receives £95 2s. 7¼d. [illegible].

45. What should I pay for police taxes on a real rental of £85, at the rate of £10 10s. per hundred £s. levied on 4-fifths of the real rental? Ans. £7 2s. 9½d. ⅕

46. A bankrupt pays his creditors 2/6 per £, and pays them in all £1864 16s. 8d.; what was his debt? Ans. £149182 13s. 4d.

47. If the quartern loaf cost 10d. when wheat is £2 16s. per quarter; what should it cost when wheat is at £4 10s. per quarter? Ans. 1/4$\frac{1}{14}$.

48. Bought 6 drums of raisins, each weighing neat 64¼ lb. for £2 2s. 8d. per cwt.; what do they come to? Ans. £7 6s. 3¼d. ⅝

49. Bought a pipe of wine for £99 16s. which leaked out 15 gals.; how must I sell the remainder per gallon to gain £6 10s. 6d. on the whole? Ans. 19s. 1¾d. [illegible]

50. Bought a property for £8967 16s. and paid for improvements £1032 4s.; what must it be sold for to clear £5 per cent. on the out laid money? Ans. £10500.

51. Shipped for Belfast 9866 yds. flannel, which cost me 1/6 per yard, duty and freight $1\frac{1}{2}$ per yard; my agent sold it for 2/2 per yd., whether did I gain or lose by this transaction, and how much after paying £$1\frac{1}{2}$ per cent. commission, and having the money remitted at £108 Irish, per £100 sterling? Ans. gained £173 3s. $9\frac{5}{54}$d.

52. How many yds. of superfine black cloth, at £1 12s. per yd. are equal in value to 7 chests of tea, each 76 lb. neat, at 6/6 per lb.? Ans. 108 yds. 1 nail.

53. How much butcher meat at 5/6d. per stone, is equal in value to 50 stones of moulded candles at 11/6 per stone? Ans. $104\frac{6}{11}$ stone.

54. If 9 yards of cloth cost ten guineas; what must I pay for 8 pieces each 36 yds.? Ans. £336.

55. A grazier purchased at a fair, ten score of lambs at 12/ per head, and eight score at 14/ per head, but not liking his bargain he mixed the two lots, and sold them off for 13/ per head; whether did he gain or lose by this transaction? Ans. gained £2.

56. A grocer was selling one kind of sugar at 3 lb. for 1/, and another kind at 2 lb. for 1/, but mixing together 200 lb. of each, he sells it at 5 lb. for 2/; whether did he gain or loss by this transaction? Ans. lost 6/8d.

57. How much water must be mixed with 50 gals. of rum at 17s. per gallon; to reduce it to 13/ per gal.? Ans. $15\frac{5}{13}$ gals.

58. Bought a puncheon of brandy, and drew off 20 gals. which I sold for £1 8s. per gall. and replaced it with water; how should I sell it thus reduced to clear the same as before? Ans. £1 1s. 4d.

59. I have the offer of rum for 15/ per gall. which stands three waters in punch, and at 21/ per gall. which stands five waters in punch; which is the cheapest and how much? Ans. at 21/, cheapest by 1/6.

60. If a steel-yard be 3 feet long, and have its fulcrum $1\frac{1}{2}$ inches from one end; what weight suspended from the shortest arm will counterpoise $8\frac{1}{2}$ lb. suspended from the longest arm? Ans. 1 cwt. 2 qr. $27\frac{1}{2}$ lb.

61. If 2 men perform a piece of work in 30 days; how long will 20 men require to do it? Ans. 3 days.

62. If a journey can be performed in 16 days of 16 hours, how

many days of 12 hours would it require to perform it at the same rate of travelling? Ans. 24d. 4h.

63. What length of riband 3 inches wide will cover a square yard of cloth? Ans. 12 yards.

64. What must I pay for 150 pigs of lead, each weighing 1 cwt. 3 qrs. at £14 10s. per fother of 19½ cwt.? Ans. £195 3s. 10$\frac{2}{13}$d.

65. How much cochineal can I buy for £131 4s. at the rate of £24 for 15 lb.? Ans. 82 lb.

66. Bought 18 frails of raisins, each weighing gross 22 lb. tare 4 lb. a frail, what do they cost at 18/ for 24 lb. neat? Ans. £49 14s. 6d.

67. How many yards of shalloon 2 qrs. wide will line 40 yards camlet 5 qrs. wide? Ans. 100.

68. If one compositor can set the types for a sheet in 8 days and another in 12 days, in what time can they do it together? Ans. 5⅖ days.

69. Two numbers are to one another as 18 to 14, the least is 60; what is the greatest? Ans. 77⅐.

70. If 3 men can cut down a field of oats in 48 days of 11 hours each, and if 4 women can do it in the same time; how long will 4 men and 2 women require to do it? Ans. 26 days 2 h.

71. Desirous of ascertaining the height of a tree I set up my staff, which was 3 feet 4 inches long, and found that its shadow was just 3½ times its length, and the shadow of the tree 81 times its length; what was the height of the tree? Ans. 77 feet 1⅐ inches.

72. If I spend £3 10s. in 20 days; how long will £500 serve me? Ans. 7 yrs. 43 w. 1⅐ day.

73. What is the weight of a silver cup which cost £26 14s. 7½d., at 11/4½d. per ounce? Ans. 3 lb. 11 oz.

74. What must I pay for 4 bags of hops, weighing gross 9 cwt. 2 lb., tare 4 lb. per bag, at 5 guineas per cwt. neat? Ans. £46 11s. 10½d.

75. "Whereas a noble and a mark just 15 yards did buy,
How many ells of the same cloth for £50 had I."
Ans. 750 ells, from *Fisher*.

COMPOUND PROPORTION.

COMPOUND PROPORTION is when five terms are given to find a sixth, or seven to find an eight, or nine to find a tenth, &c.

RULE.—Consider what term is the same as that which the question demands, and place it for the third term of the proportion; then take other two terms both of a kind, and with them and the third term make a stating as directed in Simple Proportion; proceed in like manner with every other pair of terms till they are all arranged. Then multiply the continued product of the second terms by the third for a dividend, and divide by the continued product of the first terms; the quotient is the answer in the same name as the third term, and if there is a remainder, reduce it as directed in Simple Proportion.

EXAMPLE.—If 6 horses in 9 days plough 30 acres of land working 12 hours a day; how many acres will 18 horses plough in 144 days, working on an average 10 hours a day?

$$\left.\begin{array}{lrcr} \text{Horses} & 6 & : & 18 \\ \text{Days} & 9 & : & 144 \\ \text{Hours} & 12 & : & 10 \end{array}\right\} \begin{array}{c}\text{Acres} \\ ::30\end{array}$$

$$\begin{array}{rr} 108 & 1440 \\ 6 & 18 \\ \hline 648 & 25920 \\ & 30 \\ \hline \end{array}$$

$$\begin{array}{l} 648)777600(1200 \text{ Acres.} \\ \quad\ 648 \\ \hline \quad 1296 \\ \quad 1296 \\ \hline \qquad 00 \end{array}$$

Or by canceling thus.

$$\frac{30\times10\times\overset{\overset{2}{\cancel{12}}}{\cancel{144}}\times\overset{2}{\cancel{18}}}{\cancel{6}\times\cancel{9}\times\cancel{12}}=2\times2\times10\times30=1200 \text{ Acres.}$$

In stating this question it is readily perceived that the question demands acres, I therefore write the 30 acres for the third term of the proportion, then taking other two terms of a kind, viz. the 6 horses and the 18 horses, I say, if 6 horses plough 30 acres, will 18 plough more or fewer in the same time? It is evident that they must plough more, I therefore place the 6 first and the 18 in the middle; next taking 9 days and 144 days, I say, if 9 days plough 30 acres will 144 plough more or fewer at the same rate of working, and seeing that more will be done in 144 days, I write 9 in the first place, and 144 in the second place; again, taking 12 hours and 10 hours, I say, if 12 hours plough 30 acres will 10 plough more or fewer, and finding that 10 will give fewer I write 12 in the first term and 10 in the second term. The terms being thus arranged in proper order, I multiply $30\times10\times144\times18$ for the dividend and divide by the product of 12, 9, 6, the quotient is 1200 acres, being the same denomination as the third term.

The method by canceling often saves much labour, besides its simplicity excludes in a great measure the risk of errors in the work.

The operation is performed thus, having arranged the terms as in the former stating, I write all the numbers belonging to the dividend above a line, with the sign of multiplication between them; and those belonging to the divisor below the line in like manner. I then divide 18 above, and 9 below the line by 9, writing the quotient 2 above 18, I next divide 144 above and 12 below, each by 12, writing the quotient 12 above 144; I then divide twelve above and 6 below by 6, and write the quotient 2 above 12. The numbers below the line being all canceled, I multiply together 2, 2, 10, 30 which remain uncanceled above, and their product is the answer, the same as by the other method.

EXERCISES.

1. If £100 in one year gain £4 interest, what will £760 gain in 3½ years? Ans. £106 8s.
2. At £5 per cent. per annum; what principal will yield £195 in 3 years? Ans. £1300.
3. If £1300 in three years gain £156 interest; what is the rate per cent. per annum? Ans. £4 per cent.
4. In what time will £4650 gain £744, at 4 per cent. per annum? Ans. 4 years.
5. At what rate of interest will £650 amount to £747 10s. in three years? Ans 5 per cent.
6. If 8 reapers cut 15 acres of wheat in 16 days; in how many days will 40 reapers cut 240 acres, working at the same rate? Ans. 51⅕.
7. If seven reapers cut 16 acres of oats in 14 days, working 13 hours per day; how many acres will 78 reapers cut in 21 days, working 11 hours a day? Ans. 226⅔ acres.
8. When hay sells for 10d. per stone, and oats for 2s. 6d. per bushel, I pay £5 for keeping 2 horses 30 days; what will be the expense of keeping 8 horses for a year, when hay sells for 1/ per stone, and oats for 3/ per bushel? Ans. £350 8s.
9. If 24 men build a wall 20 yards long, 4 yds. high, 2 yds. thick, in 30 days, when the days are 12 hours long; in how many days will 120 men build a wall 200 yards long, 5½ yards high, and 4 yards thick, working 8 hours a-day? Ans. 240 days.
10. If 120 pioneers cut a trench 230 yards long, 8 feet deep,

12 feet wide at the top, and 9 feet wide at the bottom, in 8 days, working 10 hours a-day; how many pioneers must be employed to cut a trench 786 yards long, 12 feet deep, 16 feet wide at the top, and 14 feet wide at the bottom, in 10 days, working 12 hours a-day? Ans. 850[illegible].

11. If 300 men in ten days, cut 3000 acres of meadow, working 12 hours a-day; in how many days will 36 men cut 426 acres, working 14 hours a-day? Ans. $10\frac{1}{7}$ days.

12. If a vessel $3\frac{1}{2}$ feet deep, five feet long, and 4 feet wide contain 700 gallons; what must be the depth of a vessel which is 6 feet long, and 3 feet wide, to contain 978 gals.? Ans. 5 feet $5\frac{1}{5}$ in.

13. If 7 tailors, in three days, finish 8 suit of clothes, working 11 hours a-day; in how many days will 250 tailors finish 20000 suit of clothes, working 14 hours a-day? Ans. 165 days.

14. If 64 men consume £8s. worth of bread in 14 days, when wheat is 8/ per bushel, how much money will be required to supply bread to an army of 60000 men for 280 days, when wheat is 63/ per quarter? Ans. £147656 5s.

15. If an iron bar 5 feet long, $2\frac{1}{2}$ inches broad, and $1\frac{3}{4}$ inches thick, weigh 45 lb.; how much will a bar of pure gold weigh, which is 2 feet long, 5 inches broad, and $3\frac{1}{4}$ inches thick, the specific gravity of the iron being to that of the gold as 7788 is to 19640? Ans. 181 lb. 6 oz. 17 dwt. 4[illegible] gr.

16. If 2880 paving stones, each 15 inches long and 9 wide, be sufficient to pave 20 yards long, and 15 yards wide of a street; how many paving stones each 18 inches long and 14 wide, will pave $2\frac{1}{2}$ miles of a street 12 yds. wide? Ans. $271542\frac{6}{7}$.

17. If it require fifteen guineas to support 6 students for 3 weeks; how much money will support 2000 students for 24 weeks, including also £6 15s. each for class fees? Ans. £55500.

18. If 80000 cwt. of ammunition must be removed from a magazine in 9 days, and at the end of 6 days only 4500 cwt. is removed by 18 horses; how many additional horses must be employed to carry off the remainder in the 3 remaining days? Ans. 586.

19. If £20s. worth of wine is sufficient for a 100 men when wine is £30 per hhd.; what is the value of a hhd., when £4s. worth is sufficient for 25 men at the same rate of drinking? Ans. £24.

20. A garrison of 2500 men had provisions when besieged for

60 days, at the rate of 24 oz. per day to each man, but a reinforcement was immediately thrown into it of 360 men, with notice that they must hold out for 90 days; how much a day must each man have that the provisions may last that time? Ans. 13 [illegible] oz.

21. If £100 in a year gain £5; what will £750 gain in 75 days at the same rate? Ans. £7 14s. 1¼d. $\frac{19}{73}$.

22. If a family of ten persons require 14/ worth of bread in a week, when wheat sells at £4 a quarter; how much money will it require to purchase bread for the inhabitants of a city containing 60000 inhabitants, for a year, when wheat sells at £3 a quarter? Ans. £169250.

23. If 200 lb. at London make 190 lb. at Hamburgh, and 38 lb. at Hamburgh 50 lb. at Bologna; how many lbs. at London are equal to 100 lbs. at Bologna? Ans. 80. lb.

24. If 100 lb. at London make 109½ lb. at Amsterdam, and 50 lb. at Amsterdam make 58 lb. at Lyons, and 116 lb. at Lyons make 96 lb. at Rochelle, and 48 lb. at Rochelle make 59 lb. at Toulouse, and 118 lb. at Toulouse make 105 lb. at Leipsic; how many lb. at London are equal to 1000 lb. at Leipsic? Ans. 869 $\frac{3469}{4599}$ lb.

25. If 100 ells at London make 60 aunes at Amsterdam, and 100 aunes at Amsterdam make 120 aunes at Hamburg, and 60 aunes at Hamburg make [illegible]0 aunes at Geneva, and 60 aunes at Geneva make 33 canes at Rome, and 66 canes at Rome make 160 varas at Madrid, and 80 varas at Madrid make 102 brasses at Venice; how many ells at London are equal to 800 brasses at Venice?

Ans. 1111⅑ ells.

26. If 35 ells at Vienna make 24 ells at Lyons, and 3 ells at Lyons make 5 ells at Antwerp, and 100 ells at Antwerp make 125 at Frankfort; how many ells at Frankfort make 42 ells at Vienna? Ans. 60.

27. If £455 amount to £576 6s. 8d. in 4⅓ years; what is the rate per cent. per annum? Ans. £8.

28. What length of a board 4 inches broad and 2 inches thick can be cut out of a cubic foot? Ans. 18 feet.

29. If 60 yards of carpeting ell wide, carpet a room; how many French ells of Brussels carpeting 3 quarters wide will carpet a room three times as large? Ans. 200 French ells.

VULGAR FRACTIONS.

A FRACTION is a quantity, which represents one, or more of those equal parts, into which, any integer, or whole is divided.

A FRACTION is expressed by writing the number of equal parts which it represents above a line, and the number of those parts into which the integer is divided below the same line, thus, $\frac{4}{5}$, and is read four-fifths; and signifies four of those equal parts of which five make an integer, or whole.

The upper number of a fraction is called the NUMERATOR, and the under number the DENOMINATOR, thus $\frac{4}{5}$, 4 is the Numerator, and 5 the Denominator. They are also called the TERMS of the fraction.

A PROPER FRACTION has the numerator less than the denominator, as $\frac{6}{7}$.

AN IMPROPER FRACTION has the numerator equal to, or greater than the denominator, as $\frac{2}{2}$ or $\frac{8}{5}$.

A SIMPLE FRACTION is that which, unconnected with any other fraction, refers immediately to its integer, as $\frac{7}{8}$, $\frac{3}{5}$.

A COMPOUND FRACTION is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{1}{2}$.

A COMPLEX FRACTION is that which has a fraction or mixed number either in its numerator, or denominator, or in both, as $\frac{\frac{1}{3}}{4}$, or $\frac{6}{\frac{7}{8}}$, or $\frac{4\frac{1}{6}}{12}$, or $\frac{9\frac{3}{4}}{11\frac{2}{5}}$.

A MIXED NUMBER is a whole number with a fraction annexed, as $6\frac{2}{3}$.

A whole number may be reduced to a fractional form, by writing 1 for its denominator, as $8 = \frac{8}{1}$.

REDUCTION OF VULGAR FRACTIONS.

REDUCTION OF VULGAR FRACTIONS is that operation by which they are changed from one denomination to another, or from one denominator to another, or from one form to another, without altering their value.

Case 1.

To reduce a whole number to a fraction having a given denominator.

Rule.—Multiply the proposed number, by the given denominator, the product is the numerator of the required fraction, under which write the given denominator.

Example.—Reduce 11 to a fraction whose denominator is 12.

$11 \times 12 = \frac{132}{12}$. Ans. I here multiply 11 by 12, the product is 132 the numerator, under which I write 12, and the fraction is $\frac{132}{12}$.

EXERCISES.

1. Reduce 5 to a fraction whose denominator is 6. Ans. $\frac{30}{6}$.
2. Reduce 7 to a fraction whose denominator is 8. Ans. $\frac{56}{8}$.
3. Reduce 12 to a fraction whose denominator is 15. Ans. $\frac{180}{15}$.
4. Reduce 75 to a fraction whose denominator is 100. Ans. $\frac{7500}{100}$.
5. Reduce 43 to a fraction whose denominator is 111. Ans. $\frac{4773}{111}$.
6. Reduce 19 to a fraction whose denominator is 20. Ans. $\frac{380}{20}$.

Case 2.

To reduce a mixed number to an improper fraction.

Rule.—Multiply the whole number by the denominator of the fraction, adding its numerator to the product, under which write the fraction's denominator.

Example.—Reduce $5\frac{7}{8}$ to an improper fraction.

$(5 \times 8) + 7 = \frac{47}{8}$ I here multiply 5 the whole number, by 8 the denominator of the fraction, the product is 40, to which I add 7, the numerator of the fraction, the sum is 47, under which I write 8, the fraction's denominator, and we have $\frac{47}{8}$ the improper fraction required.

EXERCISES.

1. Reduce $6\frac{2}{3}$ to an improper fraction. Ans. $\frac{20}{3}$.
2. Reduce $12\frac{6}{7}$ to an improper fraction. Ans. $\frac{90}{7}$.
3. Reduce $14\frac{2}{5}$ to an improper fraction. Ans. $\frac{72}{5}$.
4. Reduce $18\frac{9}{10}$ to an improper fraction. Ans. $\frac{189}{10}$.
5. Reduce $16\frac{7}{8}$ to an improper fraction. Ans. $\frac{135}{8}$.
6. Reduce $3\frac{5}{12}$ to an improper fraction. Ans. $\frac{41}{12}$.

Case 3.

To Reduce an improper fraction to a whole, or mixed number.

Rule.—Divide the numerator by the denominator, the quotient is the whole, or mixed number.

EXAMPLE.—Reduce $\frac{37}{5}$ to a whole or mixed number.

$37 \div 5 = 7\frac{2}{5}$ I here divide 37 by 5, the integral quotient is 7, and 2 over, under 2, I write the denominator 5, and the entire quotient is $7\frac{2}{5}$, the mixed number required.

EXERCISES.

1. Reduce $\frac{35}{7}$ to a whole, or mixed number. Ans. 5.
2. Reduce $\frac{96}{8}$ to a whole, or mixed number. Ans. 12.
3. Reduce $\frac{49}{5}$ to a whole, or mixed number. Ans. $9\frac{4}{5}$.
4. Reduce $\frac{125}{12}$ to a whole, or mixed number. Ans. $10\frac{5}{12}$.
5. Reduce $\frac{1698}{112}$ to a whole, or mixed number. Ans. $15\frac{9}{56}$.
6. Reduce $\frac{18972}{567}$ to a whole, or mixed number. Ans. $33\frac{29}{63}$.

CASE 4.

To reduce a compound fraction to a simple fraction.

RULE.—Multiply all the numerators together, and all the denominators together for the numerator and denominator of the simple fraction.

EXAMPLE.—Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $3\frac{1}{2}$ to a simple fraction.

$\frac{2}{3} \times \frac{3}{5} \times \frac{7}{2} = \frac{42}{30} = \frac{7}{5}$, or by canceling, thus,

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{5} \times \frac{7}{\cancel{2}} = \frac{7}{5} \text{ as before.}$$

I first reduce the mixed number $3\frac{1}{2}$ to the improper fraction $\frac{7}{2}$. I then write the fractions with the sign of multiplication between them. The continued product of the numerators is 42, the new numerator, and the continued product of the denominators is 30, the new denominator, and $\frac{42}{30}$ divided by 6 gives $\frac{7}{5}$ the fraction required. By canceling, I here cancel 2 and 3 in the numerators, and 2 and 3 in the denominators, which leaves $\frac{7}{5}$ as before.

EXERCISES.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction. Ans. $\frac{1}{4}$.
2. Reduce $\frac{1}{8}$ of $\frac{4}{5}$ of $\frac{2}{7}$ to a simple fraction. Ans. $\frac{1}{35}$.
3. Reduce $\frac{1}{5}$ of $\frac{2}{7}$ of $3\frac{1}{2}$ to a simple fraction. Ans. $\frac{1}{5}$.
4. Reduce $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{7}{18}$ of $\frac{6}{14}$ of 8 to a simple fraction. Ans. $\frac{21}{20}$.
5. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $9\frac{3}{4}$ to a simple fraction. Ans. $\frac{13}{12}$.
6. Reduce $\frac{3}{5}$ of $\frac{9}{10}$ of $\frac{5}{9}$ of $\frac{2}{15}$ of $\frac{15}{16}$ of $5\frac{1}{3}$ to a simple fraction. Ans. $\frac{1}{5}$.

CASE 5.

To reduce a complex fraction to a simple one.

RULE.—Reduce both the numerator and denominator to a simple fraction, then multiply the numerator of each of these fractions by the denominator of the other, for the simple fraction required.

EXAMPLE.—Reduce $\frac{6\frac{2}{5}}{8}$, and $\frac{12\frac{3}{4}}{16\frac{1}{3}}$ to simple fractions.

$$\frac{6\frac{2}{5}}{8}=\frac{\frac{32}{5}}{\frac{8}{1}}=\frac{32}{40}=\frac{4}{5}. \qquad \frac{12\frac{3}{4}}{16\frac{1}{3}}=\frac{\frac{51}{4}}{\frac{49}{3}}=\frac{153}{196}.$$

I here reduce $6\frac{2}{5}$ to the improper fraction $\frac{32}{5}$, and 8 to $\frac{8}{1}$ which gives the complex fraction $\frac{\frac{32}{5}}{\frac{8}{1}}$. I then multiply 32 the numerator of the upper fraction by 1, the denominator of the under fraction gives 32, and 8 the numerator of the under fraction by 5 the denominator of the upper fraction, gives 40; therefore $\frac{32}{40}=\frac{4}{5}$ the simple fraction required. The second example is performed in the same manner.

EXERCISES.

Reduce $\frac{9\frac{3}{4}}{12}$, $\frac{8\frac{3}{5}}{14}$, $\frac{12\frac{6}{7}}{16}$, and $\frac{14\frac{9}{16}}{23}$, to simple fractions.

Ans. $\frac{13}{16}$, $\frac{43}{70}$, $\frac{45}{56}$, $\frac{233}{368}$.

Reduce $\frac{3\frac{3}{4}}{6\frac{1}{2}}$, $\frac{9\frac{1}{8}}{10\frac{1}{7}}$, $\frac{12\frac{1}{3}}{14\frac{3}{5}}$, and $\frac{16\frac{2}{3}}{17\frac{7}{12}}$, to simple fractions.

Ans. $\frac{15}{26}$, $\frac{511}{568}$, $\frac{185}{219}$, $\frac{600}{633}$.

CASE 6.

To reduce one fraction to another of equal value having a given numerator.

RULE.—As the numerator of the given fraction, is to the new numerator, so is its denominator, to the new denominator.

EXAMPLE.—Reduce $\frac{9}{14}$ to a fraction of equal value, whose numerator is 30.

As $9 : 30 :: 14 : 46\frac{2}{3}$, therefore the new fraction is $\frac{30}{46\frac{2}{3}}$.

This operation being performed by proportion requires no explanation.

EXERCISES.

1. Reduce $\frac{9}{10}$ to a fraction having its numerator 45. Ans. $\frac{45}{50}$.
2. Reduce $\frac{14}{20}$ to a fraction having its numerator 86. Ans. $\frac{86}{122\frac{6}{7}}$.
3. Reduce $\frac{6}{7}$ to a fraction having its numerator 54. Ans. $\frac{54}{63}$.
4. Reduce $\frac{18}{19}$ to a fraction having its numerator 108. Ans. $\frac{108}{114}$.
5. Reduce $\frac{25}{34}$ to a fraction having its numerator 225. Ans. $\frac{225}{306}$.
6. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ to a fraction having its numerator $8\frac{3}{4}$.

Ans. $\frac{8\frac{3}{4}}{16\frac{13}{32}}$.

Case 7.

To reduce one fraction to another of equal value, having a given denominator.

RULE.—As the denominator of the given fraction, is to the new denominator, so is its numerator, to the new numerator.

EXAMPLE.—Reduce $\frac{7}{8}$ to a fraction of equal value having its denominator 56. As 8 : 56 :: 7 : 49, therefore $\frac{49}{56}$ is the new fraction required.

This case is also performed by Simple Proportion, and requires no explanation here.

EXERCISES.

1. Reduce $\frac{4}{5}$ to a fraction having its denominator 35. Ans. $\frac{28}{35}$.
2. Reduce $\frac{9}{10}$ to a fraction having its denominator 85. Ans. $\frac{76\frac{1}{2}}{85}$.
3. Reduce $\frac{21}{36}$ to a fraction having its denominator 84. Ans. $\frac{49}{84}$.
4. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{5}$ to a fraction having its denominator 90. Ans. $\frac{18}{90}$.
5. Reduce $\frac{1}{4}$ of $\frac{2}{5}$ of $\frac{1}{6}$ of $3\frac{1}{3}$ to a fraction having its denominator 6. Ans. $\frac{\frac{1}{3}}{6}$.
6. Reduce $\frac{3}{4}$ of $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{5}{9}$ to a fraction having its denominator 27. Ans. $\frac{3}{27}$.

Case 8.

To reduce a fraction to its lowest terms.

RULE.—Divide its terms by any number which divides both without a remainder, divide the quotients in the same manner, and continue this process until they are divisible only by 1, the fraction is then in its lowest terms.

EXAMPLE.—Reduce $\frac{432}{1080}$ to its lowest terms.

$$\frac{432}{1080} \div 4 = \frac{108}{270} \div 6 = \frac{18}{45} \div 9 = \frac{2}{5} \text{ lowest terms.}$$

I here divide the terms by 4, which gives $\frac{108}{270}$, then by 6, gives $\frac{18}{45}$, and then by 9, the quotients of this last division are $\frac{2}{5}$; and as no number greater than 1, will now divide its terms, the fraction is in its lowest terms.

EXERCISES.

1. Reduce [illegible], [illegible], and [illegible] to their lowest terms. Ans. [illegible], $\frac{65}{108}$, $\frac{23}{35}$.
2. Reduce [illegible], $\frac{9720}{11880}$, and [illegible] to their lowest terms. Ans. [illegible], $\frac{9}{11}$, $\frac{7}{12}$.
3. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{15}{17}$, and $\frac{1}{3}$ of $\frac{3}{8}$ of $\frac{6\frac{2}{3}}{7\frac{1}{3}}$ to their lowest terms. Ans. $\frac{6}{17}$, $\frac{1}{12}$.

CASE 9.

To reduce fractions to a common denominator.

RULE.—Multiply each numerator into all the denominators except its own, for the new numerators, and multiply all the denominators together for the common denominator.

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{5}$ and $\frac{5}{6}$ to a common denominator.

$$\left.\begin{array}{l} \left.\begin{array}{l} 2\times5\times6=60 \\ 3\times3\times6=54 \\ 5\times5\times3=75 \end{array}\right\} = \text{New numerators.} \\ 3\times5\times6=90=\text{Common denomin.} \end{array}\right\} \tfrac{60}{90}, \tfrac{54}{90}, \tfrac{75}{90}, \text{ or } \tfrac{20}{30}, \tfrac{18}{30}, \tfrac{25}{30}.$$

I here multiply 2, the first numerator, by 5, and 6, the other denominaters gives 60, then 3 the second numerator by 3 and 6 the other denominators gives 54, again 5 the last numerator by 5 and 3 the other denominators gives 75, therefore 60, 54 and 75 are the new numerators, and the denominators 3, 5 and 6 multiplied together give 90 the common denominator; therefore $\frac{60}{90}$, $\frac{54}{90}$, $\frac{75}{90}$ is the fraction required, having a common denominator. But we can divide the terms of all these fractions by 3, which gives $\frac{20}{30}$, $\frac{18}{30}$, $\frac{25}{30}$, their lowest common denominator.

EXERCISES.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.
Ans. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$.
2. Reduce $\frac{3}{5}$, $\frac{5}{7}$, and $\frac{5}{8}$ to a common denominator.
Ans. $\frac{168}{280}$, $\frac{200}{280}$, $\frac{175}{280}$.
3. Reduce $\frac{1}{5}$, $\frac{2}{7}$ of $\frac{3}{4}$, and $\frac{1}{8}$ of 5 to a common denominator.
Ans. $\frac{168}{840}$, $\frac{180}{840}$, $\frac{525}{840}$.
4. Reduce $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{6}{7}$, $\frac{2}{5}$, $\frac{5}{12}$ of $\dfrac{6\frac{1}{2}}{9}$ to a common denominator.
Ans. $\frac{2520}{7560}$, $\frac{6480}{7560}$, $\frac{3024}{7560}$, $\frac{2275}{7560}$.
5. Reduce $\frac{4}{5}$ of $\frac{6}{7}$, $\frac{1}{3}$ of $\frac{1}{5}$, and $\frac{3}{4}$ of $\frac{1}{5}$ of $2\frac{1}{2}$ to a common denominator.
Ans. $\frac{576}{840}$, $\frac{56}{840}$, $\frac{315}{840}$.
6. Reduce $\frac{2}{3}$ of 4, $\frac{1}{5}$ of 6, and $\frac{9}{10}$ of $\dfrac{7\frac{1}{4}}{8\frac{1}{3}}$ to a common denominator.
Ans. $\frac{800}{300}$, $\frac{360}{300}$, $\frac{261}{300}$.
7. Reduce $\frac{9}{10}$, $\frac{4}{5}$, $\frac{6}{15}$, $\frac{4}{25}$, and $\frac{7}{30}$ to a common denominator.
Ans. $\frac{135}{150}$, $\frac{120}{150}$, $\frac{60}{150}$, $\frac{24}{150}$, $\frac{35}{150}$.

CASE 10.

To reduce fractions from one denomination to another without altering their value.

RULE.—Convert the given fraction into a compound fraction of the denomination required, then reduce it to a simple fraction.

EXAMPLES.—Reduce $\frac{2}{5}$ of a £ to the fraction of a farthing; and $\frac{2}{5}$ of a farthing to the fraction of a £.

$$1st.\ \frac{2}{\cancel{5}} \text{ of } \frac{\overset{4}{\cancel{20}}}{1} \text{ of } \frac{12}{1} \text{ of } \frac{4}{1} = \frac{384}{1}\ \text{Ans.}$$

$$2d.\ \frac{\cancel{2}}{5} \text{ of } \frac{1}{\underset{2}{\cancel{4}}} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} = \frac{1}{2400}\ \text{Ans.}$$

1st. In working this example I write it down $\frac{2}{5}$ of $\frac{20}{1}$ because 20 shillings make a £, then of $\frac{12}{1}$ because 12 pence make a shilling, then $\frac{4}{1}$ because 4 farthings make one penny, I then cancel the 5 and the 20, and reducing to a simple fraction by case 4, gives $\frac{384}{1}$ the fraction required.

In the second example I compare it in the same manner, beginning with the farthings, thus, $\frac{2}{5}$ of $\frac{1}{4}$, because $\frac{1}{4}$ is the fourth of a penny, then, of $\frac{1}{12}$, because a penny is the twelfth part of a shilling, then of $\frac{1}{20}$ because a shilling is the twentieth part of a £; then canceling the 2 and the 4 and reducing to a simple fraction, gives $\frac{1}{2400}$ the fraction required.

EXERCISES.

1. Reduce $\frac{7}{8}$ of a £, to the fraction of a penny. Ans. $\frac{210}{1}$.
2. Reduce $\frac{2}{7}$ of a farthing, to the fraction of a £. Ans. $\frac{1}{3360}$.
3. Reduce $\frac{4}{5}$ of a £, to the fraction of a farthing. Ans. $\frac{768}{1}$.
4. Reduce $\frac{2}{3}$ of a £, to the fraction of a guinea. Ans. $\frac{40}{63}$.
5. Reduce $\frac{2}{5}$ of a guinea, to the fraction of a £. Ans. $\frac{21}{50}$.
6. Reduce $\frac{5}{7}$ of a crown, to the fraction of a guinea. Ans. $\frac{25}{147}$.
7. Reduce $\frac{8}{9}$ of a lb. Troy, to the fraction of a grain. Ans. $\frac{5120}{1}$.
8. Reduce $\frac{4}{5}$ of a grain, to the fraction of a lb. Apoth. Ans. $\frac{1}{7200}$.
9. Reduce $\frac{1}{7}$ of a ton, to the fraction of a dram. Ans. $\frac{81920}{1}$.
10. Reduce $\frac{2}{7}$ of a lb., to the fraction of a last of wool. Ans. $\frac{1}{15288}$.
11. Reduce $\frac{1}{5}$ of a nail, to the fraction of a yard. Ans. $\frac{1}{80}$.
12. Reduce $\frac{2}{3}$ of a foot, to the fraction of a league. Ans. $\frac{1}{23760}$.
13. Reduce $\frac{5}{9}$ of a square inch, to the fraction of an acre. Ans. $\frac{1}{11290752}$.
14. Reduce $\frac{5}{12}$ of a gal., to the fraction of a chaldron. Ans. $\frac{5}{3072}$.
15. Reduce $\frac{15}{16}$ of a quart, to the fraction of a ton of wine. Ans. $\frac{5}{5376}$.
16. Reduce $\frac{2}{3}$ of a hhd. ale, to the fraction of a pint. Ans. $\frac{864}{3}$.

CASE II.

To reduce a quantity to a fraction of any given denomination.

RULE.—Reduce the given quantity to its lowest name for a numerator, and the proposed integer to the same name for the denominator of the required fraction.

EXAMPLE.—Reduce 2 qrs. 16 lb. to the fraction of a cwt.

Qrs.	lb.	cwt.
2	16	1
28		4
72		4
		28
		112

$$\frac{72}{112}=\frac{9}{14} \text{ Ans.}$$

I here reduce 2 qrs. 16 lb., to lbs., which gives 72 for the numerator, and one cwt. reduced to lbs. gives 112 for the denominator, the fraction is therefore $\frac{72}{112}$, which reduced to its lowest terms, is $\frac{9}{14}$.

EXERCISES.

1. Reduce 4/8¼ to the fraction of a £. Ans. $\frac{15}{64}$.
2. Reduce 5/3 to the fraction of a guinea. Ans. $\frac{1}{4}$.
3. Reduce 6 oz. 10 dwts. 6 grs. to the fraction of a lb. Ans. $\frac{521}{960}$.
4. Reduce $\frac{4}{5}$ of $\frac{5}{7}$ of 2 oz. 3 drs. 2 scr. to the fraction of a lb. Ans. $\frac{59}{504}$.
5. Reduce 3 qrs. 10 lb. 12 oz. to the fraction of a ton. Ans. $\frac{379}{8960}$.
6. Reduce 2 tods 1 stone 6 lb. to the fraction of a sack. Ans. $\frac{19}{91}$.
7. Reduce 2 m. 6 fur. 10 p. to the fraction of a league. Ans. $\frac{89}{96}$.
8. Reduce 3 qrs. 2 n. to the fraction of an ell Eng. Ans. $\frac{7}{10}$.
9. Reduce 2 roods 30 poles to the fraction of an acre. Ans. $\frac{11}{16}$.
10. Reduce 2 qrs. 5 bu. 2 p. to the fraction of a chaldron. Ans. $\frac{43}{64}$.
11. Reduce 14 gal. 2 qts. to the fraction of a hhd. of wine. Ans. $\frac{29}{126}$.
12. Reduce 1 kil. 8 gal. 2 qts. to the fraction of a barrel of ale Ans. $\frac{53}{64}$.
13. Reduce 15 d. 16 h. 40 m. to the fraction of a solar year. Ans. $\frac{1356000}{31556928}$.
14. Reduce 3 h. 28 min. to the fraction of a day. Ans $\frac{13}{90}$.

Case 12.

To value vulgar fractions.

Rule.—Divide the numerator by the denominator as in compound division, the quotient is the value of the fraction.

Example.—What is the value of $\frac{7}{8}$ of a £.

£	s.	d.
8|7	0	0
0	17	6

or thus

```
     7
    20   s.  d.
8)140(17  6
   8
   60
   56
    4
   12
   48
   48
```

I here divide the numerator considered as £7 by the denominator, thus, the eights in 7 go no times, multiply by 20 gives 140, the eights in which, go 17 times and 4 remainder, and multiplied by 12 gives 48, and divided by 8 gives 6, therefore 17/6. is the value of the fraction by both methods.

EXERCISES.

Value.

	Answer.		Answer.
1. $\frac{3}{5}$ of a £.	12/	10. $\frac{3}{8}$ of a yd.	1 qr. 2 n.
2. $\frac{5}{7}$ of a guinea.	15/	11. $\frac{1}{3}$ of a French ell.	2 qrs.
3. $\frac{7}{12}$ of a crown.	2/11.	12. $\frac{4}{5}$ of an English ell.	1 yd.
4. $\frac{14}{15}$ of a sovereign.	18/8.	13. $\frac{2}{3}$ of a Flemish ell.	2 qrs.
5. $\frac{7}{8}$ of a lb. troy.	10 oz. 10 dwt.	14. $\frac{7}{16}$ of a qr. of corn.	3 bu. 2 p.
6. $\frac{6}{7}$ of a dr. apoth.	2 sc. 11$\frac{3}{7}$ g.	15. $\frac{4}{7}$ of a ton of wine.	1 p. 18 g.
7. $\frac{3}{4}$ of a ton.	15 cwt.	16. $\frac{1}{9}$ of a hhd. of beer.	6 gal.
8. $\frac{9}{10}$ of a mile.	7 f. 8 p.	17. $\frac{2}{3}$ of a day.	16 hours.
9. $\frac{3}{5}$ of an acre.	2 r. 16 p.	18. $\frac{9}{10}$ of a mark Scots.	1s. ster.

ADDITION OF VULGAR FRACTIONS.

Addition of vulgar fractions is the method of collecting several given fractions into one sum.

Rule.—Reduce all the given fractions to simple fractions, to the same denomination, and to a common denominator, then add the numerators, and under their sum, write the common denominator, for the sum required.

1. Example.—Add together $\frac{1}{2}$, $\frac{1}{3}$ of $\frac{3}{4}$, and $\frac{1}{3}$.

$\frac{1}{2}+\frac{1}{3}$ of $\frac{3}{4}+\frac{1}{3}=\frac{1}{2}+\frac{1}{4}+\frac{1}{3}=\frac{6}{12}+\frac{3}{12}+\frac{4}{12}=1\frac{1}{12}$. Ans.

I first reduce the compound fraction $\frac{1}{3}$ of $\frac{3}{4}$ to a simple fraction, which in its lowest terms is $\frac{1}{4}$, I then reduce $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$ to a common denominator, which gives $\frac{6}{12}+\frac{3}{12}+\frac{2}{12}$, I then add the numerators 6, 3, 2, gives 11 for the numerator, under which I write 12 the common denominator; therefore the sum of $\frac{1}{2}+\frac{1}{3}$ of $\frac{3}{4}+\frac{1}{6}$, is $\frac{11}{12}$.

2. Example.—Add together $\frac{1}{2}$ of $\frac{1}{3}$ of a £, $\frac{5}{9}$ S., and $\frac{2}{3}$ of $\frac{3}{4}$ of a penny.

$$\left.\begin{array}{l} \frac{1}{2} \text{ of } \frac{1}{3}\ £ = \frac{1}{6}\ £ = \frac{40}{1}\text{d.} \\ \frac{5}{9}\text{s.} \quad = \quad \frac{20}{3}\text{d.} \\ \frac{2}{3} \text{ of } \frac{3}{4}\text{d.} = \frac{6}{12} = \frac{1}{2}\text{d.} \end{array}\right\} \text{ then } \frac{40}{1}+\frac{20}{3}+\frac{1}{2}=\frac{240}{6}+\frac{40}{6}+\frac{3}{6}=\frac{283}{6}\text{d.} = 47\tfrac{1}{6}\text{d.} = 3\text{s. } 11\tfrac{1}{6}\text{d. Ans.}$$

I first reduce all the fractions to simple fractions of a penny, which gives $\frac{40}{1}$, $\frac{20}{3}$ and $\frac{1}{2}$, I then reduce these fractions to a common denominator, which gives $\frac{240}{6}+\frac{40}{6}+\frac{3}{6}$, and these added give the improper fraction $\frac{283}{6}$, and reduced gives $47\frac{1}{6}$d. = 3s. $11\frac{1}{6}$d. Ans. Examples of this kind are generally wrought, by valuing the fractions and adding their values.

EXERCISES.

Add together.

1. $\frac{7}{13}$, $\frac{4}{13}$, $\frac{2}{13}$. — Ans. 1.
2. $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$. — Ans. $\frac{6}{7}$.
3. $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$. — Ans. 2.
4. $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$. — Ans. $1\frac{7}{12}$.
5. $\frac{1}{5}$, $\frac{2}{3}$ of $\frac{4}{5}$, $\frac{1}{2}$ of $\frac{2}{3}$, and $\frac{2}{5}$. — Ans. $1\frac{49}{120}$.
6. $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{7}$, and $\frac{2}{3}$ of $\frac{3}{5}$. — Ans. $2\frac{13}{420}$.
7. $\frac{1}{8}$, $\frac{2}{3}$ of $\frac{5}{6}$, and $\frac{7}{8}$ of $\frac{1}{7}$ of 3. — Ans. $1\frac{1}{18}$.
8. $\frac{1}{12}$, $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{3}{5}$. — Ans. $2\frac{11}{24}$.
9. $40\frac{3}{7}$, $62\frac{5}{8}$, and $35\frac{3}{4}$. — Ans. $138\frac{45}{56}$.
10. $\frac{3}{4}$ £, $\frac{3}{4}$sh. $\frac{3}{4}$d. — Ans. £$\frac{253}{320}$ or 15/9$\frac{3}{4}$d.
11. $\frac{3}{4}$ guin. $\frac{5}{6}$£ and $\frac{3}{5}$s. — Ans. $\frac{1981}{60}$s. = £1 $13\frac{1}{60}$s.
12. $\frac{1}{7}$ guin. $\frac{2}{3}$ cr. and $\frac{1}{6}$ noble. — Ans. £$\frac{67}{180}$ = 7/5$\frac{1}{4}$d. $\frac{1}{3}$.
13. $\frac{1}{7}$ ton, and $\frac{7}{8}$ cwt. — Ans. $\frac{209}{1120}$ t. = 3 cwt. 2 qr. 26 lb.
14. $\frac{3}{8}$ yd., and $\frac{2}{5}$ Eng. ell. — Ans. $\frac{7}{8}$ yd. = 3 qr. 2 n.
15. $\frac{1}{8}$ lea., and $\frac{2}{3}$ of $\frac{3}{4}$ miles. — Ans. $\frac{4}{4}$ m. = 1 mile.
16. $\frac{5}{8}$ ton $\frac{2}{3}$ hhd. $\frac{2}{5}$ ank. — Ans. $\frac{439}{504}$ t. = 3 hhd. 14 gal. 2 qt.

SUBTRACTION OF VULGAR FRACTIONS.

Subtraction of vulgar fractions is the method of finding the difference between two fractions.

Rule.—Reduce the fractions to a common denominator as directed for addition, then take the numerator of the one

from the numerator of the other, and under their difference write the common denominator.

EXAMPLE.—From $\frac{7}{8}$ of $\frac{9}{10}$ take $\frac{5}{7}$.

$\frac{7}{8}$ of $\frac{9}{10}-\frac{5}{7}=\frac{63}{80}-\frac{5}{7}=\frac{441}{560}-\frac{400}{560}=\frac{41}{560}$ Ans.

In this example I first reduce $\frac{7}{8}$ of $\frac{9}{10}$ to the simple fraction $\frac{63}{80}$, I then reduce $\frac{63}{80}$ and $\frac{5}{7}$ to a common denominator, which gives $\frac{441}{560}$ and $\frac{400}{560}$, and $\frac{441}{560}-\frac{400}{560}=\frac{41}{560}$ the difference of the given fractions.

EXERCISES.

1. From $\frac{7}{8}$ take $\frac{3}{8}$.	Ans. $\frac{1}{2}$.
2. —— $\frac{2}{3}$ take $\frac{2}{5}$.	Ans. $\frac{4}{15}$.
3. —— $\frac{1}{5}$ take $\frac{1}{6}$.	Ans. $\frac{1}{30}$.
4. —— $\frac{9}{14}$ take $\frac{1}{2}$ of $\frac{5}{7}$.	Ans. $\frac{2}{7}$.
5. —— $\frac{2}{3}$ of $\frac{1}{8}$ take $\frac{1}{4}$ of $\frac{2}{9}$.	Ans. $\frac{1}{36}$.
6. —— $\frac{1}{8}$ of $3\frac{1}{2}$ take $\frac{3}{5}$ of $\frac{1}{6}$.	Ans. $\frac{27}{80}$.
7. —— $4\frac{1}{2}$ take $2\frac{1}{8}$.	Ans. $2\frac{3}{8}$.
8. —— $6\frac{1}{7}$ take $3\frac{35}{128}$.	Ans. $2\frac{109}{128}$.
9. —— £$\frac{3}{5}$ take $\frac{7}{13}$ cr.	Ans 9/8.
10. —— $\frac{9}{10}$ cr. take $\frac{3}{14}$ guin.	Ans. 0.
11. —— $\frac{125}{364}$ lb. troy, take $\frac{36}{85}$ oz.	Ans. $3\frac{5394}{7735}$ oz.
12. —— $\frac{7}{8}$ yd. take $\frac{3}{20}$ Eng. ell.	Ans. 2 qr. 3 n.
13. —— $\frac{1}{5}$ leag. take $\frac{7}{15}$ mile.	Ans. 1 f. 2 p. 3 yds. 2 f.
14. —— $\frac{5}{9}$ hhd. take $\frac{2}{3}$ of $\frac{3}{4}$ tierce.	Ans. 14 gallons.
15. —— $\frac{9}{16}$ qr. take 3 $\frac{3}{4}$ bu.	Ans. 3 pecks.
16. —— $\frac{5}{9}$ bar. take $\frac{2}{9}$ fir.	Ans. 2 fir.

MULTIPLICATION OF VULGAR FRACTIONS.

To multiply any number or fraction by a fraction, is to find such a part of it as the multiplier is of unity.

RULE.—Reduce all to simple fractions; then multiply all the numerators together for the numerator, and all the denominators together for the denominator of the product.

EXAMPLE.— $\frac{2}{15}$ of $6\frac{1}{2}\times\frac{3}{7}$.

$\frac{2}{15}$ of $6\frac{1}{2}\times\frac{3}{7}=\frac{26}{30}\times\frac{3}{7}=\frac{78}{210}=\frac{39}{105}=\frac{13}{35}$. Ans.

I first reduce $\frac{2}{15}$ of $6\frac{1}{2}$ to the simple fraction $\frac{26}{30}$, I then multiply 26 the numerator by 3, and 30 the denominator by 7, the products are $\frac{78}{210}$ which reduced to its lowest terms gives $\frac{13}{35}$ the product required.

EXERCISES.

REQUIRED THE PRODUCT OF

	Ans.		Ans.
1. $\frac{9}{16} \times \frac{16}{17}$.	$\frac{9}{17}$.	9. $35\frac{5}{8} \times 15\frac{7}{15}$.	551.
2. $\frac{10}{21} \times \frac{7}{8}$.	$\frac{5}{12}$.	10. £$3\frac{8}{13} \times 2\frac{7}{25}$.	£8 $4\frac{56}{65}$s.
3. $\frac{3}{8}$ of $\frac{4}{7} \times \frac{1}{6}$ of $\frac{2}{5}$.	$\frac{1}{70}$.	11. $\frac{3}{4}$ yd. $\times \frac{4}{15}$.	$\frac{1}{5}$ yd.
4. $\frac{3}{5}$ of 6 $\times \frac{6}{7}$ of $\frac{1}{3}$.	$1\frac{1}{35}$.	12. $\frac{5}{24}$ mile $\times \frac{1}{3}$ of $\frac{3}{5}$.	$13\frac{1}{3}$ pole.
5. $\frac{12}{17} \times \frac{5}{12}$ and $\frac{17}{35}$	$\frac{1}{7}$.	13. $\frac{7}{8}$ lb. troy by $\frac{4}{35}$.	1 oz. 4 dwt.
6. $\frac{11}{12} \times 1\frac{1}{11}$.	1.	14. $4\frac{1}{3}$ hhd by $\frac{4}{63}$.	$17\frac{1}{3}$ gal.
7. $65\frac{1}{4} \times \frac{16}{33}$.	$31\frac{7}{11}$.	15. $\frac{15}{34}$ day by $3\frac{7}{9}$.	1 d. 16 h.
8. $175 \times \frac{2}{15}$ of $\frac{1}{30}$.	$\frac{7}{9}$.	16. $\frac{1}{2} \times \frac{2}{3} \times \frac{8}{15} \times \frac{5}{12} \times \frac{3}{8}$.	$\frac{1}{36}$.

DIVISION OF VULGAR FRACTIONS.

DIVISION OF VULGAR FRACTIONS is the method of finding how often, or what part of a time one fraction contains another.

RULE.—Reduce all to simple fractions, then invert the divisor, and proceed as in multiplication, the result is the quotient required.

EXAMPLE.—Divide $\frac{18}{35}$ by $\frac{6}{7}$ of $\frac{1}{4}$.

$$\frac{18}{35} \div \frac{6}{7} \text{ of } \frac{1}{4} = \frac{\overset{6}{\cancel{18}}}{35} \times \frac{14}{\cancel{3}} = \frac{84}{35} = \frac{12}{5} = 2\frac{2}{5} \text{ Ans.}$$

I here reduce $\frac{6}{7}$ of $\frac{1}{4}$ to the simple fraction $\frac{3}{14}$, which being inverted becomes $\frac{14}{3}$, I then multiply 18 by 14, and 35 by 3, which reduced to their lowest terms gives $\frac{12}{5} = 2\frac{2}{5}$ for the quotient.

EXERCISES.

REQUIRED THE QUOTIENT OF

1. $\frac{4}{5} \div \frac{6}{5}$.	Ans. $\frac{2}{3}$.	9. $\frac{6}{7}$ of $\frac{3}{4} \div \frac{27}{35}$.	Ans $\frac{5}{6}$.
2. $\frac{3}{4} \div \frac{7}{8}$.	Ans. $\frac{6}{7}$.	10. $29 \div \frac{3}{7}$.	Ans. $67\frac{2}{3}$.
3. $\frac{8}{9} \div \frac{15}{35}$.	Ans. $2\frac{2}{27}$.	11. $17\frac{1}{2} \div 6\frac{1}{4}$.	Ans. $2\frac{4}{5}$.
4. $\frac{8}{45} \div \frac{7}{15}$.	Ans. $\frac{8}{21}$.	12. $\frac{5}{8} \div 5\frac{1}{3}$.	Ans. $\frac{15}{128}$.
5. $\frac{13}{16} \div \frac{27}{32}$.	Ans. $\frac{26}{27}$.	13. £$\frac{3}{4} \div \frac{6}{7}$.	Ans. 17/6.
6. $6\frac{3}{8} \div \frac{63}{88}$.	Ans. $8\frac{19}{21}$.	14. $\frac{1}{8}$ lb. troy $\div \frac{7}{8}$.	Ans. $\frac{1}{7}$ lb.
7. $\frac{9}{14} \div 6$.	Ans. $\frac{3}{28}$.	15. $\frac{5}{8}$ cwt. $\div 8\frac{7}{16}$.	Ans. $\frac{2}{27}$ cwt.
8. $56 \div \frac{8}{45}$.	Ans. 315.	16. $\frac{3}{8}$ of a ship $\div$ 15.	Ans. $\frac{1}{40}$.

17. 365 acres $\div 6\frac{1}{4}$. Ans. 58 ac. 1 r. 24 p.

18. What is the sum, difference, product, and quotient of $\frac{7}{8}$ and $\frac{5}{6}$?
Ans. $1\frac{17}{24}$ sum. $\frac{1}{24}$ diff. $\frac{35}{48}$ pr. $\frac{20}{21}$ quot.

VULGAR FRACTIONS.

RULE.—Having stated the question as directed in whole numbers, reduce all the terms to simple fractions, and the two first to the same denomination; then invert the first term and proceed as in multiplication, the product is the answer.

EXAMPLE.—If $\frac{7}{8}$ yd. cost £$\frac{11}{15}$; what will $8\frac{3}{5}$ En. ell cost?

$$\text{As } \overset{\text{yd.}}{\frac{7}{8}} : \frac{43}{4} :: \frac{11}{15} : \frac{\overset{2}{\cancel{8}}}{7}\times\frac{43}{\cancel{4}}\times\frac{11}{15} = £\frac{946}{105} = £9\ 0s.\ 2\tfrac{1}{4}d.\ \tfrac{1}{7}.\ \text{Ans.}$$

I first reduce $8\frac{3}{5}$ En. ells to the fraction of a yard, which gives $\frac{43}{4}$ yd., and having stated as above, I represent the 4th term or answer, by the product of the reciprocal of the first term into the other two; I then cancel the 8, and 4, and multiplying the upper terms together, and the under terms together, gives the improper fraction £$\frac{946}{105}$=£9 0s. $2\frac{1}{4}$d. $\frac{1}{7}$.

EXERCISES.

1. If I pay at the rate of £$\frac{3}{5}$ for $\frac{3}{7}$ oz.; what will a service of plate cost, which weighs 64 lb. $6\frac{3}{7}$ oz? Ans. £1084 12s.
2. A gentleman was left $\frac{5}{8}$ of a ship, and having sold $\frac{2}{3}$ of his share for £375, he wishes to know what part of her now belongs to him, what it is worth, and the value of the whole ship at the same rate?
   Ans. $\frac{5}{24}$, worth £187 10s. Ship worth £900.
3. A merchant holds $\frac{5}{18}$ of a ship worth £3560 17s., but has granted security upon it to the amount of £395 13s.; what part of her is still free? Ans. $\frac{1}{6}$.
4. How many yards of paper $\frac{5}{9}$ yd. wide will paper a room $3\frac{1}{3}$ yds. high, and $34\frac{1}{8}$ yds. in circuit, deducting 16 yards for doors, windows, and fire place? Ans. 189.
5. A grocer bartered $9\frac{7}{8}$ cwt. sugar at $7\frac{1}{4}$d. per lb., for tobacco at 3/$3\frac{1}{2}$ per lb.; how much tobacco should he receive for his sugar? Ans. 1 cwt. 3 q. 7lb.
6. If 25 men can perform a piece of work in $14\frac{3}{8}$ days; how long will they take to do it, when each brings a boy to assist him who does $\frac{1}{7}$ of a man's work? Ans. $12\frac{37}{64}$ days.
7. If three-fourths of a yard of silk cost $\frac{1}{5}$ of $\frac{5}{6}$ £; what will $17\frac{5}{8}$ yards cost at the same rate? Ans. £3 18s. 4d.
8. How much cloth $\frac{5}{8}$ yd. broad will make a cloak equal to one in which there are $3\frac{1}{2}$ yds., 7 qrs. broad? Ans. 9 yds. $3\frac{1}{5}$ qr.
9. Bought $3\frac{1}{2}$ pieces of silk each $24\frac{5}{8}$ En. ells, at 6/4 per yard; what did the whole come to? Ans. £34 2s. $3\frac{3}{4}$d. $\frac{1}{4}$.

10. Paid £$2\frac{1}{8}$ for the carriage of $3\frac{1}{4}$ cwt; what must I pay for the carriage of $15\frac{5}{8}$ cwt. at that rate? Ans. £10 4s. $3\frac{3}{4}$d. $\frac{1}{13}$
11. If 48 masons can build a wall in $24\frac{3}{8}$ days; how long will they take to build the same wall when assisted by 5 apprentices each working $\frac{3}{4}$ of a man's work? Ans. $22\frac{14}{23}$.
12. If a farmer can engage men at 1/7 per day, and women at 1/2 per day; which is the most profitable, supposing a woman can do $\frac{5}{8}$ of a man's work?

Ans. women cheapest by $\frac{1}{114}$.

COMPOUND PROPORTION IN VULGAR FRACTIONS.

RULE.—State the question as directed in whole numbers. Reduce the numbers in the first and second columns to the same denomination in pairs, and all to simple fractions, then invert the fractions in the first column and proceed as in multiplication, the product is the answer, which may be valued or reduced if necessary.

EXAMPLE.—If 3 men, in $6\frac{1}{2}$ days, cut $16\frac{1}{5}$ acres of wheat; how much will they cut in $15\frac{5}{9}$ days, when assisted by their wives each working $\frac{5}{6}$ of their husbands?

$$\begin{matrix} \frac{3}{1} : 5\frac{1}{2} \\ 6\frac{1}{2} : 15\frac{5}{9} \end{matrix} :: 16\frac{1}{5} = \frac{1}{\cancel{3}} \times \frac{\cancel{2}}{13} \times \frac{11}{\cancel{2}} \times \frac{\overset{28}{\cancel{140}}}{\cancel{9}} \times \frac{\overset{3}{\cancel{81}}}{\cancel{5}} = \frac{924}{13} =$$

$71\frac{1}{13}$ Acres. Ans.

Having stated the question, I represent the answer by the reciprocal of the fractions in the first column multiplied into all the other fractions, then by canceling and multiplying, I have the improper fraction $\frac{924}{13} = 71\frac{1}{13}$ acres.

EXERCISES.

1. If £$60\frac{2}{5}$, in $\frac{3}{5}$ year, gain £$1\frac{1}{5}$; what will £$376\frac{1}{5}$ gain in $2\frac{1}{4}$ years? Ans. £$28\frac{17}{604}$.
2. If 32 men, in $18\frac{2}{7}$ days of $10\frac{1}{2}$ hours each, cut a trench; in how many days of $12\frac{1}{3}$ hours long, can 96 men perform the same work? Ans. $5\frac{7}{37}$ days.
3. If a thousand men were found sufficient to fortify a camp, with a ditch and rampart, in $30\frac{3}{8}$ days, working $9\frac{1}{2}$ hours a day; in how man days will 5640 men fortify a camp three

times as large as the former, working $11\frac{4}{7}$ hours a day, and the ground harder in the proportion of $\frac{5}{9}$ to $\frac{5}{8}$?

Ans. $19\frac{118}{3807}$ days.

4. If a bar of iron 7 feet long, $3\frac{1}{8}$ inches broad, and $\frac{9}{10}$ inches thick, weigh $40\frac{1}{2}$ lb; what will a bar of steel weigh, which is 6 feet long, $3\frac{1}{2}$ inches broad, and $1\frac{1}{8}$ inches thick, supposing the specific gravity of the steel be to that of the iron as 109 to 108?

Ans. $49\frac{1}{20}$ lb.

DECIMAL FRACTIONS.

A Decimal Fraction is that whose value decreases from unity in the same ratio as the value of whole numbers decreases to unity; and is always distinguished by a point on its left.

The Notation, and Numeration of decimals is the same as whole numbers, which will be seen from the following scale.

Decimal Numeration Scale.

222222.22222

Beginning with the place of units, the first figure on the left is tens, the second hundreds, &c. as already explained. Again beginning with units, the first figure on the right is tenth, the second hundredth, the third thousandth parts of unity, &c. and may be read 2 tenths, 2 hundredths, 2 thousandths, 2 ten thousandths, 2 hundred thousandths.

But it may still be read easier thus, 22222 hundred thousandth parts of unity. It is here evident that the first figure on the left of unity is ten times its single value, and that the first figure on the right of unity is the tenth part of its single value, and so on of all other figures equally distant from the place of units.

Ciphers on the right hand of decimals do not alter their value for ·2 and ·20 are both two tenths, neither do ciphers on the left of whole numbers alter their value thus 02· and 2. are both equal to two.

But since decimals decrease in a tendfold ratio as they recede from the decimal point, it is evident that ciphers on the left of them must diminish their value in the same proportion as they increase whole numbers when placed on the right, for ·02 is only the tenth part of ·2 and ·002 is only the hundredth part of ·2.

Decimals are either TERMINATE or INTERMINATE.

A TERMINATE DECIMAL is one which extends only to a limited number of places, as .5, .25, .875, &c.

INTERMINATE DECIMALS are such as would extend ad infinitum, and are divided into the four following classes.

1st. PURE REPEATERS, which are distinguished by a point over them, thus $\cdot\dot{6}$.

2d. MIXED REPEATERS, which are distinguished by a point over the repeating figure, thus $\cdot 32\dot{7}$.

3d. PURE CIRCULATES, which are distinguished by a point over the first and last figures, thus $\cdot\dot{2}85\dot{1}$.

4th. MIXED CIRCULATES, which are distinguished by a point over the first and last figures of the circle, thus $\cdot 63\dot{4}2\dot{6}$.

CIRCULATES ARE SIMILAR when they have the same number of places, thus $\cdot\dot{6}4\dot{2}$ and $\cdot\dot{0}3\dot{5}$.

CIRCULATES ARE COTERMINOUS when they begin and end at the same distance from the decimal point, thus, $\cdot 5\dot{4}3\dot{7}$ and $\cdot 2\dot{4}9\dot{1}$.

From the explanation given of decimal notation it is evident that the several operations of addition, subtraction, multiplication, and division must be performed the same way, and by the same rules as whole numbers.

REDUCTION OF DECIMALS.

REDUCTION OF DECIMALS is the method of converting vulgar fractions into decimals; lower denominations, into the decimal of higher denominations; and decimals into vulgar fractions, or integers of lower denominations without altering their value.

CASE 1.

To reduce a vulgar fraction into a decimal fraction.

RULE.—Divide the numerator by the denominator, annexing ciphers when necessary; the number of decimal places in the quotient, is always equal to the number of ciphers annexed to the numerator.

EXAMPLES.—Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{20}$ to equivalent decimal fractions.

$\frac{1}{2} = \cdot 5$
$\frac{1}{3} = \cdot \dot{3}$
$\frac{1}{20} = \cdot 05$

In the first example, I annex, or suppose a cipher annexed to the numerator which makes it 10, and dividing by 2 the denominator gives 5 in the quotient without remainder, and because a cipher was annexed the quotient figure is ·5 tenths.

In the second example, I suppose a cipher annexed to the numerator which makes it 10, and dividing by 3 gives ·3 in the quotient, and one of a remainder, to which if a cipher be again annexed, we would have the same quotient as before and the same remainder, the ·3 is therefore a pure repeater, and marked as in the example.

In the third example, I suppose two ciphers annexed to the numerator because annexing one does not make it divisible by 20, but having annexed two, it becomes 100, in which 20 is contained 5 times without remainder, but we have annexed two ciphers to the numerator, we must therefore have two decimal places in the quotient, and this defect is supplied by prefixing a cipher to the quotient figure as in the example.

EXERCISES.

1. Reduce $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{8}$ to decimal fractions.
Ans. $.25$, $.2$, $.1\dot{6}$, $.\dot{1}4285\dot{7}$, $.125$.

2. Reduce $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, to decimal fractions.
Ans. $.\dot{1}$, $.1$, $.\dot{0}\dot{9}$, $.08\dot{3}$.

3. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{6}$ to decimal fractions. Ans. $.\dot{6}$, $.75$, $.4$, $.8\dot{3}$.

4. Reduce $\frac{7}{8}$, $\frac{4}{9}$, $\frac{6}{11}$, $\frac{5}{12}$ to decimal fractions.
Ans. $.875$, $.\dot{4}$, $.\dot{5}\dot{4}$, $.41\dot{6}$.

5. Reduce $\frac{6}{13}$, $\frac{9}{14}$, $\frac{7}{15}$, $\frac{9}{16}$ to decimal fractions.
Ans. $.\dot{4}6153\dot{8}$, $.6\dot{4}2857\dot{1}$, $.4\dot{6}$, $.5625$.

6. Reduce $\frac{7}{320}$, $\frac{119}{120}$, $\frac{7}{960}$ to decimals.
Ans. $.021875$, $.991\dot{6}$, $.00729\dot{1}\dot{6}$.

CASE 2.

To reduce lower denominations, to decimals of a higher denomination.

RULE.—Write the given denominations under each other beginning with the least; divide each denomination beginning with the least by the number in it which makes one of the next higher denomination, writing the quotient opposite that higher denomination, which divide as before; proceed in the same manner with every denomination, the last quotient is the answer.

EXAMPLE.—Reduce 15s. 8¾d. to the decimal of a £.

4	3.
12	8.75
20	15.7291$\dot{6}$
	.786458$\dot{3}$ Ans.

In this example I write 3 qrs., 8d., 15s. under each other according to the rule, I then divide the farthings, supposing ciphers annexed, by 4, and write the quotient 75 opposite to pence; I then divide the line of pence by 12, writing the quotient .7291$\dot{6}$ opposite shillings; and lastly I divide the line of shillings by 20, writing the quotient below, but instead of annexing ciphers when I come to the end of the line of shillings, I annex $\dot{6}$ the repeating figure, this last quotient is the answer, and the $\dot{3}$ on the right is a repeater.

EXERCISES.

1. Reduce /6¾ to the decimal of a shilling. Ans. .5625.
2. Reduce 6/6½ to the decimal of a £. Ans. .32708$\dot{3}$.
3. Reduce 15/7¾ to the decimal of a £. Ans. .782341$\dot{6}$.
4. Reduce 9/8¼ to the decimal of a £. Ans. .484375.
5. Reduce 4 oz. 5 dwt. 6 gr. to the decimal of a lb. troy. Ans. .355208$\dot{3}$.
6. Reduce 6 oz. 4 dr. 1 sc. 12 gr. to the decimal of a lb. Ans. .547$\dot{2}$.
7. Reduce 16 lb. 6 oz. 10 drs. to the decimal of a cwt. Ans. .1465541294$\dot{6}$2857$\dot{1}$.
8. Reduce 3 qrs. 4 lb. 10 oz. 4 drs. to the decimal of a cwt. Ans. .791434151$\dot{7}$85714$\dot{2}$.
9. Reduce 5 cwt. 1 qr. 16 lb. 12 oz. 14 dr. to the decimal of a ton. Ans. .2700020926339$\dot{2}$8571$\dot{4}$.
10. Reduce 3 qrs. 2 n. to the decimal of a yard. Ans. .875.
11. Reduce 3 fur. 8 pol. 4 yds. 1 foot to the decimal of a mile. Ans. .40246$\dot{2}\dot{1}$.
12. Reduce 2 r. 30 p. 20 yds. to the decimal of an acre. Ans. .6916$\dot{3}$22314049586776859504$\dot{1}$.
13. Reduce 12 gal. 3 qts. 1 pt. 3 gills to the decimal of a hhd. of wine. Ans. .20585$\dot{3}$1746$\dot{0}$.
14. Reduce 1 bar. 1 fir. 6 gal. to the decimal of a hhd. of ale. Ans. .9$\dot{4}$.
15. Reduce 6 bu. 3 p. 1 gal. to the decimal of a quarter. Ans. .859375.
16. Reduce 2 w. 5 d. 6 h. 45 m. to the decimal of a month. Ans. .6886160$\dot{7}$1428$\dot{5}$.

17. Reduce 5 h. 48 m. 57 sec. to the decimal of a day. Ans. .2423263$\dot{8}$.

18. Reduce 30′ 45″ 15‴ to the decimal of a degree. Ans. .512569$\dot{4}$.

19. Reduce 10 bol. 3 fir. 2 p. 3 l. to the decimal of a chalder. Ans. .6826171875.

20. Reduce 1 imperial qr. to the decimal of a chalder wheat measure. Ans. .126186494+.

21. Reduce 1 imperial qr. to the decimal of a chalder barley measure. Ans. .086498806+.

Case 3.

To reduce shillings, pence and farthings, to the decimal of a £ mentally.

Rule.—Half the number of shillings is the first decimal place, and the farthings in the remainder increased by 1 for every 24 give the second and third places, but if there is only one figure of farthings prefix a cipher to it for the second place. To complete the decimal; reduce the two last places, or their excess above 25, 50, 75, &c. considered as pence, into farthings, and increase them by 1 for every 24 gives the 4th and 5th places; continue this process till the decimal terminate or repeat.

Examples.—Reduce 9/8½ to the decimal of a £ mentally.

9/8½ = .48541$\dot{6}$

I here take 4 the half of 9 for the 1st place, which leaves 1/8½ = 82 qrs. and increased by 1 for every 24 gives 85 for the 2d and 3d places, and $85 - 75 = 10 \times 4 = 40 + 1 = 41$ for the 4th and 5th places; and $41 - 25 = 16 \times 4 = 64 + 2 = 66$, and if the process were continued 6 would repeat, therefore .48541$\dot{6}$ is the decimal sought.

EXERCISES.

Reduce mentally to the decimal of a £.

1. 2/4¼.	Ans. .117708$\dot{3}$.	7. 16/6¾.	Ans. .828125.
2. 3/8¾.	Ans. .186458$\dot{3}$.	8. 19/4½.	Ans. .96875.
3. 4/6½.	Ans. .22708$\dot{3}$.	9. 18/2¼.	Ans. .909375.
4. 5/8¾.	Ans. .286458$\dot{3}$.	10. 14/6.	Ans. .725.
5. 6/6.	Ans. .325.	11. 12/9.	Ans. .6375.
6. 13/8.	Ans. .68$\dot{3}$.	12. 17/9¾.	Ans. .890625.

Case 4.

To value a finite decimal.

Rule.—Reduce the given decimal to next inferior denomination, and point off as many decimal places as are in the multiplicand; which reduce and point as before, proceed thus till the lowest name; the numbers on the left of the points express the answer.

Example.—Value .8975 of a £.

```
 .8975
    20
17.9500
    12
11.400
     4
 1.6
```

It is evident that the process here is the same as reducing from a greater name to a less in whole numbers. In the first product we point off 4 places of decimals, because there are 4 in the multiplicand; but as ciphers are of no value on the right, we cut them off, and multiply .95 by 12, and point off two decimal places in the product; again cutting off the cipher we multiply .4 by 4, and point off one decimal place in the product, the answer is 17/11¼ .6.

EXERCISES.

1. Value .765 of a shilling. Ans. 9d. .18.
2. —— .875 of a £. Ans. 17s. 6d.
3. —— .98576 of a £. Ans. 19s. 8½d. .3296.
4. —— .75465 of a guinea. Ans. 15s. 10d. .1718.
5. —— .5432 of a crown. Ans. 2s. 8½d. .368.
6. —— .68054 of a lb. troy. Ans. 8 oz. 3 dwt. 7 gr. .9104.
7. —— .8945 of a lb. apoth. Ans. 10 oz. 5 dr. 2 scr. 12 gr. .32.
8. —— .187545 of a cwt. Ans. 21 lb. 1 dr. .29024.
9. —— .4375 of a last of wool. Ans. 5 sa. 3 tds. 1 cl.
10. —— .74635 of a yard. Ans. 2 qrs. 3 n. .9416.
11. —— .5265 of a mile. Ans. 4 f. 8 p. 2 yds. 1 f. 11 in. .04.
12. —— .4246 of an acre. Ans. 1 r. 27 p. 28 yds. 2 f. .826.
13. —— .84235 of a hhd. of wine. Ans. 53 gal. .06805.
14. —— .6354 of a hhd. of ale. Ans. 1 k. 1 f. 7 gal. 1 qt .2464.
15. —— .2985 of a quarter. Ans. 2 bu. 1 p. 1 gal. .104.
16. —— .38495 of a chalder. Ans. 6 bo. 2 p. 2 lip. .1888.
17. —— .3569 of a common year. Ans. 130 d. 6 h. 2 m. 39 s. .84.
18. —— .985 of a French ell. Ans. 5 qr. 3 n. 1.44 inch.

Case 5.

To value the decimal of a £ mentally.

Rule.—Double the figure next the point for shillings, then the 2d and 3d places diminished by 1 for every 25 are farthings, and complete the answer.

EXAMPLE.—Value .796 £ mentally. Ans. 15s. $11\frac{1}{4}$d.

I here double 7 the first figure, which gives 14s., and the 2d and 3d figures being 96, I take 3 from them leaves 93 qrs. equal to $1/11\frac{1}{4}$, which added to 14s. gives $15/11\frac{1}{4}$. answer.

EXERCISES.

Value Mentally. £	Ans.	Value Mentally. £	Ans.
1. .657.	$13/1\frac{3}{4}$.	7. .38125.	$7/7\frac{1}{2}$.
2. .425.	8/6.	8. .025.	0/6.
3. $.78\dot{3}$.	15/8.	9. $.0458\dot{3}$.	0/11.
4. $.5791\dot{6}$.	11/7.	10. $.548958\dot{3}$.	$10/11\frac{3}{4}$.
5. .9875.	19/9.	11. $.82708\dot{3}$.	$16/6\frac{1}{2}$.
6. $.626041\dot{6}$.	$12/6\frac{1}{4}$.	12. .490625.	$9/9\frac{3}{4}$.

CASE 6.

To value a single repetend.

RULE.—Reduce as if it were finite, but carry at 9 in the products of the repetend, and terminate the repeating figures of each under the right of the first product, then carry at 9 when adding the first column; if there are ciphers on the right of the multiplier, annex the repeating figure to the product instead of them.

EXAMPLE.—Value $.2423263\dot{8}$ of a day.

$$
\begin{array}{r}
.2423263\dot{8} \\
24 \\
\hline
96930555 \\
484652777 \\
\hline
5.81583,333 \\
60 \\
\hline
48.95,000 \\
60 \\
\hline
57.00
\end{array}
$$

In multiplying by 4, I have 32 for the first product, which divided by 9, gives 3 and 5 over, I write 5 and carry 3. Next multiplying by 2 gives 16÷9=1 and 7 over, I write 7 under 2, but this 7 must also be repeated to the right, bringing it under 5, the sum of the first column is 12÷9 leaves 3. Before multiplying again I cut off all the repeating figures excepting one, and multiplying by 6, the first product is 18÷9 leaves 0, which in this case is the repeating figure, and must be annexed to the product for the cipher on the right of the multiplicand; again rejecting ciphers on the right, and multiplying by 60 leaves no remainder; therefore 5 h. 48 m. 57 sec. is the answer by the rule.

EXERCISES.

VALUE OF		Ans.	VALUE OF		Ans.
1. $.91\dot{6}$s.	=	11d.	6. $.97\dot{2}$ yd.	=	2 f. 11 in.
2. $.8\dot{3}$ crown.	=	4/2.	7. $.99930\dot{5}$ day.	=	23 h. 59 m.
3. $.991\dot{6}$ £.	=	19/10.	8. $.08\dot{3}$ t. wine.	=	21 gal.
4. $.9\dot{4}$ hhd. ale.	=	1 b. 1 f. 6 gal.	9. $.08\dot{3}$ league.	=	2 fur.
5. $.8\dot{3}$ year.	=	10 months.	10. $.67\dot{4}$ Fl. ell.	=	$2.02\dot{3}$ qrs.

11. Value $.547\dot{2}$ lb. Ap. Ans. 6 oz. 4 dr. 1 sc. 12 gr.
12. Value $.41\dot{6}$ cwt. Ans. 1 qr. 18 lb. 10.6 oz.
13. Value $.805208\dot{3}$ lb. troy. Ans. 9 oz. 13 dwt. 6 gr.
14. Value $.67\dot{5}$ Eng. ell. Ans. 3 qr. $1.5\dot{1}$ nail.

CASE 7.

To value a circulating decimal.

RULE.—Reduce as if terminate, only adding to the 1st products on the right, what would have been carried to them had the circle been repeated, and terminating the circulating figures of each product under the right of the first product, and when adding carry to the right hand column as if the circle had been repeated.

EXAMPLE.—Value $.8\dot{7}1428\dot{5}$ of a cwt.

$$
\begin{array}{r}
.8\dot{7}1428\dot{5} \\
4 \\
\hline
3.4\dot{8}57142\dot{2} \\
28 \\
\hline
36857142 \\
97142857 \\
\hline
13.39,99999 \\
16 \\
\hline
240 \\
400 \\
\hline
6.4,0
\end{array}
\qquad
\begin{array}{lrrr}
 & \text{qr.} & \text{lb.} & \text{oz.} \\
\text{Ans.} & 3 & 13 & 6.4
\end{array}
$$

Had the circle been repeated 71 would have stood on the right of 5, which multiplied by 4 would have given 2 to carry to the product of 5 making it 22. In the product $\dot{8}5$ &c. is the circle, and had it been repeated 85 would have stood on the right of 2, and multiplied by 8 gives 6 to carry to the first product; again multiplying by 2, we have 1 to carry to the 1st product, and in the product by 2 the first figure of the circle on the left is 7, which we repeat under 5 on the right. In the sum of the products 9 repeats, we therefore cut them all off but one before multiplying, and work as in last case.

EXERCISES.

1. Value $.\dot{1}4285\dot{7}$ of a guinea. Ans. 3/
2. Value $.\dot{2}9\dot{6}$ of a moidor. Ans. 8/
3. Value $.\dot{2}8571\dot{4}$ of a cwt. Ans. 1 qr. 4 lb.
4. Value $.6\dot{4}2857\dot{1}$ of a cwt. Ans. 2 qr. 16 lb.
5. Value $.\dot{4}2857\dot{1}$ of a ton wine. Ans. 1 hhd. 45 gal.
6. Value $.2\dot{0}3\dot{7}$ of a hhd. ale. Ans. 1 fir. 2 gal.

CASE 8.

To Reduce a finite decimal, to a vulgar fraction.

RULE.—Write the given decimal for the numerator, and to 1, annex as many ciphers as there are figures in the decimal for the denominator.

Example.—Reduce .65 to a vulgar fraction.

$.65 = \frac{65}{100} = \frac{13}{20}$. I here write 65 for numerator, and 1 with two ciphers annexed for denominator, because there were two decimal places, and reducing to the lowest terms gives $\frac{13}{20}$.

EXERCISES.

	Answer.
1. Reduce .5, .45, .75, .05 to vulgar fractions.	$\frac{1}{2}, \frac{9}{20}, \frac{3}{4}, \frac{1}{20}$.
2. Reduce .88, .648, .752., .955 to ditto.	$\frac{22}{25}, \frac{81}{125}, \frac{94}{125}, \frac{191}{200}$.
3. Reduce .25, .125, .5125, .45125 to do.	$\frac{1}{4}, \frac{1}{8}, \frac{41}{80}, \frac{361}{800}$.
4. Reduce .186, .435, .0055, .6955 to do.	$\frac{37}{200}, \frac{87}{200}, \frac{11}{2000}, \frac{1391}{2000}$.

Case 9.

To reduce a pure repeater, or circulate to a vulgar fraction.

Rule.—Write 9 under each repeating figure for the denominator; then reduce to the lowest terms.

Examples.—Reduce $\cdot\dot{3}$ and $\cdot\dot{7}5\dot{6}$ to vulgar fractions.

$\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$ Ans.

$\cdot\dot{7}5\dot{6} = \frac{756}{999} = \frac{28}{37}$ Ans.

In the first example, writing 9 under 3 makes it $\frac{3}{9}$ and reduced $\frac{1}{3}$.

In the second example writing 9 under each figure of $.\dot{7}5\dot{6}$ makes it $\frac{756}{999}$, and reduced $\frac{28}{37}$.

EXERCISES.

1. Reduce $.\dot{4}$, $.\dot{5}$, $.\dot{6}$, $.\dot{7}$, $.\dot{8}$, $\cdot\dot{9}$ to vulgar fractions.
Ans. $\frac{4}{9}, \frac{5}{9}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, \frac{1}{1}$.
2. Reduce $.\dot{0}\dot{6}$, $.\dot{4}\dot{5}$, $.\dot{6}\dot{3}$, $.\dot{1}\dot{8}$ to vulgar fractions.
Ans. $\frac{2}{33}, \frac{5}{11}, \frac{7}{11}, \frac{2}{11}$.
3. Reduce $.\dot{1}6\dot{2}$, $.\dot{3}8461\dot{5}$, $\cdot\dot{5}7142\dot{8}$ to vulgar fractions.
Ans. $\frac{6}{37}, \frac{5}{13}, \frac{4}{7}$.
4. Reduce $.\dot{7}5\dot{9}$, $.\dot{8}7\dot{3}$, $.\dot{6}35\dot{4}$, $.\dot{1}4\dot{4}$, to vulgar fractions.
Ans. $\frac{253}{333}, \frac{97}{111}, \frac{706}{1111}, \frac{16}{111}$.

Case 10.

To reduce a mixed repetend, or circulate, to a vulgar fraction.

Rule.—Subtract the finite part from the whole decimal for the numerator; and for the denominator write 9 for each repeating figure, to which annex a cipher for every figure in the finite part; and reduce to lowest terms.

Examples.—Reduce $\cdot 32\dot{7}$ and $\cdot 29\dot{5}\dot{4}$ to vulgar fractions.

$\cdot 32\dot{7} - 32 = 295 = \frac{295}{900} = \frac{59}{180}$.

$\cdot 29\dot{5}\dot{4} - 29 = 2925 = \frac{2925}{9900} = \frac{13}{44}$.

In the first example 32 the finite part of the decimal is subtracted from $\cdot 32\dot{7}$ the whole decimal, the remainder 295 is the numerator, and its denominator is 9 for the repeating figure, and two ciphers for the two finite places.

In the second example the numerator is found as before, and the denominator is 9900, because there are two places finite, and two repeating.

EXERCISES.

1. Reduce $.7\dot{5}$, $.8\dot{3}$, $.2\dot{1}4285\dot{7}$ to Vulgar Fractions.
   Ans. $\frac{34}{45}$, $\frac{5}{6}$, $\frac{3}{14}$.
2. Reduce $.0\dot{4}\dot{5}$, $.58\dot{3}$, $.56\dot{8}\dot{1}$ to Vulgar Fractions.
   Ans. $\frac{1}{22}$, $\frac{7}{12}$, $\frac{25}{44}$.
3. Reduce $.547\dot{2}$, $.11\dot{3}\dot{6}$, $.91\dot{6}$ to Vulgar Fractions.
   Ans. $\frac{197}{360}$, $\frac{5}{44}$, $\frac{11}{12}$.
4. Reduce $.\dot{5}92\dot{5}$, $.5\dot{3}$, $.4\dot{2}\dot{7}$ to Vulgar Fractions.
   Ans. $\frac{16}{27}$, $\frac{8}{15}$, $\frac{47}{110}$.
5. Reduce $.6\dot{4}2857\dot{1}$, $.007291\dot{6}$, $.41\dot{6}$ to Vulgar Fractions.
   Ans. $\frac{9}{14}$, $\frac{7}{960}$, $\frac{5}{12}$.

ADDITION OF DECIMALS.

Case 1.

To add terminate decimals.

Rule.—Write like places directly under each other, then add as in integers, placing the decimal point in the sum, directly under those in the numbers added. Proof as in whole numbers.

Example.—Add together 65.673, 2.05, 678.5432, .432, 76, 43.62.

```
 65.673
  2.05
678.5432
   .432
 76.
 43.62
--------
857.3182
```

Observing the order in which this example is written down, it will be seen that like places stand directly under each other, both on the right and left of the points, that is, tens under tens, tenths under tenths, &c.

EXERCISES.

1. Add together ·764, .972, .0372, .0623. Ans. 1.8355.
2. Add together 61.204, .62, 764, 672.07912. Ans. 1497.90312.
3. Add together 18.18, .7324, 6769, 4.945, .3361. Ans. 6793.1935.
4. Add together .6703, 682, .421, 3.3016, .874. Ans. 687.2669.
5. Add together 94.0065, 3.7, 83.695, 4375. Ans. 4556.4015.
6. Reduce and add $13\frac{1}{2}$, $123\frac{1}{4}$, $68\frac{3}{4}$, $12\frac{4}{5}$, $9\frac{1}{8}$, $731\frac{7}{20}$. $55\frac{5}{16}$, $99\frac{9}{10}$, $133\frac{24}{25}$, $7\frac{7}{8}$. Ans. 1255.8225.
7. Reduce and add $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{5}$, $\frac{4}{5}$ of $\frac{3}{8}$, $\frac{1}{5}$ of $25\frac{2}{5}$. Ans. 6.08.

CASE 2.

To add repeating decimals.

RULE.—Extend the repeating figure in each number one place to the right of the longest finite part, and carry at 9 in the first column.

EXAMPLE.—Add together $6.7\dot{3}$, $15.85\dot{4}$, $.7689\dot{5}$, $98.\dot{6}$.

$$\begin{array}{r} 6.7\dot{3}333 \\ 15.85\dot{4}44 \\ .7689\dot{5} \\ 98.\dot{6}6666 \\ \hline 122.0234\dot{0} \end{array}$$

In this example the longest finite part is .7689, and the repeating figures are all extended one place farther to the right than it, and out of the column thus formed we always carry at 9.

EXERCISES.

1. Add together $674.\dot{2}$, $76.\dot{3}$, $45.68\dot{5}$, $.7548\dot{6}$. Ans. $796.9959\dot{7}$
2. Add ——— $4.987\dot{6}$, $54.7\dot{2}$, $.7968\dot{1}$, $17.6\dot{8}$. Ans. $78.19555\dot{8}$.
3. Add ——— $12.2\dot{6}$, $7.62\dot{3}$, $54.68\dot{7}$, $.0069\dot{4}$. Ans. $74.5847\dot{2}$.
4. Add ——— $56.\dot{6}$, $2.59\dot{3}$, $.7273\dot{1}$, $.0\dot{6}$, $.2137\dot{3}$. Ans. $60.2677\dot{1}$.
5. Reduce and add $59\frac{1}{8}$, $16\frac{1}{6}$, $18\frac{1}{12}$, $24\frac{7}{8}$, $6\frac{7}{240}$. Ans. $124.2791\dot{6}$.
6. Reduce and add £2 0s. $0\frac{1}{4}$d., £3 0s. 7d., £7 3s. 4d., £5 6s. 8d., £12 13s. 4d., £14 14s. $3\frac{1}{2}$d., £10 15s. $6\frac{1}{4}$d., £1 19s. $11\frac{3}{4}$d. Ans. £$57.686458\dot{3}$.
7. Reduce and add £3 0s. $5\frac{1}{4}$d., £6 4s. 7d., £8 7s. $7\frac{3}{4}$d., £10 9s. 4d., £5 13s. 4d., £6 17s. 5d., £3 18s. 8d., £1 19s. 11d. Ans. £$46.5\dot{6}$.

CASE 3.

To add circulating decimals.

RULE.—Extend repeaters and circulates as far to the right

of the longest finite part, as there are units in the least common multiple of the number of figures in the several circles; and in adding, carry to the right hand column the tens from the 1st column on the right of the longest finite part; then proceed as in case 1st.

EXAMPLE.—Add together $6.754\dot{3}$, $8.\dot{4}\dot{5}$, $12.7\dot{6}5\dot{4}$, $7.85\dot{3}\dot{6}$.

$$\begin{array}{r} 6.754\dot{3}33333 \\ 8.\dot{4}\dot{5}4545454 \\ 12.7\dot{6}5\dot{4}65465 \\ 7.85\dot{3}\dot{6}36363 \\ \hline 35.827\dot{9}8061\dot{6} \end{array}$$

In this example I first find the least common multiple of 2, 3, 2 the number of places in the several circles, which is 6; I then extend both repeaters and circulates 6 places to right of the longest finite part, which is here 4 in the upper line, they are thus made both coterminous and similar; I then add the column on the right of the longest finite part; and find there is 1 to carry, which I add to the right hand column, and then proceed as in case 1st.

EXERCISES.

1. Add together $.68\dot{2}\dot{5}$, $.21\dot{3}7\dot{2}$, $.132\dot{4}\dot{5}$, $.2634\dot{6}$.

   Ans. $1.2921\dot{7}0188\dot{3}$.

2. Add together $.\dot{4}632\dot{1}$, $.81\dot{5}3\dot{2}$, $.1549\dot{2}\dot{6}$, $.75\dot{3}\dot{2}$, $.67545$.

   Ans. $2.8621\dot{5}02328160744\dot{0}$.

3. Add together $.4365\dot{3}$, $.8\dot{2}\dot{3}$, $.5\dot{4}\dot{3}$, $.\dot{6}87\dot{1}$, $.2\dot{3}\dot{6}$, $.\dot{4}123\dot{1}$.

   Ans. $3.1393\dot{4}950667958524769766\dot{1}$.

4. Reduce and add together $\frac{1}{14}$, $\frac{9}{11}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{71}$, $\frac{7}{8}$.

   Ans. $4.52\dot{0}00721\dot{5}$.

5. Add together $.41\dot{3}\dot{2}$, $.\dot{5}423\dot{6}$, $.\dot{6}23145\dot{2}$.

   Ans. $1.5\dot{7}874300920108600614127912273118012397$-
   $833918291995097050370031822089214\dot{8}$.

SUBTRACTION OF DECIMALS.

CASE I.

To subtract terminate decimals.

RULE.—Write the numbers with like places under each other, and subtract as in whole numbers, then point the remainder as in addition. Proof as in whole numbers.

EXAMPLE.—Subtract 6.3754 from 8.23142.

8.23142	I here write the given numbers, units under units,
6.3754	tenths under tenths, &c. it will be obvious that there
1.85602	is no figure under the right hand place of the upper

number, we therefore write 2 or say 0 from 2 because we might have written a cipher under it without altering the value of the decimal, as has been already explained. The remaining part of the operation is the same as in whole numbers, without regarding the point till we place it in the remainder.

EXERCISES.

1. From 97.654 take 71.896. Ans. 25.758.
2. From .68321 take .586215. Ans. .096995.
3. From .8 take .674321. Ans. .125679.
4. From $6\frac{1}{8}$ take $\frac{3}{4}$ of $\frac{3}{4}$ decimally. Ans. 5.625.
5. From £6 2s. 6d. take £3 15s. 6d. decimally. Ans. £2.35.
6. What is the difference between .4 and .3682? Ans. .0318.
7. What is the difference between .3742 and .8? Ans. .4258.
8. Subtract .09 from 90. Ans. 89.91.
9. Subtract .006 from .401. Ans. .395.
10. Subtract £.1235 from £1. Ans. £.8765.

CASE 2.

To subtract repeaters.

RULE.—Extend them one place farther to the right than the longest finite part, and whenever the repeater of the minüend is less than that of the subtrahend take one from it before you borrow.

EXAMPLE from $.624\dot{2}$ take $.47\dot{3}$.

$$\begin{array}{r} .624\dot{2} \\ .4733 \\ \hline .150\dot{8} \end{array}$$

In this example .624 is the longest finite part, the repeaters are therefore extended one place to the right of it; and because $\dot{2}$ the repeater in the minuend is less than $\dot{3}$ the repeater in the subtrahend, one must be taken from the 2 before borrowing, then 3 from 11 leaves $\dot{8}$, also a repeater, after which proceed as in whole numbers.

EXERCISES.

1. Subtract $.36\dot{5}$ from $.873\dot{6}$. Ans. $.508\dot{1}$.
2. Subtract $.679\dot{2}$ from $.89\dot{4}$. Ans. $215\dot{2}$.
3. Subtract $.87\dot{6}$ from $.945\dot{3}$. Ans. $.068\dot{6}$.

4. Subtract $\cdot 456\dot{3}$ from .46. Ans. $.00\dot{3}\dot{6}$.

5. Subtract £7 6s. 8d. from £9 3s. 4d. decimally. Ans. £1.83.

CASE 3.

To subtract circulating decimals.

RULE.—Make circulates similar and coterminous as in Addition; and if the first figure in the subtrahend on the right of the longest finite part be greater than the one above it, take one from the right hand figure of the minuend before subtracting, then proceed as in whole numbers.

EXAMPLE.—Subtract $.67\dot{2}6\dot{5}$ from $.856\dot{3}\dot{2}$.

$.856\dot{3}\dot{2}3232$
$.67\dot{2}6\dot{5}2652$
$.183\dot{6}7057\dot{9}$

I here extend each of the circulates six places to the right of the longest finite part, because 6 is the least common multiple of 2 and 3 the number of places in the two circles. And because 6 the first figure in the subtrahend on the right of the longest finite part is greater than the figure above it, I take one away from the right hand figure of the minuend before subtracting, and then proceed as in whole numbers.

EXERCISES.

1. From $.63\dot{4}\dot{9}$ take $.3\dot{4}6\dot{9}$. Ans. $.28\dot{8}0025\dot{4}$.
2. From $.53\dot{2}1\dot{4}$ take $2\dot{4}36\dot{1}$. Ans. $.28\dot{8}52778070599\dot{9}$.
3. From .3456 take $.186\dot{3}2978\dot{4}$. Ans. $.1592\dot{7}0215\dot{6}$.
4. From $.976\dot{5}43\dot{8}$ take $.5\dot{2}168\dot{7}$.
Ans. $.454\dot{8}566375132698221671\dot{6}$.
5. From 4 cwt. 2 qr. 24 lb. take $2\frac{3}{23}$ cwt. Ans. $2.5\dot{7}7922\dot{0}$ cwt.

MULTIPLICATION OF DECIMALS.

CASE 1.

When both factors are terminate.

RULE.—Write the factors and multiply as in whole numbers, then point off as many decimals in the product as there are in both factors; but when there are not so may figures in the product, prefix ciphers to supply the deficiency.

EXAMPLES.—Multiply 4.634 by 3.248, and .057 by .05.

1st.	2d.
4.634	.057
3.248	.05
37072	.00285
111216	
13902	
15.051232	

In the first example, there are three decimal places in each factor, I therefore point off six decimal places in the product.

In the second example there are five decimal places in the factors, and only three figures in the product, I therefore prefix two ciphers to the product, and place the decimal point on the left of them.

EXERCISES.

REQUIRED THE PRODUCT OF

1. ·645 × ·65. Ans. ·41925.
2. ·063 × ·432. Ans. ·027216.
3. ·2136 × ·023. Ans. ·0049128.
4. 4·721 × 6·42. Ans. 30·30882.
5. 134·12 × 12·6. Ans. 1689·912.
6. ·0673 × 147. Ans. 9·8931.
7. ·31426 × ·513. Ans. ·16121538.
8. 34·124 × ·0051. Ans. ·1740324.
9. ·398765 × 10,100. Ans. 3.98765, and 39.8765.
10. ·4621 × 1000, 100000. Ans. 462.1, and 46210.

CASE 2.

When the multiplicand is a repeater, or circulate.

RULE.—For a repeater, carry at 9 on the right in each product, and add as directed for repeaters; for a circulate, to the right hand figure of each product, add what is to carry from the left of the circle, then make the circles similar, and add as directed in addition.

EXAMPLES.—Multiply $\cdot 64\dot{3}$ by 542, and $\cdot 3\dot{4}\dot{6}$ by ·763.

1st.	2d.
$\cdot 64\dot{3}$	$3\dot{4}\dot{6}$
·542	·763
1286	1039
25733	20787
321666	242525
$\cdot 34868\dot{6}$	$\cdot 2643\dot{5}\dot{2}$

In the first example, I take the nines out of the several products of 3 on the right of the multiplicand, and because three in the second product, and 6 in the 3d product are also repeaters, I extend them under 6, the right hand figure of the first product, and when adding the right hand column I carry at 9.

In the second example, multiplying 4 on the left of the circle by 3 gives 1 to carry to the first product on the right; and multiplying by 6 gives 2 to carry to the first product on the right by 6; again multiplying by 7 we have three to carry to the first product on the right, because $4 \times 7 = 28$, and if we had gone back another figure in the circle, we would have had 4 to carry to 28 making it 32. I then extend the circles till they are similar, which is already explained in addition, I then add carrying one from the left to the right of the circle.

EXERCISES.

REQUIRED THE PRODUCT OF

1. $.65\dot{4} \times .326$. Ans. $.21334\dot{8}$.
2. $7.3\dot{2} \times .835$. Ans. $6.1140\dot{5}$.
3. $7.\dot{4}\dot{3} \times .862$. Ans. $.6408\dot{3}\dot{9}$.
4. $.6\dot{3}4\dot{6} \times .928$. Ans. $.5889\dot{4}0\dot{7}$.
5. $.4\dot{8}43\dot{7} \times .5734$. Ans. $.27774\dot{2}59\dot{6}$.
6. $374.\dot{6}\dot{8} \times .0072$. Ans. $2.6977\dot{4}\dot{5}$.
7. $.7\dot{6}3\dot{2} \times 5700$. Ans. $4350.\dot{6}0\dot{0}$.
8. $.46\dot{7}89\dot{5} \times 4.632$. Ans. $2.16729\dot{3}29\dot{7}$.
9. $73.6\dot{3}7\dot{1} \times 81.43$. Ans. $5996.27\dot{2}07\dot{7}$.
10. $.426\dot{3} \times 4.768$. Ans. $2.032757\dot{3}$.

CASE 3.

To multiply when both factors repeat or circulate.

RULE.—Reduce the multiplier to a vulgar fraction, then multiply the other factor by the numerator, and divide the product by the denominator.

EXAMPLE.—Multiply $2.6\dot{7}\dot{4}$ by $.3\dot{2}\dot{4}$.

$$
\begin{array}{r}
2.6\dot{7}\dot{4} \\
.3\dot{2}\dot{4} - 3 = \underline{321} \\
2674 \\
53494 \\
\underline{802424} \\
990)858.5\dot{9}\dot{3}(.86726\ \&c. \\
\underline{7920} \\
6659 \\
\underline{5940} \\
7193 \\
\&c.
\end{array}
$$

In this example the multiplier is reduced to a vulgar fraction, of which 321 is the numerator, and 990 the denominator, we then multiply by the numerator as in last case, and divide by the denominator, the quotient may be carried out by using the circulating figures in the dividend till it repeat, or circulate.

EXERCISES.

1. Multiply $3.4\dot{1}\dot{6}$ by $.24\dot{3}$. Ans. $.831\dot{2}6599\dot{3}$.
2. Multiply $8.21\dot{7}$ by $.3\dot{8}\dot{7}$. Ans. $3.187\dot{5}0168\dot{3}$.
3. Multiply $.05\dot{3}\dot{4}$ by $3.6\dot{5}\dot{2}$. Ans. .0195170288746+.
4. Multiply $.8\dot{7}54\dot{3}$ by $16.\dot{4}625\dot{1}$. Ans. 14.41190+.
5. Multiply $257.6\dot{3}4\dot{1}$ by $.\dot{6}748\dot{2}$. Ans. 173.8584049+.
6. Multiply $37.2\dot{4}\dot{6}$ by $.8\dot{9}23\dot{6}$. Ans. 33.237779+.
7. Multiply $.\dot{4}231\dot{5}$ by $5.76\dot{3}9\dot{4}$. Ans. 2.43903727+.
8. Multiply $.4\dot{5}762\dot{3}$ by $7.2\dot{3}4365\dot{4}$. Ans. 3.310616+.
9. Multiply £$3.4\dot{2}3\dot{5}$ by $12.8\dot{4}372\dot{1}$, and value the product.
   Ans. £43 19s. 4¾d. .95+.
10. Multiply $32.3\dot{2}5\dot{6}$ cwt. by $.0065\dot{3}\dot{4}$, and value product.
    Ans. 23 lb. 10 oz. 8 dr. .293+.
11. Multiply $6.7\dot{2}\dot{4}$ by $64.\dot{3}$. Ans. $432.60\dot{5}9\dot{2}$.
12. Multiply $5.7\dot{6}$ guineas by $.46\dot{3}$, and value the product.
    Ans. £2 16s. 1¼d. $.178\dot{6}$.

CONTRACTION.

To multiply so as to retain any proposed number of decimals in the product.

Rule.—Invert the multiplier, and write its units place under that decimal place in the multiplicand which you wish the last in the product; then multiply, rejecting always the figures of the multiplicand on the right of the figure you are multiplying by, only carrying from their product thus, 1 at 5, 2 at 15, 3 at 25, 4 at 35, &c. writing the several products directly under each other on the right, then add as integers.

Example.—Multiply 42.36532 by 32.84325 retaining 4 decimal places in the product.

```
42.36532
52348.23
--------
12709596
  847306
  338923
   16946
    1271
      85
      21
---------
1391.4148
```

Because it is required here to retain 4 decimal places in the product, 2 the units place of the multiplier is written directly under 3 the fourth decimal place in the multiplicand, the whole numbers of the multiplier are then written on the right of units place, and the decimal part on the left of it as in the example. As the right hand figures of the multiplicand and multiplier are directly under each other, we multiply by the first figure in the common way, then by the second figure, beginning with the 3 immediately above it, and writing the product 6 under the right hand figure of

the last line of products, again multiplying by the 3d figure and beginning with the one immediately above it the product is 40, but multiplying the two figures on the right according to the rule we have 2x8=16, which gives 2 to carry, and 3x8+2=26 gives 3 to carry which added to 40 makes 43, we therefore write 3 under the right hand figure of the other products; proceed in the same manner with every other figure in the multiplier, then add the products, and point off as many decimals as the question requires.

EXERCISES.

1. Mult. 63.7251 by 6.3425 retaining four decimal places in the product. Ans. 404.1765.
2. Mult. 5.79432 by .643242 retaining five decimal places in the product. Ans. 3.72714.
3. Mult. 671.6342 by .0436 retaining six decimal places in the product. Ans. 29.283252.
4. Mult. 27.6342893 by 52.76298 retaining three decimal places in the product. Ans. 1458.068.
5. Mult. .73241 by .47659869 retaining seven decimal places in the product. Ans. .3490657.
6. Multiply 56.34587 by 37.2498 retaining two decimal places in the product. Ans. 2098.88.
7. Multiply .83962 by 8.4689 retaining four decimal places in the product. Ans. 7.1108.
8. Multiply 687.314 by 63.2405 retaining five decimal places in the product. Ans. 43466.08102.

DIVISION OF DECIMALS.

Case 1.

When the divisor is terminate.

Rule.—Divide as in whole numbers, and when all the figures in the dividend are taken down, if there is a remainder, annex ciphers when the dividend is terminate, but the repeating or circulating figures when interminate, and divide till the quotient terminate, repeat, or circulate, or till it is sufficiently extended; the decimal places in the quotient are always equal to the excess of the decimal places in the dividend above those in the divisor, and when there are not so many figures in the quotient, prefix ciphers to supply the deficiency.

Example.—Divide $5.45\dot{3}$ by 4.3251.

```
4.3251)5.4533(1.2608
       43251
       ------
       112823
        86502
       -------
        263213
        259506
        -------
         370733
         346008
         -------
          24725
```

In this example 5.453 is not divisible by 4.3251, therefore $\dot{3}$ is repeated, and then the divisor is contained once in it, to the remainder the repeating figure is again annexed before dividing, and so on. If the dividend had been terminate, ciphers would have been annexed instead of the repeating figure at the several steps. To ascertain the number of decimal places in the quotient, count the decimal places in the given dividend, to which add the number of figures annexed during the process the sum of which in this example is 8, and there are 4 decimal places in the divisor, therefore $8-4=4$ the number of decimal places in the quotient. The operation here might still have been carried further by annexing the repeating figure, and dividing which, would have given more decimals in the quotient.

EXERCISES.

1. Divide 689.348 by 46.23.	Ans. 14.911269+.
2. Divide 7984.32 by 9.324.	Ans. 856.319176+.
3. Divide 462.65 by 82.64.	Ans. 5.5983785+.
4. Divide 1.7455174 by .324.	Ans. 5.38739+.
5. Divide .243267 by 76.254.	Ans. .003190219+.
6. Divide 208.90625 by 18.75.	Ans. $11.141\dot{6}$+.
7. Divide $652.63\dot{5}\dot{6}$ by .4624.	Ans. 1411.40929+.
8. Divide $12.\dot{7}\dot{5}$ by 8.634.	Ans. 1.477597+.
9. Divide $20.\dot{3}5\dot{8}$ by 82.9765.	Ans. .24535089+.
10. Divide $.0425\dot{3}$ by 6.3241.	Ans. .00672559+.
11. Divide $.6\dot{3}78\dot{2}$ by .4623.	Ans. 1.379675+.
12. Divide 26.56 by 9214.	Ans. .00288257+.
13. Divide $.\dot{6}3\dot{5}$ by 693.	Ans. .000917223+.
14. Divide $.\dot{7}\dot{8}$ by 6.82.	Ans. .1155247+.
15. Divide $.\dot{7}$ by .235.	Ans. 3.309692+.
16. Divide $846.\dot{2}$ by 1000, and by 10000.	Ans. $.846\dot{2}$ and $.0846\dot{2}$.

CASE 2.

When the divisor repeats, or circulates.

Rule.—Reduce the divisor to a vulgar fraction, then multiply the dividend by the denominator, and divide the product by the numerator, the quotient is the answer.

EXAMPLE.—Divide $6.4\dot{2}\dot{3}$ by $.\dot{4}6\dot{7}$.

$.\dot{4}6\dot{7} - 4 = \frac{463}{990}$

$$
\begin{array}{r}
6.4\dot{2}\dot{3} \\
990 \\
\hline
57809 \\
57809 \\
\hline
463)6358.\dot{9}(13.7 \\
463 \\
\hline
1728 \\
1389 \\
\hline
3399 \\
3241 \\
\hline
158, \text{\&c.}
\end{array}
$$

In this example the divisor is reduced to a vulgar fraction, and the dividend is multiplied by its denominator, and the product divided by the numerator. The quotient may be carried out any length by annexing the repeating 9 to the remainders and dividing.

EXAMPLES.

1. Divide 6.324 by $.5\dot{6}$. Ans. 11.16.
2. Divide 9.852 by $.65\dot{3}$. Ans. 15.07959+.
3. Divide $69.7\dot{5}$ by $8.7\dot{3}\dot{2}$. Ans. 7.988201+.
4. Divide 7.842 by $.\dot{6}74\dot{5}$. Ans. 11.6251046+.
5. Divide 846.3 by $.\dot{7}$. Ans. 1088.1.
6. Divide $7423.$ by $\dot{6}$. Ans. 11134.5.
7. Divide $54.2\dot{6}3\dot{4}$ by $.\dot{3}$. Ans. $162.\dot{7}90\dot{3}$.
8. Divide $67.0\dot{3}39\dot{3}$ by $3.4\dot{3}7\dot{6}$. Ans. 19.5.
9. Divide 8612.75 by $109.\dot{6}$. Ans. $78.5\dot{3}57142\dot{8}$.
10. Divide $42.\dot{6}3\dot{7}$ by $8.4\dot{2}\dot{5}$. Ans. 5.0606955+.
11. Divide 12.62 by $6.2\dot{5}$. Ans. 2.017406+.
12. Divide $14.4\dot{3}\dot{2}$ by $.9874$. Ans. 14.61538+.

CONTRACTION.

To divide so as to retain any proposed number of decimal places in the quotient.

RULE.—Consider how many figures altogether must be in the quotient, and divide by the same number of figures on the left of the divisor, but instead of annexing a figure to the remainder, omit one on the right of the divisor at each step till the whole be exhausted, only carrying from them as in contracted multiplication; and point off the required number of decimals in the quotient.

EXAMPLE.—Divide 64.37852 by 5.24636 retaining 3 places of decimals.

```
5.24636)64.37852(12.271,
        52464
        -----
        11914
        10493
        -----
         1421
         1049
         ----
          372
          367
          ---
            5
            5
```

I here see, that there will be 2 places of whole numbers, and 3 places of decimals in the quotient, I therefore divide first by 5 figures on the left of the divisor rejecting the last place, but in multiplying I carry 1 from its product because it amounts to 5 and upwards, to find the second quotient figure I reject 3 from the right of the divisor and carry 1 from its product to the product of 6, and in finding each subsequent quotient figure I reject one on the right of the divisor till they are all exhausted as in the example, I then point off 3 decimal places in the quotient as was required in the question.

EXERCISES.

1. Divide 687.43 by 94·821, retaining only one decimal place in the quotient. Ans. 7.2.
2. Divide 63.8251 by 3.472 retaining 2 decimal places in the quotient. Ans. 18.38.
3. Divide 687.456 by .678243 retaining 3 decimal places in the quotient. Ans. 1013.583.
4. Divide 187.763 by 56.3249 retaining 4 decimal places in the quotient. Ans. 3.3335.
5. Divide 897.5462 by 189.7654 retaining 5 decimal places in the quotient. Ans. 4.72976.
6. Divide 6.543245 by .898763 retaining 6 decimal places in in the quotient. Ans. 7.280278.
7. Divide .4368215 by .54637482, retaining 7 decimal places the quotient. Ans. .7994905.
8. Divide .0257684 by .3542541 retaining 8 decimal places in the quotient. Ans. .07273988.
9. Divide 43.687245 by 6764.5362 retaining 6 decimal places in the quotient. Ans. 006458.

SIMPLE PROPORTION IN DECIMALS.

RULE.—Reduce the lower denominations in each term to decimals; then state the question, and work as in whole num-

bers, observing to place the decimal points correctly in the products, and quotient.

EXAMPLE.—If 3 yds. and 3 qrs. of broad cloth cost £3 16s. 6d. ; what must be paid for 38 yds. 2 qrs. 3 n. ?

```
     yds.      yds.         £
  As 3.75 : 38.6875 :: 3.825
                       3.825
                   ---------
                    1934375
                    773750
                   3095000
                  1160625      £
  3.75)147.9796875(39.46125
       1125            20
       ----        --------
        3547        9.22500
        3375           12
        ----       --------
         1729        2.700
         1500          4
         ----      --------
          2296         2.8
          2250
          ----
            468
            375
            ---
             937
             750
             ----
             1875
             1875
```

In this example I reduce 3 quarters, and 2 qrs. 2 nails each to the decimal of a yard, and 16/6 to the decimal of a £; I then state the question, and work as in whole numbers, the quotient is £39.46125, and valuing the decimal gives £39 9s. 2¼d. .8

EXERCISES.

1. How much must be paid for a dozen of silver spoons weighing 25 oz. 6 dwts. 16 gr. at the rate of £1 2s. 6d. for 2½ oz ? Ans. £11 8s.
2. What will 32¼ yards of linen cost, at the rate of 26½ yards for £3 16s. 3d. ? Ans. £4 12s. 9½d+.
3. A bankrupt owes in all £1490 5s. 10d., his estate is worth £931 8s. 7¾d. ; how much should X receive to whom he owes £253.$\dot{3}$? Ans. £158 6s. 8d+.
4. If 75 yards cost £$\frac{3}{16}$; what will 19$\frac{1}{1}\frac{1}{8}$ yards cost at that rate ? Ans. £4 18s. 5¼d. .0256.
5. If I pay £197 11s. 3d. for ⅔ of ⅓ of a ship, what is ¼ of her worth ? Ans. £164 12s. 8½d. 048.
6. How much must be paid for 2 cwt. 1 qr. 6⅔ lb. of raisins ; when 3 cwt. 1 qr. 14 lb. cost nine guineas ? Ans. £6 9s. 4d.

7. Bought 42 hhds. of rum old wine measure for £1984 10s.; how must I sell it per imperial gallon to clear £267 15s. by the whole? Ans. £1 0s. 5d. .2+.
8. How must I sell wheat per imperial quarter to be equal to £1 19s. 6d. per Linlithgow boll? Ans. £3 19s. 8¾d. .99+.
9. What is the price of oats per imperial quarter, at the rate of 15/6½ per Linlithgow boll? Ans. £1 1s. 6d. .112+.
10. When barley sells at £2 1s. 6d. per imperial quarter; what is it worth per chalder? Ans. £23 19s. 9½d. .21+.
11. If wheat sell for £4 10s. 6d. per imperial quarter; what is that per Linlithgow boll. Ans. £2 4s. 9¾d. .57+.
12. If a clergyman is allowed 10 chalders of barley, and 8 chalders of oats for stipend, what is the value of his living when the fiar price for barley is £1 17s. 8d., and for oats £1 7s. 10½d. per imperial quarter. Ans. £346 12s. 7¾d. .5+.
13. If a clergyman is allowed 5 chalders of wheat, 6 of barley, and 7 of oats for stipend; what may he spend daily, after laying up £60 for his family, the fiar prices being for wheat £3 18s. 6d., for barley £1 16s. 6½d., and for oats £1 7s. 3d. per imperial quarter? Ans. £0 18s. 2½d. .576+.
14. If the average price of wheat per imperial quarter be £4 4s. 9d.; what is that per quarter Winchester measure? Ans. £4 2s. 1¾d. .71+.
15. When ale sells for £6 per hhd. imperial measure; what is that per gallon old bear measure? Ans. 2s. 3d. .12+.
16. If rum be fraudulently retailed at 6d. per Scotch gill instead of the imperial gill; how much is gained on a hhd. by this illegal traffic? Ans. £17 3s. 5¾d. .759+.

COMPOUND PROPORTION IN DECIMALS.

RULE.—Prepare the terms as in simple proportion, then state the question as in whole numbers, and work as directed in decimals.

EXAMPLE.—If 7 men and 2 boys can perform 16$\frac{3}{13}$ roods of ditching, in 8¾ days; how many days will 13 men and 5 boys require to perform 25¼ roods, supposing a boy does ½ of a man's work?

```
                              Days.
Men    14.25     :      7.5 :: 8.4
Roods  16.53̇      :     25 3̇
       ------          ----
         475             25
        7125            375
       8550            150
      1425             ----
      -------          190.ø
      235.600            8.4
                       ------
                        760
                       1520  Days.
           235.6)1596.0(6.774
                 18240
                 17480
                  9880
                   456
```

Having reduced the vulgar fractions into decimals, the question is then stated as in whole numbers. In the work, instead of reducing $.5\dot{3}$ into a vulgar fraction, I have taken it as $.5\frac{1}{3}$ which is simpler, and in pointing, remember $.\dot{3}$ taken fractionally, does not increase the number of decimal places in the product. In the middle term the operation by $.\dot{3}$ is performed in the same manner, which gives only one decimal place in the product, and being 0 it is cut off. The division here is neither terminated nor the decimal in the quotient valued.

EXERCISES.

1. If 7 men in $6.\dot{5}\dot{4}$ days of $9.\dot{3}$ hours each, cut $7\frac{3}{4}$ acres of wheat; in how many days of $12\frac{1}{3}$ hours each will 9 men cut $15\frac{5}{8}$ acres? Ans. 7.68995+. days.
2. If 4 men shear 600 sheep in $5.\dot{6}$ days of $10\frac{3}{4}$ hours each; how many days will 24 men require to shear 15000 sheep, working 9.625 hours each day? Ans. 26.37+ days.

Take also the examples in Compound Proportion in Vulgar Fractions Decimally.

DISTRIBUTIVE PROPORTION.

Distributive Proportion is the application of simple proportion to company accounts: Or it is that rule by which we divide any number into parts having the same proportion to one another, which any proposed numbers have to one another.

This rule is generally divided into Single, and Compound Distributive Proportion.

SINGLE DISTRIBUTIVE PROPORTION.

RULE.—As the sum of the several stocks, is to each particular stock, so is the whole gain or loss, to each particular gain or loss: OR THUS, As the sum of the given numbers, is to each particular number, so is the number to be divided, to the parts required.

PROOF.—The sum of the answers is equal to the number to be divided when the work is right.

EXAMPLE.—Three merchants A, B, and C, join in company, A put in £750; B £460, and C £500, they gain £684; how much must each receive?

A's B's C's	£	£	£	£	
750+460+500=	1710 :	750 ::	684 :	300=	A's share.
	1710 :	460 ::	684 :	184=	B's share.
	1710 :	500 ::	684 :	200=	C's share.
				684=	Proof.

I here add together the stock of A, B, and C, the sum is £1710, I then say as this sum, is to A's stock £750, so is the whole gain £684, to A's share of the gain, and working by simple proportion the answer comes out £300=A's gain. In the second stating we put B's stock in the middle, the first and third terms always remaining the same, we have £184 for B's gain. In the third stating we have C's stock in the middle, and the answer comes out £200 C's gain. But the sum of these answers is £684, which was the sum gained, therefore the work is correct.

I have purposely suppressed the operation in this example, as the pupil should now be well acquainted with the method of performing it.—I shall illustrate the second rule also.

EXAMPLE.—Divide the number 1296 into parts having to each other the same proportion as 2, 6, 10.

$2 + 6 + 10 = 18 : 2 :: 1296 : 144 =$ first part.
$18 : 6 :: 1296 : 432 =$ second part.
$18 : 10 :: 1296 : 720 =$ third part.

sum of parts $= 1296 =$ sum to be divided.

The operation here is the same as in last examples. It may also be proved, that 144 : 432 : : 2 : 6, and that 432 : 720 : : 6 : 10, therefore 1296 is divided proportionally to the given numbers 2, 6, 10.

EXERCISES.

1. Two merchants A and B, put equal sums into a common stock, and gain £257; how much of the gain must each receive? Ans. £128 10s. each.
2. Two merchants in company gain £91, K's stock was £300, and L's £400; what must each have of the gain? Ans. A £39, B £52.
3. Three merchants enter into partnership, the stock of X was £240, of Y £480, and of Z £720, they gain £480; what should each receive? Ans. X £80, Y £160, Z £240.
4. Messrs. Purse, Risk, Meanwell, and Luckless engage in a speculation by which they lost £546 10s., their stocks were in the proportion of 4, 5, 6, 9; what must each sustain of the loss? Ans. P. £91 1s. 8d., R. £113 17s. 1d., M. £136 12s. 6d., and L. £204 18s. 9d.
5. Three persons enter into company, their stocks are in the proportion of 3, 5, 7, they gain £1845; required the share of each? Ans. £369, £615, £861.
6. The valuation of a parish is £17100, and an assessment of £261 is granted to the poor, there are five heritors whose valuations are £8271, £4626, £3150, £801, and £252; what part of the assessment must each pay? Ans. £126 4s. 9¾d. $\frac{87}{95}$, £70 12s. 1¾d. $\frac{7}{95}$, £48 1s. 6¾d. $\frac{15}{19}$, £12 4s. 6¾d. $\frac{72}{95}$, £3 16s. 11¾d. $\frac{44}{95}$.
7. A common containing 783 acres is to be divided among three gentlemen, C, D, E, in the proportion of 4, 6, 8, required the share of each. Ans. C 174, D 261, E 348 acres.
8. Three fleshers, P, Q, R, rent a field for £96 18s., and agree to pay rent in proportion to the stock each puts on, now P put on 36 sheep, Q 54, and R 231; what part of the rent should each pay? Ans. P £10 17s. 4 $\frac{16}{107}$d., Q £16 6s. 0 $\frac{24}{107}$d., R £69 14s. 7 $\frac{67}{107}$d.
9. A bankrupt's effects amount to £877 10s., he owes to M £220 16s. 6d., to N £312, to O £117 12s. 6d., to P £106 12s. 6d., to Q £200 6s., and to R £124 12s. 6d.; how much does he pay per £, and how much does each creditor receive, supposing the agent's charges and other expenses amount to £66? Ans. 15/ per £, and M £165 12s. 4½d., N £234, O £88 4s. 4½d., P £79 19s. 4½d., Q £150 4s. 6d., R £93 9s. 4½d.
10. Four merchants purchase a ship, of which A paid £645,

B £426, C £780, and D £972; they receive for freight £417 7s. 9d.; what must each partner have of the profits, after deducting £123 7s. 9d. for insurance, repairs, &c.? Ans. A £67 3s. 5$\frac{1}{2}$d. $\frac{274}{941}$, B £44 7s. 3$\frac{1}{2}$d. $\frac{890}{941}$, C £81 4s. 7$\frac{3}{4}$d. $\frac{827}{941}$, D £101 4s. 6$\frac{3}{4}$d. $\frac{321}{941}$.

11. The stock of an insurance company consists of a thousand shares; what must Z who holds 20 shares pay, to cover a loss of £5660? Ans. £113 4s.

12. A merchant bequeathed his whole property amounting to £13600, among his three nephews S, T, and W, giving to S £6200, to T £4800, and to W £2600, but before his death his estate was worth £25700 and his will not being altered; what should each receive after deducting £1285 for duty, &c.? Ans. S £11130 7s. 4$\frac{2}{4}$d. $\frac{16}{17}$, T £8617 1s. 2$\frac{2}{4}$d. $\frac{8}{17}$, W £4667 11s. 5$\frac{1}{2}$d. $\frac{10}{17}$.

COMPOUND DISTRIBUTIVE PROPORTION.

Compound Distributive Proportion is when the unequal stocks of the several partners remain in company for unequal times.

Rule.—As the sum of the products of each particular stock and time, is to each particular product, so is the whole gain or loss, to each particular gain or loss. Proof as in last case.

Example.—Two merchants enter into company, B put in £60 for 4 months, and C £80 for six months, they clear £12 16s.; how must it be divided?

$60 \times 4 = 240 =$ product of B's stock and time.
$80 \times 6 = 480 =$ ——— C's stock and time.
$\overline{720} =$ Sum of products.
As 720 : 240 : : £12 16s. : £4 5s. 4d. = B's gain.
As 720 : 480 : : £12 16s. : £8 10s. 8d. = C's gain.
£12 16s. 0d. Proof.

I here multiply B's stock by the time it continued in company, the product is 240; I then multiply C's stock and time in the same manner, the product is 480; and the sum of these products is 720, I then say by the rule, as 720, the sum, is to 240 the first product, so is £12 16s. the whole gain, to B's share. In the second stating C's share of gain comes out by making the second product the middle term.

EXERCISES.

1. Two young gentlemen agree to pay the expense of an excursion in proportion to their rentals, and the time they have been in possession of them, A has possessed a rental of £6780 for 6 years, and B of £4920 for 8 years, they spend £2400; what must each pay?
   Ans. A £1219 15s. 9½d. $\frac{334}{667}$.; B £1180 4s. 2¼d. $\frac{333}{667}$.
2. Three graziers rent a field for £24, into which A put 40 bullocks for 4 months, B 30 for two months, C 36 for 5 months; what part of the rent should each pay?
   Ans. A £9 12s.; B £3 12s.; C £10 16s.
3. Three merchants enter into copartnery, A put into the common stock £4000 for 12 months, B £3000 for 15 months, and C £5000 for 8 months, they clear £790; what must each receive, allowing A £125 for managing the concern? Ans. A £365, B £225, C £200.
4. Three merchants enter into copartnery for 12 months, A put in at first £364, and 4 months after £40; B put in at first £408, and 7 months after £86; C put in at first £148, and 3 months after £86, and 5 months after that £100, they gain £1436; what must each receive of the profits? Ans. A £519 5s. 7½d. $\frac{1140}{3241}$, B £589 19s. 0¼d. $\frac{2567}{3241}$, C £326 15s. 3¾d. $\frac{2505}{3241}$.
5. Three merchants A, B and C enter into copartnery for 16 months, A put in at first £100, and at the end of 8 months withdrew £40, and 4 months after put in £140; B at first put in £200, and 6 months after £50, four months after he withdrew £100; C at first put in £150, and 4 months after withdrew £50, 8 months after which he put in £100, they gained £357: how much must each have?
   Ans. A £92, B £155, C £110.

COMMISSION AND BROKERAGE.

COMMISSION AND BROKERAGE are certain fixed, or stipulated per centages allowed to agents for transacting business for others.

RULE.—When the rate amounts to, or exceeds £1 per cent.; multiply the value of the transaction by it, and divide the product by 100. Or take parts of 100 with it. But when the rate is under £1 per cent.; take parts of a £ with it, and divide the result by 100.

EXAMPLE.—Calculate the commission on £600 at £2½, and at 5/6 per cent.

600 2½ ——— 300 1200 ——— 15,00 Ans. £15.	or thus $2\frac{1}{2}=\frac{1}{40}$)600 Ans. £15.	5=¼	600 at 5/6. 6=$\frac{1}{10}$	150 15 ——— 1,65 20 ——— 13,00. Ans. £1 13s.

The division by 100 is here performed by cutting off two places from the right of the whole number.

EXERCISES.

1. What is the brokerage on £3650, at 4/6 per cent.? Ans. £8 4s. 3d.
2. What is the commission on £680, at 2 per cent. Ans. £13 12s.
3. What should my agent charge for transacting business to the amount of £960, when I allow him 1½ per cent? Ans. £14 8s.
4. What does a broker receive for selling out stock to the amount of half a million? Ans. £625.
5. If an agent transact business to the amount of £846200 per annum, and is allowed 2¾ per cent.; what is his neat income, supposing he loses by bad debts £5 per cent. on his nominal income? Ans. £22106 19s. 6d.
6. When a factor is allowed 6/8 per cent. for commission; what should he charge for transacting business to the amount of £2400? Ans. £8.
7. My agent writes me that he has transacted business on my account, to the value of £4360; what commission is he entitled to at 3¼ per cent? Ans. £141 14s.
8. Calculate the commission on £1800, at ¼, ½, ¾, and 1 per cent. Ans. £4 10s., £9., £13 10s., £18.
9. Calculate the commission on £3600, at 2, 3, 4, 5 per cent. Ans. £72. £108. £144. £180.
10. Calculate the brokerage on £1000, at 2/6, 3/4, 6/, 9/, per cent. Ans. £1 5s., £1 13s. 4d., £3., £4 10s.
11. Suppose six hundred and seventy-eight millions of stock be transferred at the Bank of England in a-year; how much money does this business put into the hands of the stock-brokers, they being allowed ⅛ per cent. both for buying, and selling? Ans. £1695000.
12. A banking company discount bills to the amount of half a million annually, and charge ¼ per cent. commission to defray the expenses of the business; now, if they employ

15 clerks with a salary on an average of 110 guineas each, and pay for their premises £250, and for coal, candle, books, &c. 210 guineas; how much stands to the credit of the company after deducting these charges? Ans. £297.

13. I have sold a consignment for £2648, on which I paid duty and other charges £568; for how much do I stand indebted to my employer, after deducting $2\frac{1}{2}$ per cent. commission? Ans. £2013 16s.

14. Purchased goods for my employer to the amount of £986, paid for packing, porterage, &c. £5 10s. 6d., Custom-house dues £15 9s. 6d.; what is the amount of the invoice, after adding $1\frac{1}{2}$ per cent. commission?
Ans. £1022 2s. $1\frac{1}{2}$d.

15. My Factor in Jamaica writes me, that he has purchased goods on my account to the value of £1640; what does his commission come to at $8\frac{1}{4}$ per cent.? Ans. £135 6s.

16. An agent purchased goods for his employer to the amount of £1540, and paid expenses of shipment £20 10s.; what does the invoice amount to, adding commission at $3\frac{3}{4}$ per cent.? Ans. £1619 0s. $4\frac{1}{2}$d.

SIMPLE INTEREST.

INTEREST is a certain rate per cent. per annum given by the borrower, to the lender for the use of his money.

The sum lent is called PRINCIPAL.

The rate agreed upon is called INTEREST.

The sum of principal and interest is called AMOUNT.

CASE 1.

To calculate the interest of any sum, for any number of years, at any rate per cent.

RULE.—Multiply the principal by the rate, and the number of years, and divide the product by 100.

EXAMPLE.—What is the interest of £684 for 4 years at $2\frac{1}{2}$ per cent.?

684
$2\frac{1}{2}$
342
1368
1710
4
68,40
20
8,00 Ans. £68 8s.

The operation here is so very simple that it requires no explanation. The division by 100 is always performed in interest by cutting off 2 places to the right which has already been explained.

EXERCISES.

1. Calculate the interest on £520 for a year, at 3 per cent. Ans. £15 12s.
2. Calculate the interest on £843 10s. for 5 years, at $4\frac{1}{2}$ per cent. Ans. £189 15s. 9d.
3. Calculate the interest on £894 for $5\frac{1}{2}$ years, at $3\frac{1}{4}$ per cent. Ans. £159 16s. $0\frac{1}{2}$d. $\frac{2}{5}$.
4. Calculate the interest on £75 for 3 years, at 5 per cent. Ans. £11 5s.
5. Calculate the amount of £320 for 6 years, at $2\frac{3}{4}$ per cent. Ans. £372 16s.
6. Calculate the amount of £40 at 5 per cent., for 21 years. Ans. £82.
7. Calculate the interest on £966 for $4\frac{1}{2}$ years, at 5, 4, 3, and 2 per cent. Ans. £217 7s., £173 17s. $7\frac{1}{2}$d., £130 8s. $2\frac{1}{4}$d. $\frac{3}{5}$, £86 18s. $9\frac{1}{2}$d. $\frac{2}{5}$.
8. Calculate the interest on £3460 for $5\frac{1}{4}$ years, at $4\frac{3}{4}$, $4\frac{1}{2}$, $4\frac{1}{4}$ per cent. Ans. £862 16s. 9d., £817 8s. 6d., £772 0s. 3d.
9. Calculate the interest on £1100 at $2\frac{3}{4}$ per cent. for 1, 2, 3, 4 years. Ans. £30 5s., £60 10s., £90 15s., £121.
10. Calculate the interest on £568 at $4\frac{1}{4}$ per cent. for $1\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{3}{4}$, $4\frac{1}{2}$ years. Ans. £36 4s. $2\frac{1}{4}$d. $\frac{3}{5}$., £54 6s. $3\frac{1}{2}$d. $\frac{2}{5}$., £90 10s. 6d., £108 12s. $7\frac{1}{2}$d.

CASE 2.

To calculate the interest for any number of months.

RULE.—Multiply by the rate, and take parts of the product for the months, then divide by 100 as in last case.

EXAMPLES.—Calculate the interest on £650, at 3 per cent. for 4 months.

	650
mo.	3
$4 = \frac{1}{3}$	)1950
	6,50
	20
	10,00 Ans. £6 10s.

Having multiplied by the per cent., then because 4 months are $\frac{1}{3}$ of a year I take $\frac{1}{3}$ of the product, and cut off two figures to the right as in last case.

EXERCISES.

1. Calculate the interest on £420 at 3½ per cent. for 1, 2, 3, and 4 months.
   Ans. £1 4s. 6d., £2 9s., £3 13s. 6d., £4 18s.
2. Calculate the interest on £1640 at 4¼ per cent. for 5, 6, 7, and 8 months.
   Ans. £29 0s. 10d., £34 17s., £40 13s. 2d, £46 9s. 4d.
3. Calculate the interest on £18376 at 2¾ per cent. for 9, 10, and 11 months.
   Ans. £179 0s. 1½d., £421 2s. 4d., £463 4s. 6⅝d. ⅛.
4. Calculate the interest on £2000 at 5 per cent. for 2¼, 3½, 4¾, 6⅗ months.
   Ans. £18 15s., £29 3s. 4d., £39 11s. 8d., £55.

Case 3.

To calculate the interest on any sum, for any number of days, at any rate per cent.

Rule.—Multiply the principal by the days, and by double the rate per cent., and divide the product by 73000. At 5 per cent. multiply the principal by the days and divide by 7300.

Example.—Calculate the interest on £650 8s. for 65 days at 4¼ per cent.

```
            650.4
               65
            32520
           39024
          42276.0
              8.5
          211380
         338208
73.000)359.3460(4.922
        292         £4 18 5½.
         673          Ans.
         657
          164
          146
           186
           146
            40
```

The operation here is performed decimally, and the quotient valued mentally, which generally saves much labour in calculating the interest for days. It is unnecessary to run out the decimal more than three places, as this always gives the farthings correctly, and the rule for valuing the decimal mentally extends only to three places.

EXERCISES.

1. Calculate the interest on £96 for 20, 30, 40, 50 days, at 5 per cent. Ans. 5/3 $\frac{9}{73}$, 7/10½ $\frac{54}{73}$, 10/6 $\frac{18}{73}$, 13/1¾ $\frac{17}{73}$.

2. Calculate the interest on £345 10s. for 60, 70, 80, 90, days, at 4½ per cent. Ans. £2 11s. 1¼d. $\frac{191}{365}$, £2 19s. 7½d. $\frac{162}{365}$, £3 8s. 1¾d. $\frac{132}{365}$, £3 16s. 8$\frac{26}{365}$d.
3. Calculate the interest on £520 15s. for 57, 64, 122, 204 days, at 3¼ per cent. Ans. £2 12s. 10¼d., £2 19s. 4¼d., £5 13s. 1½d. £9 9s. 2¼d.
4. Calculate the interest on £956 12s. for 82, 99, 110, 263 days, at 2¾ per cent. Ans. £5 18s. 2¼d., £7 2s. 8½d. £7 18s. 6½d., £18 19s. 1¼d.
5. Calculate the interest on £2321 16s. for 22, 46, 482, 631 days; at 3¾ per cent. Ans. £5 4s. 11½d., £10 19s. 5½d., £114 19s. 6¼d, £150 10s. 4¾d.
6. Calculate the interest on £237 6s. 6d. from 1st January, to 15th August at 4½ per cent. Ans. £6 12s. 3d.
7. Calculate the interest on £1000 from Whitsunday, to Martinmas at 3 per cent. Ans. £14 15s. 10¾d.
8. Calculate the interest on £500 from Candlemas to Lammas at 5 per cent. Ans. £12 6s. 6¾d.
9. Calculate the interest on £425 15s. from Martinmas to Whitsunday at 2½ per cent. Ans. £5 7s. 10¾d.
10. Calculate the interest on £845 12s. 6d. from Ladyday to Michaelmas at 3 per cent. Ans. £13 1s. 4d.
11. Calculate the interest on £1560 10s. 9d. from Midsummer till Christmas at 5 per cent. Ans. £39 6s. 8¼d.
12. Lent my friend £300 on the 17th July, at 4 per cent., which he is to repay on the 1st March; what should I then receive? Ans. £307 9s. 3½d.
13. Borrowed £750 4s. on the 12th February, which I am to repay with interest at 5 per cent. on Michaelmas day; how much money will answer this demand? Ans. £773 14s. 8d.
14. What is the interest on £5340 from 7th June till 17th December at 3¼ per cent.? Ans. £91 15s. 4½d.
15. A debt of £690 9s. 6d. was due 2d January 1828, and paid 12th August; how much did it amount to, reckoning interest at 4 per cent.? Ans. £707 7s.

BANKING-HOUSE CALCULATIONS.

Banking-House Calculations consist chiefly in the application of simple interest, to ascertain the interest due on **deposit**, and other accounts on which partial payments are made,

also on CASH and CURRENT ACCOUNTS, and the DISCOUNT ON BILLS, &c.

CASE 1.

To calculate the interest, when partial payments are made.

RULE.—Subtract the several sums paid, in the order of their dates; then multiply the original sum by the days from the time it became due till the first payment was made, and the several balances by the days between the preceding and subsequent payments; the sum of these products multiplied by double the rate, and divided by 73000 gives the interest due.

EXAMPLE.—Deposited in the Royal Bank on the 6th of July £875, which I drew as follows, 1st Sept. £210, 19th Oct. £120, 12th Dec. £245, and the balance on the 3d Feb., how much interest was then due at 3 per cent.?

		Sum.	Days.	Products.
July 6.	Deposited	875	57	49875
Sept. 1.	Drew	210		
	due	665	48	31920
Oct. 19.	Drew	120		
	due	545	54	29430
Dec. 12.	Drew	245		
	due	300	53	15900
Feb. 3.	Drew	300		127125
				6
			73.000)	762.750
				10.448
			Ans.	£10 8 11¾

I here multiply £875 by 57, the days between 6th July and 1st Sept. then subtract the £210 drawn, and multiply the remainder by 48, the days between 1st Sept. and 19th October, again subtract £120, and so on. I then add the several products, and multiply the sum by 6, double the rate, and divide this product by 73000 which is here done decimally, and the quotient valued mentally.

EXERCISES.

1. A bill for £860 due 9th March, was paid in the following manner, March 30th £250, May 2d £300, June 15th £75, July 7th £135, and the balance with interest at 4½ per cent. August 10; how much did the last payment amount to? Ans. £107 8s. 11¼d.
2. Received from the bank, on the 18th Septr. £1000, ¼ of which was paid on Michaelmas, ¼ on Christmas, ¼ on Ladyday, and ¼ on Midsummer day; how much interest is due at 3¾ per cent.? Ans. £14 15s. 10¾d.
3. A bill for £1500 was due on Candlemas, and was paid by the following instalments, viz. £500 on Candlemas day, 500£ on Whitsunday, and the balance with interest on

Lammas; how much did the last payment amount to, reckoning legal interest? Ans. £519 6s. 3¼d.

4. Borrowed from the bank on the 20th August at 4 per cent. £5650, to pay off a bond over my estate, which I repaid by the following instalments, Nov. 30th £300, January 16th £250, May 31st £2500, August 1st £2000, and the balance November 20th; what interest was then due? Ans. £191 2s. 6¾d.

5. Lodged in the bank on the 17th November £2000, and drew for it in the following order, Decr. 10th £320, January 31st £250, March 27th £600, June 18th £596, and the balance August 12th; what interest was then due at 2¾ per cent.? Ans. £22 2s. 8d.

6. About to engage in a mercantile speculation, I drew from the bank on the 19th Feb. £16000, which I refunded by the following instalments, May 30th £5300, July 20th £2700, September 10th £6500, and the balance November 30th; what interest is due at 4½ per cent? Ans. £312 8s. 6¼d.

CASE 2.

To calculate the interest on cash, and current accounts, &c.

RULE.—Take the sums in the order of their dates, add them when they are both Dr. or both Cr., and the sum is either Dr. or Cr., the same as the numbers added; but subtract when the one is Dr. and the other Cr., affixing the sign of the greatest to the remainder; multiply the several balances by the days between the transactions as in last case; and when the balance is sometimes due to, and sometimes due by the bank, extend the Dr. and Cr. products in different columns, then multiply the sum of the Dr. column by double the interest exacted, and of the Cr. column by double the interest granted by the bank, subtract these products, and divide the remainder by 73000 for the interest required.

EXAMPLE.—Required to whom the balance is due on the following account on the 1st December, and how much, including interest, the bank exacting 4, and granting 3 per cent. on balances.

Mr. A. B's. account-current with the Bank of Scotland.

DR. CR.

Feb. 26.	To Cash.	520	March 16.	By Cash.	650
April 10.	To ——	460	May 15.	By ——	570
—— 30.	To ——	230	June 30.	By ——	395
June 12.	To ——	150	July 20.	By ——	400
Aug. 15.	To ——	730	Oct. 25.	By ——	250
Sept. 24.	To ——	600	Nov. 14.	By ——	859

Dates.		Sums.	Days.	Dr. Products.	Cr. Products.
Feb. 26.	To Cash	520	18	9360	
March 16.	By ——	650			
	Cr.	130	25	—— ——	3250
April 10.	To ——	460			
	Dr.	330	20	6600	
—— 30.	To ——	230			
	Dr.	560	15	8400	
May 15.	By ——	570			
	Cr.	10	28	———	280
June 12.	To ——	150			
	Dr.	140	18	2520	
—— 30.	By ——	395			
	Cr.	255	20	———	5100
July 20.	By ——	400			
	Cr.	655	26	———	17030
Aug. 15.	To ——	730			
	Dr.	75	40	3000	
Sept. 24.	To ——	600			
	Dr.	675	31	20925	
Oct. 25.	By ——	250			
	Dr.	425	20	8500	
Nov. 14.	By ——	859			
Balance due A. B.		434	17	———	7378
Interest due to bank,		3..15..8		59305	33038
1st Dec. due A. B.		£430..4..4		8	6
				474440	198228
				198228	
				73.000)276.212	
				£3..15..8	

In this example A. B. on the 26th Feb. drew £520, he is therefore Dr. to that amount; on the 16th March he lodged £650, and this exceeds the sum for which he was Dr. by £130, he is therefore Cr. by £130; on the 10th April he drew £460, and this exceeds the sum by which he was Cr. by £330, he is therefore Dr. £330; on the 30th April he drew £230, which added to £330, makes £560 now due the bank; and so on with every other receipt and payment. Again between the date of the first draft, and first deposit are 18 days, we therefore multiply £520 by 18 and write the product in the column

of Dr. products; also between the date of the first deposit and second draft are 25 days, we therefore multiply £130 by 25 and write the product in the column of Cr. products, and so on with every balance according as it is Dr. or Cr. In this example the balance of principal is due to A. B. and the interest to the bank, therefore the difference of these sums is due to A. B. on the 1st December.

EXERCISES.

1st. Required the balance, and to whom due, on the following account, on the 30th December, the bank exacting 4½, and granting 3½ per cent. on balances.

Dr.—Mr. B. C's Cash account with the Royal Bank.—Cr.

Jan. 25.	To Cash	257	Jan. 1.	By Balance	360
Feb. 19.	To ——	480	April 12.	By Cash	275
May 6.	To ——	289	June 20.	By ——	500
Aug. 5.	To ——	1000	Sept. 30.	By ——	560
Oct. 25.	To ——	560	Nov. 16.	By ——	800
Dec. 12.	To ——	140	Dec. 24.	By ——	380

Ans. Due B. C. £135 10s. 7¾d.

2. Messrs. D. and E. in account-current with the Commercial Bank.

Dr. Cr.

Jan. 4.	To Cash	200	Jan. 30.	By Cash	150
Feb. 25.	To ——	320	March 18.	By ——	500
May 17.	To ——	400	April 7.	By ——	90
June 30.	To ——	190	July 28.	By ——	240
Aug. 19.	To ——	87	Sept. 9.	By ——	620
Oct. 31.	To ——	705	Dec. 17.	By ——	500
Dec. 8.	To ——	220			

Calculate the balance including interest, on the above account till the 20th Dec., the bank exacting 4½ and granting 3½ per cent. on balances. Ans. Due the Bank £25 8s. 6¾d.

3. Messrs. T. and S. in account-current with the British Linen Company Bank.

Dr. Cr.

Jan. 30.	To Cash	640	Jan. 1.	By Balance	720
March 18.	To ——	350	Feb. 20.	By Cash	210
April 12.	To ——	190	May 17.	By ——	430
June 24.	To ——	500	July 7.	By ——	100
Sept. 10.	To ——	470	Nov. 1.	By ——	680

Required to whom the interest is due, and how much, on the 30th November, the bank exacting 5 and granting 4½ per cent. on balances. Ans. £4 6s. 8d. due the Bank.

4. What is the interest, and to whom due, on the following account, on the 31st December, the bank exacting $4\frac{1}{4}$ and granting $3\frac{1}{2}$ per cent. on balances?

Messrs. F. & Co. in account-curt. with Sir W. F. J. H. & Co.
DR. CR.

Feb. 12.	To Cash	426	March 15.	By Cash	520
April 16.	To ——	760	May 3.	By ——	675
July 18.	To ——	230	July 30.	By ——	396
Aug. 13.	To ——	434	Sept. 10.	By ——	873
—— 28.	To ——	325	Nov. 8.	By ——	235
Oct. 19.	To ——	250	Dec. 15.	By ——	290
Dec. 1.	To ——	536			

Ans. Interest due to Bank £2 10s. $8\frac{3}{4}$d.

5. To whom is the balance due on the following account on the 30th August, and how much including interest, the bank exacting $3\frac{3}{4}$, and granting 3 per cent. on balances?

DR.—Mr. H. in account-curt. with Ramsays, Bonars & Co.—CR.

Sept. 3.	To Cash	730	Sept. 30.	By Cash	620
Oct. 19.	To ——	350	Oct. 28.	By ——	580
Nov. 20.	To ——	216	Dec. 4.	By ——	637
Jan. 18.	To ——	166	March 1.	By ——	400
Feb. 8.	To ——	430	May 14.	By ——	780
April 24.	To ——	508	July 10.	By ——	390
June 30.	To ——	650	—— 20.	By ——	486
Aug. 10.	To ——	743			

Ans. Due Mr. H. £105 8s. $2\frac{1}{4}$d.

6. To whom is the balance due on the following account, on the 24th December, and how much including interest, the bank exacting 4, and granting $3\frac{1}{4}$ per cent. on balances?

DR.—Mr. P. in account-current with the Leith Bank.—CR.

Jan. 30.	To Cash	156	Jan. 1.	By Balance	230
Feb. 3.	To ——	320	Feb. 28.	By Cash	100
March 16.	To ——	210	April 7.	By ——	260
April 30.	To ——	134	May 19.	By ——	400
June 4.	To ——	500	Aug. 3.	By ——	500
July 1.	To ——	180	—— 20.	By ——	300
Sept. 16.	To ——	420	Oct. 30.	By ——	110
Nov. 20.	To ——	632	Nov. 8.	By ——	545
Dec. 4.	To ——	225	Dec. 15.	By ——	275

Ans. Due the bank £61 9s. 11d.

7. What is the balance including interest, on the following account, and to whom due on the 29th December, the bank exacting 4½, and granting 4 per cent. on balances?

Mr. F. in account-current with the National Bank of Scotland.

Dr.					Cr.
Jan. 3.	To Cash	50	Jan. 20.	By Cash	35
Feb. 6.	To ——	80	Feb. 24.	By ——	60
March 12.	To ——	360	March 18.	By ——	430
April 9.	To ——	400	April 30.	By ——	500
—— 15.	To ——	120	June 7.	By ——	200
May 20.	To ——	390	July 1.	By ——	30
June 16.	To ——	270	Aug. 7.	By ——	320
July 20.	To ——	100	Oct. 20.	By ——	75
Sept. 18.	To ——	230	Nov. 25.	By ——	584
Nov. 14.	To ——	330	Dec. 15.	By ——	200
Dec. 5.	To ——	412	—— 21.	By ——	315

Ans. Due the Bank £5 0s. 3½d.

To calculate the interest on bonds, when partial payments are made at intervals greater than a year.

Rule.—Find the interest due at the time the payment is made, which add to the principal, and from the sum subtract the payment, the balance is the new principal.

Example.—Lent on bond 1st Feb. 1826 £2500 at 5 per cent., of which £500 was paid 8th April 1827, and £500 on 16th Sept. 1828, and the balance 25th Dec. 1829; how much did it amount to?

1826.				
Feb. 1.	Principal due . . .	£2500	0	0
	Interest for 1 year and 66 days .	147	12	0¼
1827.	Amount .	£2647	12	0¼
April 8.	Received in part . .	500	0	0
	Balance .	£2147	12	0¼
	Interest for one year and 161 days	154	14	11
1828.	Amount .	£2302	6	11¼
Sept. 16.	Received in part . . .	500	0	0
	Balance .	£1802	6	11¼
	Interest for a year and 100 days	114	16	1¾
1829.				
Dec. 25.	Balance due at this date amounts to	£1917	3	1¼

1. Borrowed on bond 15th March 1824 £1000, at 4 per cent., which was paid in the following order, May 20th 1825, £120, August 16th 1826 £250, November 20th 1827 £530, and the balance Feb. 25th 1828; what was the amount of last payment? Ans. £241 8s. $9\frac{1}{4}$d.
2. Lent on bond at $4\frac{1}{2}$ per cent. £500, on the 12th May 1826, of which I have received the following payments, July 20th 1827 £60, August 16th 1828 £180, September 29th 1829 £220, the balance is to be paid on the 30th January 1831; how much should I then receive?

   Ans. £111 4s. $4\frac{3}{4}$d.

Case 3.

To discount bills.

Rule.—Calculate the interest on the sum specified in the bill, from the day it is discounted, till three days beyond the term of the bill, for the discount; which being subtracted from the bill, leaves the proceeds.

Example.—What is the discount and proceeds on a bill for £420, dated 15th March at 4 months, and discounted 25th April, at legal interest?

This bill is due 18th July, which from 25th April is 84 days.

```
         £
        420 = Bill.
         84 = Days.
73.00)332.80
        4  11  2 = Discount.
     £415   8 10 = Proceeds.
```

It is evident both from the rule, and the operation, that the process here is simply finding the interest for the days the bill has to run, which gives the discount, and the discount taken from the bill leaves the proceeds, or sum the holder of the bill receives.

EXERCISES.

1. A bill for £400 dated 17th Sept. at 3 months was discounted on the 3d October; what is the proceeds?

   Ans. £395 14s. $6\frac{1}{4}$d.
2. A bill dated 7th July at 4 months for £250 was discounted 10th August; what did the holder receive?

   Ans. £247 9s. 7d.
3. A bill for £1000, drawn at 5 months, and dated 20th May was discounted on the 25th July at 4 per cent; how much did the holder receive, allowing also $\frac{1}{2}$ per cent. commission on the proceeds? Ans. £985 3s. $8\frac{1}{4}$d.

4. What is the discount and proceeds on a bill of £750, dated 1st February at 2 months, and discounted 15th February, at 4½ per cent.? Ans. Dis. £4 8s. 9¼d., Proc. £745 11s. 2¾d.
5. What is the discount on a bill of £600, for 63 days?
Ans. £5 3s. 6¾d.
6. Required the amount of proceeds on the following bills discounted 18th November.
K. L's. bill for £630, due 6th January.
M. N's. bill for £450, due 20th January.
P. Q's. bill for £370, due 29th January.
R. S's. bill for £240, due 15th February. Ans. £1675 6s. 3d.
7. A banker discounted a bill for £2000 which had 40 days to run, and besides legal interest, charged ¼ per cent. commission on the proceeds; how much did the holder of the bill receive, and what interest had the banker for his money?
Ans. Holder received £1979 1s. 11d. Banker's interest £9 10s. 9d. per cent.

STOCK JOBBING CALCULATIONS.

Case 1.

To calculate the value of stock sold.

Rule.—Deduct ⅛ from the price per cent., multiply the quantity of stock by the remainder, the product divided by 100 gives the value.

Example.—Calculate the value of £800 three per cent. consols, at 87⅜ per cent.

Stock	£800	The calculation in this case, after deducting an eighth for brokerage, is evidently the same as simple interest for one year.
Price 87⅜—⅛=87¼	200	
	69600	
	698,00	

Value £698

EXERCISES.

1. Value £750, three per cent. consols, sold at 87⅛ per cent.
Ans. £652 10s.
2. Value £6380, three per cent. red. sold at 86⅜ per cent.
Ans. £5518 14s.
3. Find the value of £985, four per cent. consols, sold at 98⅝ per cent. Ans. £971 9s. 1½d.
4. What is the value of £2500, Navy 5 per cents., sold at 112¼ per cent.? Ans. £2809 7s. 6d.

5. Sold £485 India stock, at 242 per cent. what did I receive? Ans £1178 1s. 10½d.
6. Sold £1600 three per cents. red. at 87¾⅛ per cent. what did I receive? Ans. £1402 8s.

CASE 2.

To calculate the price of stock bought.

RULE.—Add ⅛ to the price, then proceed as in last case.

EXAMPLE.—What is the cost of £600 three per cents. red. bought at 86⅞ per cent.?

Stock, £600	The only difference here from last case, is the adding instead of subtracting ⅛.
Price, 86⅞ + ⅛ = 87	
522,00	
Cost £522	

EXERCISES.

1. What will £500 three per cent. consols cost, when they sell at 87⅜ per cent.? Ans. £437 10s.
2. What should I pay for £780 three per cents. red., when they sell at 88¾ per cent.? Ans. £693 4s. 6d.
3. Bought £3650 four per cents. when selling at 108½ per cent.; what did it cost? Ans. £3964 16s. 3d.
4. How much must be paid for £3800 navy five per cents., at 120¼⅜ per cent.? Ans. £4595 12s. 6d.
5. What sum must I remit my broker, to purchase for me £750 bank stock, at 208½ per cent.? Ans. £1564 13s. 9d.
6. What will £250 10s. stock cost, at 88⅞ per cent.? Ans. £222 18s. 10¾d. ¼.

CASE 3.

To calculate what stock any given sum will purchase.

RULE.—Multiply the sum to be invested by 100, and divide the product by the selling price of the stock, increased by ⅛, the quotient is the quantity of stock.

EXAMPLE.—How much stock at 81⅜ per cent. will £652 purchase?

81.5)65200.0(800 Stock.	The operation here is performed decimally, and is so simple as to require no explanation.
6520	
00	

EXERCISES.

1. How much 3 per cent. consols, at 86⅝, will £650 12s. 6d. purchase? Ans. £750.
2. How much stock in the 3 per cents. red., at 85⅞ will £313 18s. purchase? Ans. £365.
3. How much India Stock, at 210¼⅜, will £2109 7s. 6d. purchase? Ans. £1000.

4. My broker invested for me £1318 6s. $8\frac{3}{4}$d. $\frac{3}{8}$ in the 3 per cents. red. when selling at $79\frac{3}{4}$; what quantity of stock should be entered to my name? Ans. £1650 10s.
5. How much stock at $84\frac{7}{8}$ will £8 10s. purchase? Ans. £10.

CASE 4.

To calculate the rate of interest arising from money invested in the Stocks.

RULE.—Multiply the dividend on a hundred £ stock by 100, and divide the product by the selling price increased by $\frac{1}{8}$, the quotient is the rate of interest.

EXAMPLE.—What rate of interest arises from investing money in the 3 per cent. consols, when selling at $87\frac{3}{8}$?

$$\frac{3\times100}{87\frac{3}{8}+\frac{1}{8}}=\frac{300}{87\frac{1}{2}}=\frac{600}{175}=3\frac{3}{7}\text{ per cent., or £3 8s. }6\frac{3}{4}\text{d. }\frac{3}{7}.$$

1. What rate of interest arises from vesting money in the 3 per cents. red., when selling at $75\frac{7}{8}$? Ans. £$3\frac{1}{4}\frac{8}{9}$. per cent.
2. When bank stock yields $10\frac{1}{2}$ per cent.; what interest is obtained when it sells at $224\frac{5}{8}$ per cent.? Ans. £4 13s. $5\frac{1}{4}$d.
3. When the 4 per cents. sell for $104\frac{7}{8}$; what rate of interest is derived from vesting money in them? Ans. £$3\frac{1}{2}\frac{7}{7}$.
4. What interest arises from vesting money in India Stock, when selling at 232, the dividends being 10 per cent.? Ans. £4 6s. 2d.
5. When 3 per cent. consols sell at $84\frac{5}{16}$; what interest do they yield for money invested in them? Ans. £3 11s. $0\frac{1}{2}$d.
6. When 3 per cents. sell at $87\frac{3}{8}$, and 4 per cents. at $104\frac{7}{8}$; which is the preferable investment for money, and how much? Ans. The fours by 7/$7\frac{1}{4}$ $\frac{5}{7}$ per cent.

PROMISCUOUS EXERCISES.

1. What must I pay for £100 a-year in the long annuities, when selling at $16\frac{1}{2}$ years purchase, brokerage $\frac{1}{8}$ per cent. on the cost? Ans. £1652 1s. 3d.
2. What annual pension can I purchase in the long annuities for £1800, when selling at 17 years purchase, brokerage $\frac{1}{8}$ per cent. on the purchase money? Ans. £105 15s.
3. What is the neat proceeds of £80 a-year in the long annuities, sold at $17\frac{1}{2}$ years purchase, brokerage $\frac{1}{8}$ on the gross proceeds? Ans. £1398 5s.
4. When the 3 per cent. consols sell for $86\frac{3}{8}$, how much money must I remit to my broker, to purchase for me an annuity of £150 a-year? Ans. £4325.

5. How must the 3 per cents. sell to yield $4\frac{1}{2}$ per cent. for money invested in them? Ans. £$66\frac{2}{3}$.
6. How must the 4 per cents. sell to give 5 per cent. for money vested in them? Ans. $80\frac{1}{4}$.
7. How much is gained by purchasing £1000 stock at $75\frac{5}{8}$, and selling it at $78\frac{7}{8}$? Ans. £32 10s.
8. By selling £800 stock in the 3 per cents. reduced at $85\frac{5}{8}$, and investing the proceeds in 3 per cent. consols at $85\frac{7}{8}$; how much of this latter stock do I hold? Ans. £$797\frac{461}{687}$.
9. If the 3 per cents. sell for $84\frac{7}{8}$ what should the 4 per cents. sell for, to give the same interest? Ans. £$113\frac{1}{6}$.
10. What must I pay for 12 exchequer bills of £500 each, bearing interest at 2 pence per cent. per day, bought 60 days after date, at a premium of 2/ per cent., and brokerage 1/ per cent.? Ans. £6039.
11. If the 3 per cents. reduced sell for $87\frac{3}{8}$, and the 3 per cent. consols for $86\frac{7}{8}$ on the 1st March; which is the preferable investment, and how much? Ans. Red. by £[illegible].
12. How much per cent. should the 3 per cents. red. sell higher than the 3 per cent. consols from 5th July to 10th Oct.? Ans. £0 14s. $6\frac{1}{4}$d. $\frac{1}{47}$.
13. How much per cent. should the 3 per cent. consols sell higher than the 3 per cents. red., from 10th October till 5th January? Ans. £0 15s. $9\frac{3}{4}$d. $\frac{3}{23}$.
14. Bought £3000 omnium, on which 30 per cent. had been paid, at a premium of 1 per cent.; what should I pay, allowing brokerage $\frac{1}{8}$ per cent.? Ans. £933 15s.
15. Calculate the neat proceeds of £4250 reduced script, sold at $83\frac{1}{8}$ per cent., deducting 4 instalments of £600 each, still unpaid, brokerage $\frac{1}{8}$ per cent.? Ans. £1127 10s.

INSURANCE OFFICE CALCULATIONS.

Insurance is a contract between parties, by which the insurers, for the consideration of a stipulated per centage, engage to exempt the insured from any assigned risk, to which he is exposed.

Case 1.

To calculate the premium and policy duty on land insurance.

RULE.—Multiply the sum to be insured by the premium both in £'s, and divide the product by 100, the quotient is the premium.

To find the policy duty. Multiply the duty on 100, by the number of 100s, the product is the duty, which added to the premium gives the full expense.

EXAMPLES.—What is the expense of insuring my household furniture, books, &c., against accidents by fire, to the amount of £1870, premium 3/ and policy duty 3/ per cent.

1870 × .15 ÷ 100 =	£2	16	1¼	premium.
19 × 3 ÷ 20 =	2	17	0	policy duty.
	£5	13	1¼	full expense.

The sum to be insured is here multiplied by .15 the decimal of 3/, and the product divided by 100 gives £2 16s. 1¼d.

Again we calculate the policy duty on 19 hundred because the duty is charged on the part of 100 the same as on 100, that is £70 pays the same policy duty as 100.

EXERCISES.

1. What is the expense of insuring £850 against risk by fire, premium 2/, and policy duty 3/ per cent.? Ans. £2 4s.
2. What is the expense of insuring £568 10s. on furniture, books, clothes, &c. against accidents by fire, premium 3/, and policy 3/ per cent.? Ans. £1 15s. 0½d. ⅔.
3. What must I pay for insuring £6840 on a spinning mill, against accidents by fire, premium 5/, and policy duty 3/ per cent.? Ans. £27 9s.
4. Insured £1500 on goods on the canal between Edinburgh and Glasgow against accidents, premium 1/, policy duty 2/6 per cent.; what is the expense? Ans. £2 12s. 6d.
5. Insured £2320 on my crop in the barn, and barn-yard against accidents by fire, premium 3/, and policy 3/ per cent.; what must I pay? Ans. £7 1s. 7¼d.
6. What must I pay for insuring my farm stock, valued at £2160, premium 2½, policy 3/ per cent.? Ans. £57 6s.
7. What is the expense of insuring £25680 on a distillery, premium 5/, policy 3/ per cent.? Ans. £102 15s.
8. What is the expense of insuring £6000 on a ship and cargo, in harbour, premium 3/, and policy 2/6 per cent.? Ans. £16 10s.

CASE 2.

To Calculate Sea Insurance.

This case is performed by the same rule as last case, only when the rate is in guineas increase the sum to be insured by $\frac{1}{20}$ of itself, before applying the rule.

EXAMPLE.—Find the expense of insuring £1860 on a ship and cargo, from Greenock to Jamaica, premium 5 guineas per cent. policy 5/ and commission ½ per cent.

$\frac{1}{20}$)1860	5=¼)19	½)1860	£97 13=Premium.
93	£4 15	9,30	4 15=Policy duty.
1953		20	9 6=Commission.
5		6,00	£111 14=Whole expense.
97,65			
)20			
13,00			

It will be observed here that the $\frac{1}{20}$ is added only when finding the premium.

EXERCISES.

1. Find the expense of insuring £1000 from Leith to London, premium 1½ per cent. policy 2/6 and commission ½ per cent. Ans. £21 5s.
2. Calculate the expense of insuring £980 on a ship and cargo, from Leith to Rotterdam, at 3 guineas per cent. policy 5/ and commission ½ per cent. Ans. £38 5s. 4¾d. ⅕.
3. Find the expense of insuring £865 on a cargo, from Malaga to London, premium 2½ per cent., policy 5/ and commission ½ per cent. Ans. £28 4s.
4. Find the expense of insuring £2500 on a ship and cargo, from Rio Janeiró to Dublin, at 6 guineas per cent. policy 5/ and commission ½ per cent. Ans. £176 5s.
5. Find the expense of insuring £65320 on a ship and cargo, from Calcutta to London, premium 8 guineas per cent. policy 5/, and commission ½ per cent. Ans. £5976 19s. 7d.
6. Insurance was effected on 600 chests of tea, valued in the policy at £30 per chest; from China to Liverpool, at £9 per cent. policy 5/, and commission ½ per cent., but upon arrival, it appeared that only 450 chests had been shipped; find how much must be returned for short interest, after deducting ½ per cent. commission, and 1/ per £ of premium for brokerage, also find the whole cost of this insurance to the owners, and how much per cent. it cost them.

Ans. returned for short interest £362 5s., Cost £1392 15s., £10 6s. 4d. per cent.

7. Insurance was effected on 80 puncheons rum from Tobago to Greenock, valued in the policy at £18 a puncheon, premium £8 per cent. to return 3½ per cent. if the ship sail with convoy and arrive safe, policy 5/ and commission ½ per cent. but upon arrival it was found that no more than 65 puncheons had been shipped on board; how much must the insurers return to the insured, deducting ½ per cent. commission, and 1/ per £ of brokerage from the return for short interest? Ans. £60 2s. 4¾d. ½.

8. Insurance was effected on 100 pipes of wine, from Lisbon to Leith, valued in the policy at £38 per pipe, premium 7 guineas per cent. to return 3½ per cent. if the ship sailed with convoy, and arrived safe, which she did; policy 5/, and commission ½ per cent.; what is the neat expense of this insurance, and how much per cent.?

Ans. neat expense £174 16s., £4 12s. per cent.

Case 3.

To calculate how much must be insured to cover any given sum.

Rule.—Multiply the proposed sum by 100, divide this product by the difference between £100 and the sum of the premium, policy, and commission per cent., the quotient is the answer.

Example.—What sum must be insured to cover £650, premium 3¼ per cent. policy 5/, and commission ½ per cent.

$$\frac{650 \times 100}{\underset{100-(3\ 5+5+10)}{\pounds\ \text{s.}\quad \text{s.}\quad \text{s.}}} = \frac{65000}{96} = \pounds677\ \text{1s. 8d.}$$

The process here is so simple as to require no explanation.

1. How much must be insured to cover £1000, premium 4 guineas per cent., policy 5/, and commission ½ per cent.?

Ans. £1052 1s. 6½d. $\frac{1406}{1901}$.

2. How much must be insured to cover £860, premium 2⅝ per cent., policy 2/6, and commission ¼ per cent.?

Ans. £886 11s. 11½d. $\frac{2}{97}$.

3. Insured so as to cover £3000 on a ship and part cargo, from Leith to Cork, premium £3 7s. 6d. per cent., policy 2/6 and commission ½ per cent.; at Cork she took in goods to the value of £1500, find how much must be insured to cover the whole from Cork, to Kingston Jamaica, premium 5 guineas per cent., policy 5/ and commission ½ per cent.

Ans. £4920 4s. 3$\frac{2}{7}$d.

4. Find how much must be insured to cover £16000 on a ship

from London to Bombay, premium 9¼ per cent., policy 5/, and commission ½ per cent.; find also how much must be insured to cover her home, premium, policy, and commission as before.

Ans. Out £17777 15s. 6½d. ⅜, Home £19753 1s. 8¼d. ⅔⅝.

5. Insurance was effected to cover 90 hogsheads of tobacco, from Virginia to Liverpool, valued in the policy at £24 per hhd., premium 5 guineas per cent., policy 5/, and commission ½ per cent., but when the ship arrived it was ascertained that only 60 hhds. had been shipped; how much must the insurers return for short interest, deducting the usual brokerage and commission? Ans. £34 9s. 4¼d. $\frac{622}{1173}$.

6. Insurance was effected on £6000, to cover goods from Port Jackson, New South Wales, to London, at £8 per cent., policy 5/ and commission ½ per cent.; but it was ascertained that goods only to the amount of £4800 were shipped, and the vessel was lost; find how much the insurers must pay to indemnify the insured, the expense of recovering the loss being 2¼ per cent., deducting the usual brokerage and commission from the returns for short interest.

Ans. £5436 9s. 9¼d. .212+.

Case 4.

To calculate averages.

Rule.—As the whole value at risk, is to the whole loss, so is each person's share of the risk, to their share of the loss. And as the whole risk, is to 100, so is the whole loss, to the loss per cent.

Example.—A ship bound from Greenock to Jamaica was overtaken by a storm, in which a mast and rigging, long boat, and anchor were cut away, and to lighten the ship, goods were thrown over board to the value of £800; replacing mast and rigging cost £900, long boat £60, anchor £60. The ship and neat freight were valued at £9600, of which £5600 was insured; the cargo was valued at £12000, of which L8500 was insured; find the average loss per cent. and what each party must sustain of the loss.

Replacing mast and rigging, deducting ⅓,	£600
———— Long boat, deducting ⅓,	40
———— Anchor,	60
Goods thrown over board,	800
Total average loss, =	£1500

Ship and freight insured,	£5600	= 14100
Cargo insured,	8500	
Ship and freight risked,		4000
Cargo risked,		3500
Whole value at risk, =		£21600

As 21600 :	100 ::	1500 :	£$6\frac{17}{18}$			Loss per cent.
As 21600 :	14100 ::	1500 :	£979	3	4	Insurers pay.
As 21600 :	4000 ::	1500 :	£277	15	$6\frac{1}{2}$ $\frac{2}{3}$	Shipowners pay.
As 21600 :	3500 ::	1500 :	£243	1	$1\frac{1}{4}$ $\frac{1}{3}$	Owners of goods pay.
		Proof=	£1500	0	0	

1. A ship from Barbadoes to Leith, suffered damage by a storm in her masts and rigging, repairing which cost £300; afterwards she was overtaken by a French letter of mark, which she beat of, but in the engagement she lost her foremast and had her rigging damaged, and several shot in her hull, repairing of which cost £330, by the water admitted through the bores which the balls made, the cargo was damaged to the amount of £980: in order to encourage the sailors to a desperate resistance, the captain promised £10 to each sailor who should survive the action, and £50 to the relations of those who should be killed, there were twenty sailors, 5 of whom were killed. The ship and neat freight were valued at £6300, of which £4000 was insured, the cargo was valued at £10600, of which £8000 was insured; calculate the average loss which each party must sustain.

   Ans. Insurers pay £1263 1s. $6\frac{3}{4}$d. $\frac{6915}{10647}$, owners of ship pay £290 15s. $4\frac{0}{4}$d. $\frac{7008}{10647}$, owners of goods pay £246 3s. $0\frac{3}{4}$d. $\frac{7371}{10647}$.

2. A ship from London to Leith was driven by stress of weather and stranded; the expense of cutting her off was £40, repairing rigging cut away £120, replacing a cable £30, a new anchor £50, dock dues £10, the cargo sustained damage to the amount of £400; the ship was valued at £3000, of which £1500 was insured in London, and £700 in Leith; value of cargo £6000, of which £2000 was insured in London, and £3000 in Edinburgh; calculate what part of the general average loss each party must sustain.

Insurers in London pay . .	£233	6s.	8d.
Insurers in Leith pay . .	46	13s.	4d.
Insurers in Edinburgh pay . .	200	0s.	0d.
Owners of ship pay . . .	53	6s.	8d.
Owners of goods pay . . .	66	13s.	4d.

3. A ship bound from Bombay to London, suffered damage in her masts and rigging in a storm, which obliged her to put into St. Helena for repairs, on entering the harbour she was bilged, which caused a partial disloading of the cargo, before she could be repaired; the expense of repairing the masts and rigging was £300, and the other

charges amounted to £200; after leaving St. Helena she was taken by a Spanish gun-brig, who carried from her, goods and provisions to the value of £2000. She was afterwards retaken by a British sloop of war, and carried into Portsmouth, where the ship was sold for £9800, and the cargo for £15600, of which $\frac{1}{8}$ was claimed for salvage; the ship and neat freight were valued at £13000, of which £10000 was insured, and the cargo was valued at £18000, of which £12000 was insured; calculate how much each party must sustain of the loss.

Insurers of Ship pay,	£1867	4	$1\frac{2}{4}$	$\frac{4}{31}$.
Insurers of Cargo pay,	2080	12	$10\frac{3}{4}$	$\frac{11}{31}$.
Owners of Ship pay,	586	16	$6\frac{1}{4}$	$\frac{23}{31}$.
Owners of Cargo pay,	1040	6	$5\frac{1}{4}$	$\frac{13}{31}$.

4. Insurance was effected on 60 chests of tea, and 20 hhds. of sugar from the East Indies to England, in the course of the voyage the vessel sprung a leak, by which both the tea and sugar were damaged, and required to be sold off immediately upon the arrival of the vessel; the tea was valued in the policy at £30 per chest, and sold for £48, but if sound it would have brought £60 per chest; the sugar was valued in the policy at £42 per hhd., and sold for £24 per hhd., but if sound it would have brought £36; how much should the owners of the goods receive after deducting $2\frac{1}{2}$ per cent for recovery, the tea belonging to A B & Co., and the sugar to Y Z & Co.?

Ans. A B & Co. £351, Y Z & Co. £273.

COMPOUND INTEREST.

COMPOUND INTEREST arises from adding the interest when due, to the principal, and considering the amount as a new principal.

RULE.—Find the amount of the given sum at the time the interest is first payable, consider this amount as a new principal, and find its amount in the same manner, and so on for each payment. To find the compound interest, subtract the given principal from the last amount, the remainder is the interest.

EXAMPLE.—What is the compound interest, and amount of £1000 for 3 years, at 5 per cent.?

£ 1000	£1000	£ 1050		£1102	10		
5	50	52	10	55	2	6	
50,00	1050	1102	10	£1157	12	6	=Compd. amount.
	5		5	£1000			=Principal.
	52,50	55,12	10	£ 157	12	6	=Compd. interest.
	20	20					
	10,00	2,50					
		12					
		6,00					

The process here is so evident as scarcely to require explanation, for the first year's interest added to the principal becomes the 2d year's principal, and the second year's interest added to the 2d year's principal, becomes the 3d year's principal, and so on for any number of years.

EXERCISES.

1. What will £560 amount to in 3 years, at 5 per cent. compound interest? Ans. £648 5s. 4¾d. $\frac{1}{5}$.
2. What will £900 amount to in 4 years, at 4 per cent. compound interest? Ans. £1052 17s. 5½d. $\frac{77}{100}$.
3. What is the compound interest of £2000 for 5 years, at 3 per cent.? Ans. £318 10s. 11¾d. $\frac{1}{25}$.
4. What is the amount of £800 at compound interest, for 3 years, at 4 per cent. when the interest is payable yearly? Ans. £899 17s. 9¾d. $\frac{3}{5}$.
5. What is the amount of £800 at compound interest, for 3 years, at 4 per cent., interest payable half yearly? Ans. £900 18s. 7½d. $\frac{21}{25}$.
6. What is the amount of £800 at compound interest, for 3 years, at 4 per cent., interest payable quarterly? Ans. £901 9s. 3¼d. $\frac{72}{100}$.
7. A gentleman dying left his son, who was a minor, an estate of £30000 a-year, subject to the following restrictions, viz. £10000 a-year to support the family and defray public burdens during the minority, the remainder to be appropriated to pay off a bond of £150000 upon the property, bearing interest at 5 per cent.; the trustees after paying off the debt, improved the surplus at 3 per cent. compound interest; the heir came of age 12 years after his father's decease, on which day he gave an entertainment to his friends, and tenantry, which cost £3000, and made the following presents, to his brother and sister, each £10000, to each of his 3 trustees £5000, distributed among his

servants £2000, and gave to each of 50 poor people on his estate £40; how much ready money had he the command of after making these donations? Ans. £6263 11s. 6½d. $\frac{40}{100}$.

DISCOUNT.

DISCOUNT is an allowance made for the payment of money before it becomes due. THE PRESENT VALUE of any sum due some time hence, is that which being put to interest for the same time and rate, would amount to the sum due.

RULE.—As the amount of £100 for the given time and rate, is to £100, so is the whole debt, to the present value. Which present value deducted from the whole debt, leaves the discount: OR THUS, as the amount of £100 for the given time and rate, is to its interest for the same time and rate, so is the given sum, to its discount, which deducted from the sum leaves the present worth.

EXAMPLE.—What is the present value, and discount of £800 due six months hence, at 5 per cent. discount?

m. m. £ £
As 12 : 6 : : 5 : 2.5

As $102.5 : 100 :: 800 : \frac{800\times100}{102.5}$ = £780 9s. 9d. present worth.

£800—£780 9s. 9d. = £19 10s. 3d. discount.

Or thus,

As $102.5 : 2.5 :: 800 : \frac{800\times2.5}{102.5}$ = £19 10s. 3d. discount.

£800—£19 10s. 3d. = £780 9s. 9d. present worth.

It will be observed here that both the methods given in the rule produce the same result. I have considered it unnecessary to give the work at length, the process being so very simple.

EXERCISES.

1. What is the present worth of £750 10s. payable 3 months hence, at 4 per cent. discount? Ans. £743 1s. 4½d. $\frac{54}{101}$.
2. What is the discount on £975 5s. payable 5 months hence, at 5 per cent.? Ans. £19 18s. 3¾d. $\frac{36}{49}$.
3. What is the present worth of £1000 payable two years hence, discount 3½ per cent.? Ans. £934 11s. $7\frac{7}{107}$ d.
4. What is the neat proceeds of a bill for £600 due 4 months hence, discount 5 per cent, and commission ½ per cent.? Ans. £587 3s. 3¼d+.
5. Bought goods for £930, one third payable every 4 months, how much ready money are they worth, allowing 5 per cent. discount? Ans. £900 3s. 1½d+.

6. Sold goods for £1200 payable ⅓ in hand, and ⅓ every 4 months till the whole is paid; how much discount must I allow at 5 per cent. for having the whole paid ready money? Ans. £28 17s. 7½d+.

7. What ready money will discharge a bill of £3000 due 4 months hence at 4½ per cent. discount?
Ans. £2955 13s. 3½d. $\frac{86}{203}$.

8. If a banker discount bills to the amount of £800000 for a year, at 5 per cent.; how much does he profit by using bankers' discount instead of true discount?
Ans. £1904 15s. 2¾d.

9. What is the present worth of £560 payable 73 days hence, allowing 3 per cent. discount? Ans. £556 13s. 2¼d+.

10. I am offered £675 ready money for a parcel of goods, or £700 at 6 months; which is the best offer, allowing discount at 5 per cent.?
Ans. credit preferable by £7 18s. 6¼d+.

11. What sum due 9 months hence is equal in value to £2500 ready money, supposing 4 per cent. discount allowed?
Ans. £2575.

12. Sold goods for £966 ready money, but my merchant, anxious to make other purchases with the money, insists upon me taking his bill at 3 months; for how much should it be drawn, including discount at 6 per cent.?
Ans. £980 9s. 9½d. .4.

EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is the method used by merchants, and others, for ascertaining the time at which several debts, due at different periods, may be settled at one payment, without disadvantage to either party.

RULE.—Multiply each debt by the time before it becomes due; then divide the sum of the products, by the sum of the debts, and the quotient is the equated time for paying the whole.

EXAMPLE.—P. owes to R. £300 at 3 months, £60 at 4 months, and £600 at 6 months; when should the whole be settled at one payment?

```
300 × 3 =  900
 60 × 4 =  240
600 × 6 = 3600  m. days.
96,0     )474,0( 4  28⅛
          384
           90×30
            96
```

I here multiply the several debts by their respective terms, and divide 4740 the sum of the products, by 960 the sum of the debts, which gives 4 months and 28⅛ days, the equated time to pay the whole.

EXERCISES.

1. If £300 is payable in 2 months, £300 in 4 months, and 300 in 6 months; when may the whole be paid at once without loss to either party? Ans. 4 months.
2. Delivered to a banker the following bills, viz. A B's bill for £100 due in 20 days, B C's for £200 due in 40 days, C D's for £280 due in 60 days, and D E's for £400 due in 80 days; at how many days should he grant me a bill for the whole? Ans. 60 days.
3. A debt is to be discharged in the following manner, viz. $\frac{1}{4}$ present, $\frac{1}{6}$ at 2 months, $\frac{1}{8}$ at 3 months, $\frac{1}{12}$ at 4 months, and the rest at 6 months; what is the equated time for paying the whole at one payment? Ans. $3\frac{7}{24}$ months.
4. A debt of £100 is payable £20 present, £20 in 3 months, and £20 every month after till the whole is paid; when should it be discharged at one payment? Ans. 3 m. 18 d.
5. A is indebted to B £600 payable in 4 months, but is willing to pay £200 in hand provided the rest is forborn a proportionally longer time, to which B agreed; required the time for paying the balance? Ans. 6 months.
6. A granted his bill to B for £1000 at 8 months, but at the end of 3 months A is willing to pay £300 provided the balance be forborn so much longer, to which B consents; when should the balance be paid?
   Ans. $10\frac{1}{7}$ months from date.
7. A lodges the following bills with his banker on the 28th February, viz. B's bill for £60 payable on the 20th March, C's for £80 payable on the 15th April, D's for £100 payable on the 30th May, and E's for £140 payable on the 18th of June; when should a bill for the whole be made payable, and for how much should it be drawn, deducting $\frac{1}{4}$ per cent. for stamp and commission?
   Ans. in $77\frac{6}{19}$ days, or 17th May. Drawn for £378 2s.
8. A debt is to be paid $\frac{1}{2}$ present, $\frac{1}{4}$ at 4 months, $\frac{1}{8}$ at 6 months, $\frac{1}{16}$ at 8 months, and the rest at 10 months, required the time for discharging the whole at one payment. Ans. $2\frac{7}{8}$ months.

BARTER.

Barter is the method of exchanging one commodity for another, by way of accommodation, without direct profit to either party.

RULE.—Find the value of the goods whose price and quantity are given, then find what quantity of the other goods may be purchased for that money. OR THUS,—As the price of the goods required, is to the price of the goods given, so is the given quantity, to the required quantity.

EXAMPLE.—Bartered 72 yards of broad cloth at 19/6 per yard, for Irish linen at 3/6 per yard; how much linen should I receive?

s. d.	s. d.	£	s. yd.		s. d.	s. d. yd.
19 6	As 3 6 :	70	4 :: 1	Or thus, as	3 6 :	19 6 :: 72
6	2	20			2	2
5 17 0	7	1404			7	39
12		2				72
£70 4 0	7	)2808			7	)2808
	yds.	$401\frac{1}{7}$ Ans.			yds.	$401\frac{1}{7}$ Ans.

It is evident, when the second part of the rule can be applied that the work is shortened considerably.

EXERCISES.

1. A and B bartered, A has 6 hhds. of sugar weighing each 10 cwt. 3 qrs. 18 lb. gross, tare 1 qr. 18 lb. per hhd., worth $6\frac{1}{2}$d. per lb., which he barters with B for tea worth 6/4 per lb.; how much tea should A receive for his sugar?
   Ans. 5 cwt. 1 qr. $15\frac{8}{19}$ lb.
2. Bartered 7 punch. rum at £67 4s. each, for hollands at £1 5s. a gall.; how many gallons should I receive?
   Ans. 376 gal. 1 qt. $2\frac{6}{25}$ gil.
3. Bartered 100 yards shirting muslin at 2/6 per yard, for 100 yards printed cotton at 1/8 per yard, and the remainder in ribbons at 1/3 per yard; how many yards of ribbon should I receive? Ans. $66\frac{2}{3}$.
4. H gave K 100 dozen wine at £3 3s. per dozen, for £60 and 600 yards silk; how was it rated per yard? Ans. 8/6.
5. M has 16 cwt. 2 qrs. tobacco at £3 5s. 4d. per cwt. which he barters with N for 3 cwt. 1 qr. white pepper at 5/ per lb.; who receives the balance, and how much?
   Ans. N receives £37 2s.
6. H has 95 pairs of stockings at 3/8 a pair ready money, but in barter charges 4/, S has shoes at 6/6 a pair ready money; how should he rate them in barter to be equal with H, and how many pair should he give for the stockings?
   Ans. 7/1 $\frac{1}{11}$., and 53 $\frac{23}{39}$ pair.

7. Bartered 16 hhds spermaceti oil, each 4 cwt. 1 qr. 16 lb. gross, tare 32 lb. per hhd., at 8/ per gal. of 9 lb., for tobacco at 2/6, and tea at 5/6 per lb. and to have an equal quantity of each; how many lbs. of each do I receive?
Ans. 817⅞ lb.

8. A gave Z 20 bags cotton wool, each 3 cwt. 2 qr. 21 lb. gross, draft 1 lb. per bag, tare 5 lb. per cent., at 8½d. per lb. neat; for which Z gave him an equal quantity of broad cloth at 17/6, cassimeres at 3/3, cambrics at 9d., calicos at 6d. per yard; how many yards of each did A receive for his cotton?
Ans. $252\frac{5}{132}$ yds.

PROFIT AND LOSS.

Profit and Loss is the rule by which merchants are enabled to ascertain how to buy and sell, so as to gain or lose a certain rate per cent. &c.

Case 1.

The prime cost and selling price given, to find the gain, or loss per cent.

Rule.—As the prime cost, is to the gain or loss on it, so is £100 to the gain or loss per cent.

Example.—Bought cloth at 6/8 per yard; what would be gained per cent. by selling it at 7/, or lost by selling it at 6/3?

	d.	d.	£		d.	d.	£
As	$\cancel{8}\cancel{0}$:	$\cancel{4}$ ::	$\cancel{1}\cancel{0}\cancel{0}$	As	$\cancel{8}\cancel{0}$:	5 ::	$\cancel{1}\cancel{0}\cancel{0}$
	$\cancel{2}\cancel{0}$		£5 per cent. gain at 7/		4		5
							5
							4)25

loss at 6/3 = 6¼ per cent.

This example might have been stated thus, as 6/8 : 7/ :: £100 : £105 from which subtract £100 leaves £5; and, in the second part, as 6/8 : 6/3 :: 100 : 93¾ which subtracted from £100 leaves £6¼ as before.

EXERCISES.

1. Bought silks at 5/8 a-yard, and sold for 7/; what was the gain per cent.?
Ans. $23\frac{9}{17}$.

2. Bought a lot of tea at 4/9 per lb., and sold it being damaged for 4/; what was the loss per cent.? Ans. $15\frac{15}{19}$.

3. Bought a pipe of wine for £126, which leaked out 15 gallons, I sold the remainder for £163; what was my gain per cent., and what was a gallon sold for?
Ans. £$29\frac{23}{63}$ per cent., sold for £1 5s. $10\frac{1}{4}$d. $\frac{12}{21}$ per gal.

4. A grazier bought a lot of cattle for £1000 ready money, and sold them immediately after for £1150, for which he received a bill at 4 months, and discounted it at 5 per cent. and $\frac{1}{2}$ per cent. commission; what was his gain per cent. by this transaction? Ans. £12 10s. 2d.

5. A gentleman has a rental of £1600 a year, and took bills from his tenants at 3 months for half a year's rent, and discounted them at 5 per cent. and $\frac{1}{4}$ per cent. commission; what did he lose per cent. by this transaction?
Ans. $1\frac{1}{2}$.

6. Bought 1000 lambs at 2 for £1, and 1000 at 4 for £1, and having mixed them altogether, I sold them at the rate of 6 for £2; whether did I gain or lose, and how much per cent.? Ans. lost £83 6s. 8d. £$11\frac{1}{9}$ per cent.

Case 2.

The cost, and gain or loss per cent. given, to find the selling price.

Rule.—As £100, is to £100 increased by the gain, or diminished by the loss per cent. so is the cost, to the selling price.

Example.—If I purchase brandy at £1 5s. a gallon; how must I sell it to clear 10 per cent. and to lose 6 per cent.?

As 100 : 110 :: 25 : 27s. 6d. to gain 10 per cent.

As 100 : 94 :: 25 : 23s. 6d. to lose 6 per cent.

I have considered it unnecessary to give the operation in these accounts, they are so simple, especially by cancelling.

EXERCISES.

1. Bought wheat at £3 15s. a quarter, and sold it at 8 per cent. profit; what was it sold for a quarter? Ans. £4 1s.

2. I am willing to lose 10 per cent. on barley which cost me £2 18s. a quarter; how must I sell it?
Ans. £2 12s. $2\frac{1}{4}$d. $\frac{3}{5}$.

3. If sugar be bought at £2 10s. a cwt., how must it be sold per lb. to clear 12 per cent.? Ans. 6d.

4. A of Edinburgh, bought of B of Glasgow, 7000 yards muslin, for £583 6s. 8d. and paid for carriage 13/4; how must he sell it per yd. to gain 18 per cent.? Ans. $1/11\frac{3}{4}$ +.

5. Bought butter in Belfast for £2 12s. a cwt. cash, freight, customs, insurance, &c. 4/; how must I sell it to gain 15 per cent. and 3 months interest at 5 per cent. on the outlaid money? Ans. £3 5s. $1\frac{1}{4}$d.
6. Bought cheese at £112 a ton cash; how must I sell it per cwt. at the end of two months, and take a bill at three months to clear ten per cent.? Ans. £6 6s. $8\frac{3}{4}$d. $\frac{1}{2}\frac{3}{5}$.

Case 3.

The selling price, and gain or loss per cent. given, to find the prime cost.

Rule.—As 100 increased by the gain, or diminished by the loss per cent. is to 100, so is the selling price, to the prime cost.

Example.—Sold tea at 11/ a lb. and cleared 10 per cent., and at 8/ and lost 20 per cent.; what was the prime cost?

As 110 : 100 : : 11 = 10/ prime cost.
As 80 : 100 : : 8 = 10/ prime cost.

This rule is so plain that it requires no explanation.

EXERCISES.

1. Sold goods for 5/ by which I cleared 8 per cent.; what was the prime cost? Ans. 4/$7\frac{1}{2}$d. $\frac{2}{9}$.
2. Sold sugar at £3 4s. a cwt., and gained 12 per cent.; required the prime cost? Ans. £2 17s. $1\frac{1}{2}$d. $\frac{6}{7}$.
3. Sold tobacco at 2/11 a lb., by which I lost $12\frac{1}{2}$ per cent.; what was it bought for? Ans. 3/4d.
4. If by selling coffee at £14 a cwt. I lose $16\frac{2}{3}$ per cent., what was the prime cost a lb.? Ans. 3/.
5. Sold cloth at £1 5s. a yard, and took a bill at three months, after discounting which, I find 20 per cent. cleared; what was it bought for? Ans. £1 0s. $6\frac{3}{4}$d. $\frac{2}{3}$.
6. Bought wine, and after keeping it two years, sold it for £75 12s. a hhd., and cleared 10 per cent. per annum upon the prime cost; what did it stand me a gallon? Ans. £1.

Case 4.

When two selling prices, and the gain or loss per cent. on one of them is given, to find it on the other.

Rule.—As the price whose rate is given, is to the other, so is 100 increased by the gain, or diminished by the loss per cent. to a fourth proportional, whose excess above 100 is the gain, and its defect the loss per cent.

Example.—If by selling butter at 10d. a lb. I gain 10 per cent.; what do I gain per cent. by selling at 11d. a lb.? and

if by selling at 10d. I lose 10 per cent.; what do I gain or lose by selling at 11d. a lb.?

As 10 : 11 :: 110 : 121 & 121—100=£21 gain per cent. at 11d.
As 10 : 11 :: 90 : 99 & 100— 99=£1 loss. per cent. at 11d.

This process is so simple as to require no further illustration here.

EXERCISES.

1. Sold paper at 17/6 a ream, and gained 12 per cent.; what do I gain per cent. by selling at 18/? Ans. £15 4s.
2. Sold carpeting at 3/6 a yd., and cleared 15 per cent.; what do I gain or lose by selling at 3/ a yd.? Ans. £1 3/7 loss.
3. By selling India shawls at £60 I gain 30 per cent.; what is the gain or loss per cent. by selling at £40? Ans. £13⅓ loss.
4. If by selling wheat at £3 10s. a quarter I lose 20 per cent.; what do I gain or lose by selling at £4 a quarter? Ans. £8 4/7 loss.
5. By selling hats at £1, I lost 12½ per cent.; what do I gain or lose by selling at £1 5? Ans. £9 7s. 6d. gain.
6. A tobacconist, by selling snuff at 4/ a lb., cleared 8 per per cent.; but growing scarce, he raised to 5/ a lb.; how much did he then gain per cent.? Ans. £35.
7. A grazier was offered £10 a head, for a score of cattle, by which he would have gained 9 per cent.; but expecting markets to rise, he kept them for 6 months, and then sold them for £12 a head; how much per cent. did he gain on the prime cost, after paying £25 for grass? Ans. £17 3s. 6d.

CASE 5.

When the cash price is to be advanced, so as to allow any proposed discount or credit.

RULE.—As 100 minus its discount, is to 100, so is the cash to the advanced price.

EXAMPLE.—How must I rate broad cloth a yard to obtain £1 4, and allow a discount of 10 per cent.; and if bought at 15/, how must I sell it to gain 10 per cent., and allow a discount of 4 per cent.

As 90 : 100 :: 24 Ans. £1 6s. 8d. to allow 10 per cent. discount
As 96 : 110 :: 15 Ans. 17/2¼ to allow 4, and gain 10 per cent.

EXERCISES.

1. How must I sell mustard to have 1/6 a lb., after allowing 4 per cent. discount? Ans. 1/6¾

2. How must I sell spirits to have 11/ a gallon, and allow a discount of 12 per cent.? Ans. 12/6.
3. Bought soap at £2 16s. a cwt.; how must I sell it to gain $7\frac{1}{2}$ per cent., after it has inleaked 1 lb. in 20 of the original weight? Ans. £3 3s. $4\frac{1}{4}$d. $\frac{13}{19}$.
4. Bought coals at 9/ a ton; how must I sell them to clear 6 per cent., and give 6 months credit? Ans. 9/10 $\frac{2}{5}$.
5. Bought school books at 2/6; how must I sell them to clear 10 per cent., and give 2d. a shilling discount? Ans. 3/$3\frac{1}{2}$ $\frac{2}{5}$.
6. How must I sell herrings worth £1 10s. a barrel cash, to allow a discount of 4 per cent.? Ans. £1 11s. 3d.

Case 6.

When the whole gain or loss, and the rate per cent. are given, to find the prime cost, or selling price of the whole.

Rule.—As the rate per cent. is to 100, so is the whole gain or loss to the prime cost; which, increased by the gain, or diminished by the loss, gives the selling price.

Example.—Sold goods at 6 per cent. profit, and cleared £57; what were they bought and sold for?

As 6 : 100 :: 57 : £950 prime cost.
£950 + 57 = £1007 sold for.

EXERCISES.

1. If I lose £50 by tea at the rate of $2\frac{1}{2}$ per cent.; what was it bought and sold for?
Ans. bought for £2000, and sold for £1950.
2. By selling coffee at 3/ a lb., I gained £16 at the rate of $5\frac{1}{3}$ per cent.; what quantity was sold?
Ans. 18 cwt. 3 qr. $6\frac{2}{3}$ lb.
3. Sold sugar at 10d. a lb., by which I lost £80 at the rate of 12 per cent.; what quantity was sold, and what did it cost a cwt.?
Ans. sold 125 cwt. 2 qr. 24 lb. cost £5 6s. $0\frac{1}{2}$d. $\frac{10}{11}$.
4. Retailed cheese at 8d. a lb., by which I gained £100 at the rate of 10 per cent.; what quantity did I sell?
Ans. 14 ton 14 cwt. 2 qr. 16 lb.
5. Sold wine at £3 10s. a dozen, and gained £1000 at the rate of 15 per cent.; what was it bought and sold for, and how many dozens were sold? Ans. bought for £6666 13s. 4d. Sold for £7666 13s. 4d. Qt. $2190\frac{10}{21}$ doz.
6. Sold tobacco at £11 4s. a cwt., by which I lost £160 at the rate of 8 per cent.; what quantity did I sell, and

what was it bought and sold for? Ans. Qt. 8 ton 4 cwt. 1 qr. 4 lb. Bought for £2000. Sold for £1840.

Promiscuous Exercises.

1. Bought a house for £600, and after paying £60 for repairs, sold it for £726; what was the gain per cent.? Ans. £10.
2. By selling rum at 18/ a gallon, I gained 24 per cent.; what is the gain or loss per cent., by selling for 16/?
   Ans. £10$\frac{2}{9}$ gain.
3. Bought cheese at £3 per cwt. cash, and after keeping it 4 months, sold it for £3 10s. per cwt., but found it had lost 4 lb. per 100 lb. of the original weight; what did I gain or lose per cent., including interest? £10 6s. 8d. gained.
4. Bought 100 jars of Dutch butter, each weighing gross 30 lb., tare 8 lb. per jar, for £120; of which I retailed 50 jars at 1/4 per lb., 20 at 1/1 per lb., and the rest at 10d. per lb., the empty jars were sold for 1/ each; how much did I gain or loss upon the whole, and how much per cent.?
   Ans. gained £9 13s. 4d. £8$\frac{1}{18}$ per cent.
5. A vintner bought a puncheon of aqua, containing 130 gallons, for £58 10s., and paid for carriage £1 12s. 6d., he sold unreduced to a friend, 40 galls. at 11/, to the remainder he added a gallon of water for every 5 of spirits, and retailed it at 5d. per gill; how much did he gain per cent. upon his outlaid money? Ans. £56$\frac{1}{4}\frac{8}{7}$.
6. By selling wheat at £4 a quarter, cash, I gained 6 per cent.; how should I sell it per quarter, to clear the same, and give a discount of 5 per cent.? Ans. £4 4s. 2$\frac{1}{2}$d. $\frac{2}{15}$.
7. Sold rice at £1 17s. 4d. per cwt., by which I cleared £70, at the rate of 14$\frac{2}{7}$ per cent.; what quantity was sold, and what was it bought for per lb.?
   Ans. 300 cwt., bought for 3$\frac{1}{2}$d. a lb.
8. Bought 1000 quarters of wheat, at £3 a quarter, and after 15 months, sold it for £3 10s. a quarter; what did I gain or lose per cent., considering that the quantity had inleaked 3 per cent., deducting also interest on the cost at 5 per cent. per annum, and £107 10s. for granary rent and incidental charges? Ans. £3 6s. 8d. per cent. gain.
9. Bought 12 hhds. tobacco, weighing gross 84 cwt., tare, 12 lb. per cwt., at £14 per cwt. neat, of which I sold 25 cwt. at 14 guineas a cwt., but the market rising, I wish to sell the remainder, so as to clear 20 per cent. upon the whole, what must I charge a cwt.? Ans. £17 17s.

10. Sold a quantity of tea at 5/6 a lb., by which I lost £20 at the rate of $8\frac{1}{3}$ per cent.; required the quantity sold, and the prime cost per lb. Ans. 800 lb. cost. 6/ a lb.
11. Bought 50 jars of French grapes, weighing gross 40 lb. a jar, tare 15 lb. each, of which I retailed 20 jars at 3/ a lb., 10 at 2/6 a lb., and the remainder at 2/ a lb., and the empty jars sold for 2/ each, I have gained 10 per cent. by this transaction, what was the prime cost of the whole?
Ans. £146 11s. $9\frac{3}{4}$d. $\frac{3}{11}$.
12. Sold a quantity of silks, $\frac{1}{4}$ of the whole at 5/ a yard, by which I gained at the rate of 25 per cent., the remaining $\frac{3}{4}$ brought only 3/6 a yard; whether did I gain or lose, and how much per cent. by this transaction?
Ans. lost $3\frac{1}{8}$ per cent.

MERCANTILE COMPOSITIONS

OR

ALLIGATION.

Mercantile Composition is the method of finding the value, or quality of any part of a mixture, of which the price, or quality of the several ingredients is given; or of finding the quantity of any proposed ingredients necessary to make a composition of any assigned price, or quality.

Case 1.

When the quantity, and the price, or quality of the several ingredients are given, to find the price, or quality of any proposed part of it.

Rule.—Multiply each ingredient by its price, or quality, and the sum of the products, divided by the sum of the ingredients is the answer.

Examples.—1st, A grocer made a composition of 20 lb. tea at 4/6, 16 lb. at 6/, and 12 lb. at 8/; what should he charge for a lb. of the mixture?

2d, A goldsmith melted together 6 lb. of gold 20 caracts fine, 12 lb. of 22 caracts fine, and 16 lb. of 23 caracts fine; of what fineness is a lb. of the mixture?

1st.		2d.	
$20 \times 4\frac{1}{2} =$	90	$6 \times 20 =$	120
$16 \times 6 =$	96	$12 \times 22 =$	264
$12 \times 8 =$	96	$16 \times 23 =$	368
48	)282(5/1½	34	)752(22 $\frac{2}{17}$
	240		68
	&c.		&c.

In the first example the price of the ingredients is given to find the price of the mixture. In the second example the quality of the several ingredients is given to find the quality of the mixture.

EXERCISES.

1. An apothecary made the following composition of medicines, viz. 6 lb. at 6/, 4 lb. at 5/, 8 lb. at 4/6, and 6 lb. at 3/; what is the value of a lb. of the mixture? Ans. 4/7.
2. A grocer mixed 4 cwt. sugar at 50/, with 6 cwt. at 60/ and 8 cwt. at 75/; what is a cwt. of the composition worth? Ans. £3 4s. 5¼d. ⅓.
3. A spirit dealer mixed 10 gallons aqua at 7/ a gallon, with 12 at 7/6, 6 at 8/, 4 at 9/6, and 4 gallons of water; what is a gallon of the composition worth? Ans. 6/10.
4. A corn chandler mixed 3 qrs. of beans at 44/, with 6 qrs. of peas at 50/, 8 qrs. of oats at 32/, and 4 qrs. of barley at 40/; what is a qr. of the mixture worth? Ans. £2 0s. 4½d. $\frac{2}{7}$.
5. A tobacconist mixes 12 lb. of snuff at 4/ a lb. with 16 lb. at 6/, 20 lb. at 8/3, and 6 lb. at 18/; what is a lb. of the mixture worth? Ans. 7/8½ ⅔.
6. A silversmith melted together 7 lb. of silver 16 caracts fine, 8 lb. of 18 caracts fine, 12 lb. of 20 caracts fine, and 5 lb. aloy; of what fineness was the mass? Ans. 15½.

CASE 2.

To find how much of each simple is in any proposed quantity of the compound.

RULE.—As the whole mixture, is to the proposed quantity, so is the several simples, to the quantity of each.

EXAMPLE.—A merchant mixed 11 gallons brandy with 3 of port wine and 2 of whisky; how much of each is in 2 gallons of the mixture?

	Gal.	Gal.	Gal.	
11 as	16 :	2 ::	11 :	1 gallon 1 quart 1 pint brandy.
3 as	16 :	2 ::	3 :	0 gallon 1 quart 1 pint wine.
2 as	16 :	2 ::	2 :	0 gallon 1 quart 0 pint whisky.
16 sum				

The process here is so simple that it appears unnecessary to give the work at length.

EXERCISES.

1. An oilman mixed together 16 gal. train oil, 36 gal. seal oil, and 60 gal. spermaceti oil; how much of each kind is in 14 gallons of this composition?

   Ans. 2 gal. tr. $4\frac{1}{2}$ gal. se. $7\frac{1}{2}$ gal. sp.

2. A goldsmith melted together 12 lb. of gold, 4 lb. of silver, and 2 lb. of brass; what weight of these metals is in a vase of 5 lb. made of this mixture?

   Ans. 3 lb. 4 oz. gold, 1 lb. 1 oz. 6 dwt. 16 gr. silver, 6 oz. 13 dwt. 8 gr. brass.

CASE 3.

To increase or diminish the simples in any proposed ratio.

RULE.—As the sum of the given simples is to the proposed sum, so is the several given simples, to those required.

EXAMPLE.—A spirit merchant had an order for 30 gal. aqua, 15 gal. rum, 6 gal. brandy, and 5 gal. hollands, but the quantity was afterwards restricted to 42 gallons, how much of each should he send?

30	as 56 : 42 :: 30 : 22 gal. 2 qts. aqua.
15	as 56 : 42 :: 15 : 11 gal. 1 qt. rum.
6	as 56 : 42 :: 6 : 4 gal. 2 qts. brandy.
5	as 56 : 42 :: 5 : 3 gal. 3 qts. hollands.
56	

EXERCISES.

1. A meal-man was ordered to send to a customer 3 cwt. oatmeal, 4 cwt. barley-meal, 2 cwt. flour, 5 cwt. split peas, and 6 cwt. salt, but the order was afterwards restricted to 15 cwt. and the same proportions; how much should he send of each?

   Ans. 2 cwt. 1 qr. oatmeal, 3 cwt. barley-meal, 1 cwt. 2 qr. flour, 3 cwt. 3 qrs. peas, 4 cwt. 2 qrs. salt.

2. A grocer had a commission for 68 lb. tea, 140 lb. sugar, 168 lb. rice, 84 lb. tobacco, 112 lb. soap, and 196 lb. cheese, but was afterwards ordered to send in all half a ton, and in the same proportions; how much should he send of each?

   Ans. $99\frac{1}{6}$ lb. tea, $204\frac{1}{6}$ lb. sugar, 245 lb. rice, $122\frac{1}{2}$ lb. tobacco, $163\frac{1}{3}$ lb. soap, $285\frac{5}{6}$ lb. cheese.

CASE 4.

When the rate of the simples, and also of the mixture are

given, to find the quantity of the several simples necessary to make the mixture.

RULE.—Write the rate of the simples under each other, and the mixture rate on their left, and all in the same denomination.—Link all the simple rates which are greater than the mixture rate, with all those that are less than it.—Take the difference between the rate of each simple and the mixture rate, and place it on the right of all those with which it is linked, then the difference, or sum of the differences opposite each simple expresses the quantity of it which is required to make the composition.

EXAMPLE.—A wine-merchant wishes to mix together wines at 18/, 23/, 30/, and 36/ a gallon, and to sell the mixture at 28/; how much of each kind should he use?

28 { 18 = 2+8 = 10 gal. at 18/.
23 = 2+8 = 10 gal. at 23/.
30 = 10+5 = 15 gal. at 30/.
36 = 10+5 = 15 gal. at 36/.

Having placed the rate of the simples, and of the mixture according to the rule, I link 18 and 23, which are both less than the mixture rate, with 30 and 36 which are greater, I then take the difference between 28 and 18, which is 10, and place it opposite to 30 and 36 the rates to which 18 is linked; I then take the difference between 28 and 23 which is 5, and place it also opposite to 30 and 36 to which 23 is linked, with + between it and 10; the difference between 28 and 30, 28 and 36 are in like manner placed opposite to 18 and 23 with which they are linked; then adding these differences we have 10 gal. at 18/ and 23/, and 15 gal. at 30/ and 36/, which are the quantities by this method of linking required to make the composition worth 28/ per gallon.

EXERCISES.

1. A grocer wishes to compound teas at 4/6, 5/, and 6/6, and to sell the mixture at 5/6; how much of each kind should he take? Ans. 2 lb. at 4/6 and 5/, and 3 lb. at 6/6.
2. A wine merchant has wine at 9/, 12/ 18/ and 24/ per gallon; how much of each should he use, to make a composition worth 19/ a gallon?
Ans. 5 gal. at 9/, 12/, 18/, and 18 gal. at 24/.
3. A brewer has ales at 3/, 2/6, 1/6 and 1/ per gallon, and wishes to make a mixture at 2/3 a gal.; how much of each kind should he take?
Ans. 24 gal. at 3/ and 2/6, and 12 gal. at 1/6 and 1/.

4. A goldsmith has gold at 16, 18, 21, and 23 caracts fine; how much must he use of each to make it 20 caracts fine?
Ans. 4 at 16 and 18, and 6 at 21 and 23.

5. How much aloy must be mixed with gold 24 caracts fine to reduce it to 18 caracts fine?
Ans. 18 of gold and 6 of aloy.

6. A corn merchant sold wheat which he was to deliver at 60 lb. per bushel, but upon examining his samples he finds them to be 56, 58, and 63 lb.; how must he mix them to enable him to fulfil his engagement?
Ans. 3 at 56 and 58, 6 at 63.

Case 5.

When the rate of the simples, and of the mixture are given, and the quantity of one of the simples limited.

Rule.—Find the quantity of the several simples by linking, as in last case, then say, as the difference opposite to that simple whose quantity is limited, is to the other differences severally, so is the limited quantity to the required quantities.

Example.—How much tea at 5/, 6/8, and 9/6 per lb. must be mixed with 14 lb. at 8/, to make a mixture worth 7/ per lb.

$$84\begin{cases}60 = 30 + 12 = 42. & \text{as } 28 : 14 :: 42 : 21 \text{ lb. at } 5/\\ 80 = 30 + 12 = 42. & \text{and } 6/8.\\ 114 = 24 + 4 = 28. & \text{as } 28 : 14 :: 28 : 14 \text{ lb. at } 9/6.\\ 96 = 24 + 4 = 28. & \end{cases}$$

Therefore 14 lb. at 8/, being mixed with 21 lb. at 5/, 21 lb. at 6/8 and 14 lb. at 9/6, will make a composition worth 7s. a lb.

EXERCISES.

1. A spirit merchant has 36 gall. of spirits worth 8/ per gall., which he wishes to mix with three other kinds worth 9/, 12/, and 16/ a gall., in order to sell the composition for 11/ a gall.; how much of each should he use?
Ans. 36 gall. at 9/, and 30 gall. at 12/, and 16/.

2. How much coffee at 3/, 2/8, and 2/, must be mixed with 40 lb. at 2/4, that the composition may be worth 2/6 a lb.?
Ans. 40 lb. of each kind.

3. How much rum at 18/, 15/, and 14/ a gall., must be mixed with 10 gall. of water, to reduce the composition to 12/ a gall.?
Ans. 10 gall. of each kind.

4. A goldsmith melted 15 lb. of gold, 23 caracts fine, with three other kinds of 20, 18, and 17 caracts fine; how much of each must he use that the whole may be 19 caracts fine?
Ans. 15 lb. at 20, and 25 lb. at 18 and 17 fine.

Case 6.

When the rate of the simples, and of the mixture are given, but the quantity of the mixture limited.

Rule.—Find the quantity of the several simples by linking as in last case, then say, as their sum, is to the limited quantity, so is each of them separately, to the several quantities required.

Example.—A farmer has barley at 46 lb., 48 lb., and 52 lb. a bushel, of which he wishes to make a mixture of 40 bushels weighing 50 lb.; how much should he take of each kind?

$$50\left\{\begin{array}{l} 46 \\ 48 \\ 52 \end{array}\right. \begin{array}{l} = 2 \\ = 2 \\ = 4+2 \end{array} \begin{array}{l} = 2 \\ = 2 \\ = \underline{6} \\ \text{Sum } 10 \end{array} \quad \begin{array}{l} \text{as } 10 : 40 :: 2 : \ \ 8 \text{ bush. at } 46 \text{ lb.} \\ \text{as } 10 : 40 :: 2 : \ \ 8 \text{ bush. at } 48 \text{ lb.} \\ \text{as } 10 : 40 :: 6 : \underline{24} \text{ bush. at } 52 \text{ lb.} \\ \qquad\qquad\qquad\quad 40 \text{ Proof.} \end{array}$$

Having found the quantities by linking as in last case, I say as their sum, which is here 10, is to the limited quantity 40, so are these quantities 2, 2, 6 severally, to the limited quantities 8, 8, 24 of each.

EXERCISES.

1. An oil-man has oil at 3/4, 4/, 6/3, and 8/ a gallon; how much of each should he use to make a composition of 84 gallons, worth 5/6 a gallon? Ans. 19 gal. 2 qts. $1\frac{73}{83}$ pints at 3/4 and 4/, 22 gal. 1 qt. $0\frac{10}{83}$ pt. at 6/3 and 8/.
2. A grocer has coffee at 1/8, 2/, 2/6, 3/ per lb. which he wishes to mix, so as to have a cwt. worth 2/8 per lb.; how much of each should he use?
   Ans. 14 lb. at 1/8, 2/, and 2/6, and 70 lb. at 3/.
3. A vintner has spirits which he retails at 8/, 10/6, 12/, and 16/ per gallon; how much of each must he use to fill an anker barrel worth /5 per gill? Ans. 1 gal. $2\frac{2}{21}$ qts. at 8/ 10/6, 12/, and 5 gal. $1\frac{5}{7}$ qts. at 16/.
4. A farmer has lambs worth £5, £8, £12, and £13 a score; how many of each should he mix to have 20 score, worth £9 10s. a score. Ans. 5 score of each kind.

EXCHANGE OF FOREIGN MONIES.

Exchange is the method of ascertaining how much money of one country, is equivalent to any proposed sum of the money of another country, according to a given course of exchange.

EXCHANGE is transacted by merchants and bankers by means of "BILLS OF EXCHANGE," thus settling the mutual debts of distant countries without the expense of remitting specie.

THE PAR OF EXCHANGE, is the intrinsic value of the money of any one country compared with that of any other country.

The COURSE OF EXCHANGE is the uncertain and varying sum of the money of one country which is given for a certain and fixed sum of the money of another country.

AGIO is the difference between bank and current money, and also between the intrinsic and circulating value of foreign coins.

USANCE is the usual time which merchants and bankers in one country, allow to those of another country to pay bills of exchange.

DAYS OF GRACE, are the days allowed for paying bills, after their term is expired, before diligence is used.

Tables of the monies, weights, and measures, used in the commerce of the several countries, and states noticed in the following treatise on Exchange.

FRANCE.

MONEY, OLD SYSTEM.—12 deniers = 1 sou, 20 sous = 1 livre tournois, 3 livres = 1 ecu or crown, 24 livres or 8 ecus = 1 louis.

MONEY, NEW SYSTEM.—10 centimes = 1 decime, 10 decimes = 1 FRANC, 80 francs = 81 livres tournois, 100 francs = 101¼ livres tournois.

WEIGHTS, OLD SYSTEM.—24 grains = 1 denier, 3 deniers = 1 gross, 8 gross = 1 once, 8 onces = 1 mark, 2 marks = 1 poid de marc or livre, 100 livres = 1 quintal, 1 poid de marc = 7560 English troy grains.

WEIGHTS, NEW SYSTEM.—10 milligrammes = 1 centigramme, 10 centigrammes = 1 decigramme, 10 decigrammes = 1 GRAMME, 10 grammes = 1 decagramme, 10 decagrammes = 1 hectogramme, 10 hectogrammes = 1 kilogramme, 10 kilogrammes = 1 myriagramme, 1 gramme = 15.4441464 Eng. Troy grains.

LINEAL MEASURE, OLD SYSTEM.—12 lines = 1 inch, 12 inches = 1 foot, 6 feet = 1 toise, 1 toise = 76.734 Eng. inch.

LINEAL MEASURE, NEW SYSTEM.—10 millimetres = 1 centimetre, 10 centimetres = 1 decimetre, 10 decimetres = 1 METRE, 10 metres = 1 decametre, 10 decametres = 1 hectometre, 10 hectometres = 1 kilometre, 10 kilometres = 1 myriametre, 1 metre = 39.3694 Eng. inches.

OLD LIQUID MEASURE.—2 choppins = 1 Paris pint, 228 pints = 1 muid of wine at Paris, 384 pints = 1 queue of Champaigne, 432 pints = 1 queue Burgundy, or pipe brandy. 1 pint = 57.24 Eng. cubic inches.

OLD DRY MEASURE.—16 litrons = 1 boisseau, 12 boisseau = 1 setier, 12 setier = 1 muid; 1 boisseau = 779.564 Eng. cubic inches.

NEW SYSTEM OF LIQUID AND DRY MEASURES.—10 mililitres = 1 centilitre, 10 centilitres = 1 decilitre, 10 decilitres = 1 LITRE, 10 litres = 1 decalitre, 10 decalitres = 1 hectolitre, 10 hectolitres = 1 kilolitre, 10 kilolitres = 1 myralitre; 1 litre = 61.024429 Eng. cubic inches.

HOLLAND.

16 pfenings = 1 stiver = 5 cents = 2 grotes Flemish, 20 stivers = 1 florin or guilder = 100 cents = 40 grotes Fl., 2½ florins = 1 rix dollar = 250 cents = 100 grotes Fl., 12 pence or grotes = 1 shilling Fl., 20 Sh. = £1 Fl. = 6 florins or guilders.

Intrinsic value of the florin banco = 23.274 pence sterling.
In Amsterdam and all Holland accounts are kept both in pfenings, stivers, and guilders. Cents and florins, £s sh. and pence Flemish.

WEIGHTS.—1 lb. Dutch standard = 7625 Eng. troy grains, 1 lb. Dutch troy = 7595 Eng. troy grains, 1 lb. East India Company = 7738 Eng. troy grains, 1 lb. Antwerp for iron, silk, &c. = 7261 Eng. troy grains.

CLOTH MEASURE.—1 ell of Amsterdam = 27.176 English inches, 1 ell Flemish = 27.976 English inches.

LIQUID MEASURE.—4 pints = 1 stoop, 8 stoops = 1 stekan, 2 stekans = 1 anker, 4 ankers = 1 ahm, 1 ahm = 9297 English cubic inches.

DRY MEASURE.—36 sacks = 1 last = 178104 Eng. cubic inches at Amsterdam, 29 sacks at Rotterdam and 39 sacks at Flushing = to a last at Amsterdam.

FLANDERS AND BRABANT.

MONEY.—12 pfenings currency = 1 stiver, 20 stivers = 1 florin, 7 fl. currency = 6 fl. of exchange.

WEIGHTS AND MEASURES.—The lb. = 7261 English troy grains, the long ell for silks, &c. = 27.33 Eng. inches, the short ell for woollens = 26.947 Eng. inches, a viertel = 4705 Eng. cubic inches, and 37½ viertels = 1 last of corn.

GERMANY.

12 phenings = 1 schilling, 16 schillings = 1 mark lubs, 2 marks = 1 dollar of exchange, 3 marks = 1 rixdollar, 6 phenings lubs = 1 grote or penny Flemish, 12 pence or grotes = 1 schilling, 20 schillings = £1 Flemish, 1 schilling lubs or Hambro' = 2 pence or grotes Flemish.

Intrinsic value of a rixdollar = 59.458 pence sterling.

Accounts are kept as above in Hamburg, Holstein, Lubec, Mecklenburg.

4 pfenings = 1 kreutzer convention currency, 60 kreutzers = 1 florin or gulden, 1½ florins = 1 rixdollar current, 2 florins = 1 rix dollar effective.

Intrinsic value of a florin = 26.815 pence sterling.

Accounts are kept thus in Austria, Bavaria, Bohemia, Franconia, Hungary, Swabia, Thuringia, Trieste, and Tyrol.

8 pfenings cash = 1 marien grashen, 36 marien grashen = 1 rixdollar. } Brunswick, Hanover, and Luneburg.

Intrinsic value of the rixdollar = 44.683 pence sterling.

WEIGHTS.

1 lb. =	7476 Eng. troy grains		at Hamburg and Oldenburg.
1 lb. =	7461	do.	at Holstein, Lubec, and Wismar.
1 lb. =	7458	do.	at Mecklenberg.
1 lb. =	7511	do.	at Hanover and Luneburg.
1 lb. =	7206	do.	at Brunswick and Hildesheim.

CLOTH MEASURE.

1 ell =	22.5583 English inches		at Hamburg and Holstein.
1 ell =	22.718	do.	at Lubec.
1 ell =	22.913	do.	at Hanover and Luneburg.
1 ell =	22.469	do.	at Brunswick.

DRY MEASURE.—60 fasses = 1 last of corn = 192905 Eng. cubic inches at Hamburg, 96 sheffels = last of wheat and rye = 195696 Eng. cubic inches at Lubec and Holstein, 96 sheffels

= 1 last of oats = 229856 Eng. cub. inches at Lubec and Holstein, 96 himtens = 1 last of corn = 182208 Eng. cubic inches at Brunswick, Hanover, Luneberg, and Zell.

RUSSIA.

50 copecks = 1 poltin, 100 copecks = 1 rouble, 2 roubles = 1 ducat, 1 rouble at par = 41.32 pence sterling.

The rouble is subdivided into silver, and bank notes, each divided into 100 copecks; 1 silver rouble = 3 roubles 76 copecks bank notes.

80 alberts groshen = 1 albert dollar.	Accounts are kept thus at Libau and Riga.
1 albert dollar = 55.887 pence ster.	

Weights.—3 solotnicks = 1 loth, 32 loths = 1 lb., 40 lbs. = 1 pood, 10 poods = 1 berkowitz, 1 lb. = 6313 Eng. troy grains. 1 lb. = 6377 Eng. troy grains at Libau, 1 lb. = 7223 Eng. troy grains at Narva, 1 lb. = 6430 Eng. troy grains at Pernau, 1 lb. = 6646 Eng. troy grains at Revel, 1 lb. = 6454 Eng. troy grains at Riga.

Cloth Measure.—16 wershocks = 1 arsheen, 3 arsheens = 1 sacheen, 500 sacheens = 1 verst or mile, 1 arsheen = 28 Eng. inches. 1 ell = 23.5 Eng. inches at Narva, 1 ell = 21.6 Eng. inches at Pernau and Riga, 1 ell = 21.1 Eng. inches at Revel.

Liquid Measure.—11 czarkens = 1 kruska, 8 kruskas = 1 vedro, 40 vedros = 1 sarokoi, 1 vedro = 751.72 English cubic inches.

Dry Measure.—8 garnitzys = 1 chelwerick, 2 chelwericks = 1 pajack, 2 pajacks = 1 osmin, 2 osmins = 1 chetwert, 15 chetwerts = 1 last, 1 chetwert = 11902 Eng. cubic inches.

24 loofs = 1 last of wheat, rye, barley, and peas = 183493 Eng. cubic inches at Libau, 60 loofs = 1 last of oats = 229366 ditto ditto. 24 tons = 1 last of wheat, &c. = 237413 Eng. cubic inches at Narva. 24 tons = 1 last of wheat, &c. = 185497 Eng. cubic inches at Pernau. 24 tons = 1 last of wheat, &c. = 173366 Eng. cubic inches at Revel. 24 tons = 1 last of wheat and barley = 190872 Eng. cubic inches at Riga, 30 tons = 1 last of oats, malt, and peas = 238590 Eng. cubic inches at Riga.

SWEDEN.

Money.—12 runstycken = 1 shilling, 48 shillings = 1 rixdollar, 1 rixdollar = 4/9.251 sterling.

WEIGHTS.—1 grocer's lb. = 6563 Eng. troy grains, 1 miner's lb. = 5801 Eng. troy grains, 1 seaport lb. = 5250 Eng. troy grains, 1 Stralsund lb. = 7460 Eng. troy grains.

CLOTH MEASURE.—1 Swedish ell = 23.36 English inches, 1 Stralsund ell = 23 Eng. inches.

DRY MEASURE.—56 kannor = 1 ton, 24 ton = 1 last, 1 last = 214560 Eng. cubic inches, 96 sheffels 1 last Stralsund = 228234 Eng. cubic inches.

LIQUID MEASURE.—4 quarts = 1 stop, 2 stops = 1 kannor, 15 kannor = 1 ankar, 2 ankars = 1 eimer, 3 eimers = 1 oxhufud, 48 kannors = 1 ton, 1 kannor = 160 Eng. cubic inches, 1 Stralsund stubgen = 236 Eng. cubic inches.

DENMARK AND NORWAY.

MONEY.—16 shillings = 1 mark, 6 marks = 1 rixdollar, 1 rixdollar currency = 3/11 stg. 1 rixdollar banco = 4/9½ stg.

WEIGHTS.—7715 troy gr. = 1 lb., 16 lb. = 1 lispand, 20 lispands = 1 shippond.

LINEAL MEASURE.—12.3⅝ Eng. inches = 1 Rhindland foot, 2 Rh. feet = 1 ell.

LIQUID MEASURE.—228.2 Eng. cubic inches = 1 stubgen, 10 stubgens = 1 anker, 4 ankers = 1 ahm or tierce, 3 ahms = 1 pipe, 2 pipes = 1 fuder, 1 barrel tar = 7074 Eng. cub. in.

DRY MEASURE.—1061.125 Eng. cubic inches = 1 scheffel, 8 scheffels = 1 toende, 12 toendes = 1 last.

PRUSSIA.

MONEY.—12 pfenings = 1 good-groshen, 24 good-groshens = 1 rixdollar, 1 rixdollar currency = 3/2.192 stg., 1 rixdollar banco = 4/2.26 stg.

5 swares = 1 grote, 72 grotes = 1 rixdollar current = 39.242 pence sterling at Bremen on the Weser, and Oldenburg in Westphalia. 17¼ pfenings currency = 1 groshen, 30 groshens = 1 florin, 3 florins = 1 rixdollar current = 2/10.18 stg. at Dantzic, Koningsberg and Memel. 10 wittens = 1 stiver, 20 stivers = 1 florin = 1/2.115 stg. at Embden.

WEIGHTS.—7232 troy grs. = 1 lb., 110 lb. = 1 centner.

LINEAL MEASURE.—26.24 Eng. inches = 1 ell, 1 mile = 8472.7 Eng. yards.

Liquid Measure.—2 oessels = 1 quart, 32 quarts = 1 anker, 2 ankers = 1 eimer, 3 eimers = 1 oxhoft, 4 oxhufts = 1 fuder; 1 eimer = 4493 Eng. cubic inches.

Dry Measure.—3172·72 Eng. cubic inches = 1 scheffel, 12 scheffels = 1 malter, 2 malters = 1 wispel, 2 wispels = 1 last of oats, 3 wispels = 1 last of wheat.

POLAND.

Money.—9 pfenings = 1 groshen, 30 groshens = 1 florin or guilder, 1 florin = 6.387 pence stg.

Weights.—6236 Eng. troy gr. = 1 lb., 32 lb. = 1 stone, 5 stones = 1 centner.

Lineal Measure.—24.3 Eng. inches = 1 ell.

Liquid Measure.—2 pounds = 1 quart, 4 quarts = 1 garniec, 1 garniec = 97 Eng. cubic inches.

Dry Measure.—3121 Eng. cubic inches = 1 korkeck, 60 korkecks = 1 last.

SPAIN.

Money.—34 maravedies = 1 real, 20 reals = 1 Spanish dollar, Vellon money. 34 maravedies = 1 real, 8 reals = 1 peso or dollar of exchange, 32 reals = 1 pistole of exchange, 375 maravedies = 1 ducat of exchange, Old Plate money. 32 reals Vellon = 17 reals Old Plate. 1 dollar of exchange = 3/5.959 stg.

Weights.—36 grains = 1 adarme, 2 adarmes = 1 dracma, 8 dracmas = 1 ounce, 8 ounces = 1 mark, 2 marks = 1 pound, 25 lb. = 1 arroba, 4 arrobas = 1 quintal; 1 lb. = 7114 Eng. troy grs.

Lineal Measure.—12 inches = 1 foot = 11.1282 Eng. inches, 3 feet = 1 vara or yard, 2 varas = 1 brasa.

Liquid Measure.—32 lb. = 1 arroba, 1 arroba = 955 Eng. cubic inches.

Dry Measure.—3487 Eng. cubic inches = 1 fanega, 12 fanegas = 1 cahiz.

PORTUGAL.

MONEY.—400 rees = 1 crusado of exchange, 480 rees = 1 new crusado, 1000 rees = 1 milree, 1 milree = 5/1.842 stg.

WEIGHTS.—72 grs. = 1 oitavo, 8 oitavos = 1 ounce, 8 ounces = 1 mark, 2 marks = 1 lb., 32 lb. = 1 arroba, 4 arrobas = 1 quintal; 1 lb. = 7084 Eng. troy grs.

LINEAL MEASURE.—12 points = 1 line, 2 lines = 1 grain, 6 grains = 1 dedo, 12 dedoes = 1 foot, 2 f. 54 grs. = 1 cavado, 3⅛ feet = 1 vara, 2 varas = 1 brasa, 1 foot = 12.96 Eng. inches.

LIQUID MEASURE.—4 quartillos = 1 canada, 6 canadas = 1 alquier, 2 alquiers = 1 almude, 26 almudes = 1 pipe, 1 almude = 1040 Eng. cub. inches.

DRY MEASURE.—4 maquaras = 1 quart, 4 quarts = 1 meyo, 2 meyoes = 1 alquiere, 4 alquieres = 1 fanga, 15 fangas = 1 moyo, 1 alquiere = 817 Eng. cubic inches.

ITALY.

MONEY.

ROME.—10 bajocci = 1 paolo, 10 paoli = 1 scudo = 4/7.475, sterling.

VENICE.—12 denari piccoli = 1 soldo, 20 soldi = 1 lira = 5.221d. sterling. 12 denari di ducato = 1 grosso, 24 grossi = 1 ducat = 2/8.361 sterling.

NAPLES.—10 grani = 1 carlin, 10 carlins = 1 ducato di regno = 3/8, sterling. 10 grani = 1 carlin, 12 carlins = 1 scudo = 4/4.815 sterling.

MILAN.—12 dinari currency = 1 soldo, 20 soldi = 1 lira, 6 lire = 1 scudo = 3/11.418 sterling.

GENOA.—12 denari di lira fuori banco = 1 soldo, 20 soldi = 1 lira, 5¾ lire = 1 pezza, 8 lire = 1 scudo = 5/8 sterling.

LEGHORN and FLORENCE.—12 denari di lira = 1 soldo, 20 soldi = 1 lira moneta buona = 8.606d. sterling. 12 denari di pezza = 1 soldo, 20 soldi = 1 pezza = 4/1.455 sterling. 24 lire moneta lunga = 23 lire moneta buona.

WEIGHTS.

ROME.—24 grains = 1 scrupolo, = 3 scrupoli = 1 dramme, 8 dr. = 1 ouncie, 12 oz. = 1 libra or lb. = 5238 Eng. troy grs.

VENICE.—12 oz = 1 peso grosso = 7383 Eng. troy grs. 12 oz. 1 peso sottile = 4672 Eng. troy grs.

NAPLES.—20 acini = 1 trapeso, 30 trapesi = 1 oz. 12 oz = 1 lb. = 4966 Eng. troy grs. 2.8 lb. = 1 rotolo grosso, 100 rotoli grossi = 1 cantaro grosso. 2.5 lb. = 1 rotolo piccolo, 100 rotoli piccoli = 1 cantaro piccolo.

MILAN.—28 oz. = 1 libra grossa = 11784 Eng. troy grs. 1 peso sottile = 5060 Eng. troy grs.

GENOA.—1 peso grosso = 4907 Eng. troy grs., 1 peso sottile = 4898 Eng. troy grs., 1 rotolo cantaro = 7460 Eng. troy grs.

LEGHORN and FLORENCE.—24 grs. = 1 denari, 24 denari = 1 ounce, 12 oz. = 1 lb. = 5243 Eng. troy grs.

LINEAL MEASURE.

ROME.—1 foot = 11.61 Eng. inches, 1 palmo = 9.791 Eng. inches, 8 palmi = 1 canna, used for silk and woollens, 1 braccio for these goods = 33.384 Eng. inches, but for linens the canna is = 82.276, and the braccio = 24.992 Eng. inches.

VENICE.—1 foot = 13.677 Eng. inches. The silk braccio = 24.825, and the woollen braccio = 27.354 Eng. inches.

NAPLES.—12 onzie = 1 palmo, 8 palmi = 1 canna = 83.05 Eng. inches.

MILAN.—The braccio = 23.135 Eng. inches.

GENOA.—1 palmo = 9.725 Eng. inches, 1 canna grossa = 116.7 Eng. inches, 1 custom-house canna = 97.5 Eng. inches.

LEGHORN AND FLORENCE.—2 palmi = 1 braccio, 4 bracci = 1 canna. The wollen braccio is = 23.255, and the silk braccio is = 22.913 Eng. inches.

LIQUID MEASURE.

ROME.—4 cartocci = 1 foglietta, 4 fogliette = 1 boccale, 32 boccali = 1 barile of wine = 2556 Eng. cubic inches. 4 cortocci = 1 foglietta, 4 fogliette = 1 boccale, 28 boccali = 1 barile of oil = 2237 Eng. cubic inches.

VENICE.—16 enghistare = 1 secchia, 4 secche = 1 quartaro, 4 quartari = 1 bigoncia, 4 bigonce = 1 amphora = 38581 Eng. cubic inches. 40 miri = 1 migliago = 1000 peso grosso of oil.

NAPLES.—60 caraffi = 1 barile = 2693 Eng. cubic inches. 32 pignatte = 1 staro, 10 stari = 1 salma = 11329 Eng. cubic inches. The barile is used for wine, &c., the salma for oil.

MILAN.—8 baccali = 1 pint, 8 pints = 1 mina 2 mine = 1 staro, 3 stari = 1 branta = 3166 Eng. cubic inches.

GENOA.—50 pints = 1 barrel, 2 barrels = 1 mozzarola = 10531.3 Eng. cubic inches.

LEGHORN AND FLORENCE.—2 quartucci = 1 mezzetta, 2 mezzette = 1 buccalo, 2 buccali = 1 fiasco, 20 fiaschi = 1 barile. 1 barile of wine at Leghorn = 2564, and at Florence = 2427 Eng. cubic inches. The barile of oil at both places = 1924 Eng. cubic inches.

DRY MEASURE.

ROME.—1½ starelli = 1 staio, 3 staji = 1 quarta, 4 quarte = 1 rubbio = 16701 Eng. cubic inches.

VENICE.—4 quarters = 1 staro = 4946 Eng. cubic inches.

NAPLES.—36 tomoli = 1 carro = 112388 Eng. cubic inches.

MILAN.—2 starelli = 1 staio, 8 staji = 1 moggo, 2 moggi = 1 rubbo, 14 rubbi = 1 mina = 236445 Eng. cubic inches.

GENOA.—12 gambette = 1 quarta, 8 quarte = 1 mina = 7116 Eng. cubic inches.

LEGHORN AND FLORENCE.—8 mazzete = 1 quarta, 12 quarte = 1 staio, 3 staji = 1 sacco, 8 sacci at Leghorn, and 7½ at Florence = 1 moggio, 1 sacco = 4466 Eng. cubic inches.

TURKEY.

MONEY.—3 aspers = 1 para, 40 paras = 1 piaster = .1/1.972 sterling.

WEIGHTS.—60 drams = 1 oz., 12 oz. = 1 rottolo = 35187 Eng. troy grs.

LINEAL MEASURE.—1 archin, or long pic = 27.88 Eng. inches, 1 endassee, or short pic = 25½ Eng. inches.

5

LIQUID MEASURE.—1 almud = 519.57 Eng. cubic inches.
DRY MEASURE.—1 killot = 2162.6 Eng. cubic inches.

IRELAND.

IRELAND exchanges with BRITAIN a variable number of £.'s and sh. currency for £100, sterling, par £100, sterling for £108 6s. 8d. Irish, and 3 days of grace, the same as in Great Britain.

As the exercises in Exchange are all performed by rules already given and explained, it would be superfluous to repeat them again under this head.

EXERCISES.

1. Reduce £1000 sterling into Irish money, exchange at par. Ans. £1083 6s. 8d.
2. Reduce £1000 Irish currency, into sterling, exchange at par. Ans. £923 1s. 6½d. $\frac{1}{13}$.
3. A of London owes B of Dublin £6787, Irish; for how much sterling should B draw, exchange 12½ per cent.? Ans. £5938 12s. 6d.
4. A of Dublin owes B of London £5000 sterling; for how much Irish should the bill be drawn, exchange 8½ per cent.? Ans. £5425.
5. My annual rental in Ireland is £7658; for how much sterling should I draw upon my agent for half a year's rent, exchange 10 per cent.? Ans. £3480 18s. $2\frac{2}{11}$d.
6. A gentleman draws £6400 from a banker in London, and orders upon his agent in Belfast for the amount; for how much Irish should the order be given, exchange 15 per cent., commission ½ per cent.? Ans. £7396 16s.

WEST INDIES.

Par of exchange with Jamaica is £100 sterling = £140 currency, but in the other islands there is no fixed par.

EXERCISES.

1. How much sterling does £10000 Jamaica currency amount to, exchange at par? Ans. £7142 17s. 1½d. $\frac{4}{7}$.
2. How much Jamaica currency does £880 sterling amount to, exchange at par.? Ans. £1232.
3. My factor in Barbadoes has purchased goods on my account to the value of £4650 currency; for how much sterling should he draw, exchange 144 per cent., commission 6 per cent.? Ans. £3422 18s. 4d.

4. My agent in Martinico owes me £12680 currency; for how much sterling should I draw, exchange 220 per cent.? Ans. £5763 12s. 8½d. $\frac{10}{11}$.

5. My factor in Trinidad has sent me goods to the amount of £4650 currency; for how much sterling should he draw, exchange 200 per cent., and factorage 8 per cent.? Ans. £2511.

6. How much Jamaica currency will purchase a bill on London for £1000 sterling, premium in favour of London 12½ per cent.? Ans. £1575.

AMERICA.

AMERICA exchanges with BRITAIN a variable number of £'s currency for £100 sterling, par £100 currency = £90 sterling in the British possessions. Par of the dollar = 4/6 sterling, or 40 dollars = £9 sterling.

EXERCISES.

1. How much currency of Canada must be given for £650 sterling, exchange at par? Ans. £722 4s. 5¼d. ⅓.

2. My factor in New Brunswick owes me £896 currency, what is the amount in sterling money, exchange at par? Ans. £806 8s.

3. How much New York currency will purchase a bill of £1000 sterling on London, exchange 178 per cent., and premium 3 per cent.? Ans. £1833 8s.

4. How much sterling must I remit to Pensylvania to pay a debt of £3650 currency, exchange 168 per cent.? Ans. £2172 12s. 4½d. $\frac{2}{7}$.

5. My factor in New England writes, that he has drawn on me for £5890 sterling; how much currency does his draft amount to, exchange at 136 per cent.? Ans. £8010 8s.

6. Reduce 8640 dollars into sterling, bills on London being at a discount of 2 per cent. Ans. £1983 13s. 5¾d+.

FRANCE.

FRANCE exchanges with BRITAIN a variable number of francs and cents. per £ sterling, par 23 francs 23 cents = £1 sterling. Usance 30 days after date, and 10 days of grace.

EXERCISES.

1. How much sterling in 860 fr. 50 cents., exch. 24 fr. 50 cents. per £ sterling? Ans. £35 2s. 5¼d. $\frac{27}{43}$.

2. In £565 sterling; how many francs, exch. 25 fr. 20 cents. per £ sterling? . Ans. 14238.
3. What is the value of 3630 livers, in sterling money, exch. 31 pence per ecu.? Ans. £156 5s. 10d.
4. In 700 myriagrammes, how many cwt. avoirdupois? Ans. 137. cwt. 3 qrs. 16.164 lb.
5. My agent in Paris has purchased on my account a myriametre of silks at 5 fr. 40 cents. per metre; how much stg. should he draw for, exch. 25 fr. 5 decims per £ stg. and how should I sell it per impl. yard, to clear £300 and pay £120 of duty and expenses? Ans. draw for £2117 12s. 11¼d. $\frac{3}{17}$, Sells for 4/7½. $\frac{2558622}{3346399}$.
6. Purchased in Bourdeaux 25 myrialitres of cognac brandy, at 1 fr. 25 cents per litre, exch. 25 fr. per £ stg. paid for duty, freight, &c. 18/6 per impl. gal.; how must I sell it per gal. to clear 25 per cent.? Ans. £1 8s. 9¾d+.

HOLLAND.

Holland exchanges with Britain a variable number of shillings and grotes Flemish, per £ sterling. Par 36/7 banco per £ sterling. Usance one month after date, and six days of grace.

EXERCISES.

1. Reduce £1000 stg. into florins, exch. 37/9 Fl. per £ stg. Ans. 11325.
2. Reduce 1858 florins, 20 cents., into stg. money, exch. at par. Ans. £169 6s. 2¾d. $\frac{259}{435}$.
3. Reduce £650 Fl. banco into currency, agio 2½ per cent. Ans. £666 5s.
4. Reduce £860 Fl. currency, into banco, agio 3 per cent. Ans. £834 19s. 0¼d. $\frac{41}{103}$.
5. Reduce £480 Fl. banco, into current guilders, agio 3½ per cent. Ans. 2980 guil. 16 st.
6. Reduce 680 guilders 15 st., 8 pf. currency into stg., agio 5 per cent., exch. 36/8 Fl. banco per £ stg. Ans. £58 18s. 9¾d. $\frac{5}{7}$.
7. Reduce £3460 stg. into current florins, exch. 38/6 per £ stg., agio 4 per cent. Ans. 41561 fl. 52 cents.
8. Shipped at Amsterdam 4800 lb. Dutch standard weight of cheese, for which I paid at the rate of 2 st. 8 pf. currency per lb., how must I sell it per cwt. in Leith to gain £20 on the whole, after paying £8 10s. for duty, freight, &c., exch. 39/6 Fl. banco per £ stg., agio 5 per cent.? Ans. £1 12s. 10½d+.

9. Shipped at Flushing 100 ahms of gin at 8 florins 50 cents. currency per stekan, paid in London for custom-house dues, freight, &c., £1 1s 6d. per imperial gallon; how must I sell it per gallon, to gain £600 upon the whole, agio 2 per cent., exch. at par? Ans. £1 8s. 8½d+.

10. My agent in Holland writes that he has shipped on my account 145 sacks of linseed at Rotterdam, and 234 sacks at Flushing, for which he has paid at the rate of 30/6 Fl. currency per sack of Amsterdam; what did it cost per imperial quarter, exch. 38/6 banco per £ stg., agio 4 per cent.? Ans. £2 14s. 7¾d+.

FLANDERS AND BRABANT.

FLANDERS AND BRABANT exchange with BRITAIN a varying number of shillings and pence Flemish banco per £ stg., par 38/11½ Fl. per £ stg. Usance and days of grace, the same as at Amsterdam.

EXERCISES.

1. Reduce 68280 pfenings Brabant currency into florins of exchange. Ans. 243 fl. 17 st. $1\frac{5}{7}$ pf.
2. Reduce 1000 lb. of Flanders into cwts. avoirdupois. Ans. 9 cwt. 1 qr. $1\frac{3}{7}$ lb.
3. In 650 impl. yds. of silk; how many Flanders ells? Ans. 856.0144+.
4. Reduce 33894 impl. yds. of flannel into Brabant woollen ells. Ans. 45280.8846+.
5. Reduce 1000 viertels of Flanders into impl. quarters. Ans. 422.9232+
6. If wheat be bought at Antwerp for 8 florins 10 st. currency per viertel; how much stg. will purchase 100 lasts, exch. 39/ Flemish banco per £ stg.? Ans. £2335 3s. 3½d. $\frac{27}{31}$.

GERMANY.

HAMBURG, &c. exchange with BRITAIN a variable number of shillings and pence Flemish banco per £ stg. 2½, rixdollars Hambro' banco are equal to £1 Fl. banco. There is no permanent par between Hamburg and Britain. Usance one month after date, and 12 days of grace.

AUSTRIA, &c. exchange with BRITAIN a variable number of florins and kreutzers per £ stg., par 9 fl. 37 kreutzers.

Brunswick, &c. exchange with **Britain** a variable number of rixdollars and marien groshen cash, per £ stg. par 5 rxd. 25 groshen. Usance 14 days, and 3 days of grace.

EXERCISES.

1. Reduce 1920000 phennings into rixdollars. Ans. $3333\frac{1}{3}$.
2. Reduce 69810 marks lubs banco in £'s Fl. banco. Ans. £9308.
3. Reduce £680 stg. into marks lubs currency, exch. 35/6 Fl. banco per £ stg., agio 10 per cent. Ans. $9957\frac{3}{4}$.
4. Reduce 640320 kreutzers into stg. exch. at par. Ans. £1109 14s. $9\frac{1}{2}$d. $\frac{259}{577}$.
5. Reduce 985320 marien groshen into stg. exch. 5 rxd. 30 marien grosh. per £ stg. Ans. £4692.
6. In 100 lb. of Hamburg; how many lb. avoirdupois? Ans. $5219\frac{14}{1872}$.
7. In 480 impl. yds.; how many ells of Hambro'? Ans. 776.4257+.
8. Bought 12 last of wheat at Hambro', 6 of wheat, and 4 of oats at Lubec, and 5 of barley at Zell; how many impl. qrs. had I in all? Ans. 299.7655+.
9. It appears from the books of a deceased merchant, that there was owing by him in Hamburg 33000 marks currency, in Mecklenburg 365 rixdollars banco, in Vienna 6846 florins 30 kreutzers, in Hanover 6846 rixdollars; how much stg. will meet these demands, supposing the agio at Hambro' to be 20 per cent., and exch. 35 shillings Fl. banco per £ stg., at Vienna 10 rixdollars per £ stg., and in Hanover 6 rixdollars per £ stg.? Ans. £4004 6s. 4d.

RUSSIA.

Russia exchanges with **Britain** the rouble for a varying number of pence stg., par 41.324 pence. Usance 3 months after date, and 10 days of grace.

EXERCISES.

1. Reduce £686 stg. into roubles, exch. 3/6 per rouble. Ans. 3920.
2. Reduce 5657 r. 50 cop. into stg. exch. 3/4 per rouble. Ans. £942 18s. 4d.
3. Reduce £855 stg. into rouble bank notes, exch. 4/3 stg. per silver rouble. Ans. 15128 r. $47\frac{1}{7}$ cop.

4. Reduce 6987 albert dollars into stg. exch. 375 albert groshen per £ stg. Ans. £1490 11s. $2\frac{1}{4}$d. $\frac{3}{5}$.
5. In 10 tons avoirdupois; how many berkowitz?
Ans. 62 berk. 37 lb. 20 lo. $1\frac{7211}{8319}$ solot.
6. In 10000 lb. at Riga; how many cwt. at Leith?
Ans. 82 cwt. 1 qr. 8 lb.
7. A of London is indebted to B of Petersburg, 1000 roubles, which he wishes to remit through Amsterdam, exch. between Amsterdam and London 37/6 Fl. per £ stg., and between Amsterdam and Petersburg 48 stivers per rouble; how much stg. will pay the debt? Ans. £213 6s. 8d.
8. Shipped at Riga 40 berkowitz of hemp, at 30 rouble bank notes per pood; what does it amount to in stg. exch. 52 stivers current per silver rouble, and 36/ Fl. banco per £ stg. agio $2\frac{1}{2}$ per cent.? Ans. £749 11s. $7\frac{1}{2}$d. $\frac{3962}{5781}$.
9. Shipped for Petersburg 6000 yds. flannel at 1/10 per yd., which sold there for 2 r. 66 cop. bank notes per arsheen, exchange $6\frac{1}{2}$ silver roubles per £ stg.; what did I gain or lose by this adventure, after paying 44 guineas of expenses?
Ans. gained £265 $10\frac{38}{893}$s.
10. Purchased at Petersburgh 30 lasts of barley at 2 r. 50 cop. bank notes per cheltwert, 50 lasts of wheat at Narva, for 10 silver roubles per ton, and 20 sarokoi of linseed oil at Riga for 1 silver r. 5 cop. per vedro; for how much stg. must the bill of exch. be drawn, exch. 48 st. 8 pf. Fl. banco per silver rouble, and 38/3 Fl. per £ stg.?
Aus. £2776 13s. $10\frac{1}{2}$d. $\frac{878}{2397}$.

SWEDEN.

SWEDEN exchanges with BRITAIN a variable number of rixdollars and shillings per £ sterling. Par 4 rxd. 21 sh. Usance 2 months after sight, and 10 days of grace.

EXERCISES.

1. Reduce £500 sterling into rxd. exchange at par.
Ans. 2218 rd. 36 sh.
2. Reduce 4860 rxd. into stg. exch. $4\frac{1}{2}$ rxd. per £ stg.
Ans. £1080.
3. In 1000 lb. avoirdupois; how many sea-port lbs.?
Ans. $1333\frac{1}{3}$.
4. In 100 English ells; how many Stralsund ells?
Ans. 195 $\frac{15}{23}$.

5. Imported from Sweden 100000 sea-port lb. of iron, at 1 sh. 6 runst. per sea-port lb., paid for freight, customs, &c. £198 10s. stg.; how must I sell it per cwt. in Leith to clear £60 10s. upon the whole, exch. 4 rxd. 24 sh. per £ stg.? Ans. £1 8s. $5\frac{1}{2}$d. $\frac{4828}{5625}$.
6. In 100 lasts of wheat in Sweden; how many English qrs.? Ans. 1209.09281+.
7. In 100 tons of wine in Sweden; how many English tons? Ans. 10.99136+.

DENMARK AND NORWAY.

Denmak and Norway exchange with Britain a variable number of rixdollars and shillings currency per £ sterling. Par 5 rxd. 40s. per £ stg. Usance 2 months after sight, and 10 days of grace.

EXERCISES.

1. In £695 stg.; how many rxd. current, exch. 3/4 per rxd. Ans. 4170.
2. In 6856 rxd. banco; how much stg., exch. 3/6 stg. per rxd. current, agio 20 per cent.? Ans. £1434 15s. $2\frac{1}{4}$d. $\frac{3}{4}$.
3. In 10 cwt. avoirdupois; how many ship-ponds? Ans. 3 ship. 3 lisp. $8\frac{312}{1543}$ lb.
4. In 250 barrels Norway tar; how many impl. gallons? Ans. 6378.167+.
5. A of Edinburgh is indebted to B of Copenhagen, 9848 rxd. banco, exchange 5 rxd. 48 sh. currency per £ stg., agio 25 per cent.; how much sterling is the debt? Ans. £2238 3s. $7\frac{1}{4}$d. $\frac{6}{11}$.
6. London is indebted to Copenhagen, 12600 rxd.; how much stg. will pay the debt, the exch. between Copenhagen and Hamburg being 100 rxd. for 300 marks banco, and between Hamburg and London 36/9 Fl. banco per £ stg.? Ans. £2535 16s. $11\frac{3}{4}$d. $\frac{5}{83}$.

PRUSSIA.

Prussia exchanges with Britain a variable number of rixdollars and groshen current, for £1 stg. Par 16 rxd. 16 grosh. Usance 3 months after date, and 10 days of grace.

EXERCISES.

1. Reduce £540 stg. into Prussian rixdollars current, exch. 7 rxd. 8 good grosh. currency per £ stg. Ans. 3960.

2. Reduce 9840 rxd. Prussian banco into stg., exch. 6 rxd. 18 good grosh. current per £ stg.; agio 33$\frac{1}{3}$ per cent.
Ans. £943 14s. 0$\frac{3}{4}$d. [illegible].

3. Purchased 1000 lasts of wheat in Dantzic, at 7 fl. 15 grosh. per sheffel; how much stg. was that per impl. quarter, exchange 250 grosh. per £ Fl. and 35/6 Fl. per £ stg.?
Ans. £2 16s. 8$\frac{1}{2}$d. [illegible].

4. Purchased at Koningsberg 15 fuder of mead at the rate of 8 fl. 15 grosh. per anker; what does it amount to in stg., exch. 290 grosh. per £ Fl. and 37/6 Fl. per £ stg.?
Ans. £165 7s. 7$\frac{1}{25}$d.

5. For how much stg. must a bill of exch. be drawn to pay for 100 centners of tallow, purchased in Berlin at 9 rxd. 6 good grosh. current per centner, exch. 150 rxd. current for 255 guilders Fl. banco, and 34/9 Fl. per £ stg.
Ans. £150 16s. 9$\frac{1}{4}$d. [illegible].

POLAND.

Poland exchanges with Britain through Holland. The course of exchange varies from 240 to nearly 300 groshen per £ Flemish.

EXERCISES.

1. Reduce £100 stg. into Polish florins, exch. 38/3 Fl. per £ stg. and 260 grosh. per £ Flemish.
Ans. 1657 fl. 15 grosh.

2. Reduce 45630 florins into stg. exch. 294 grosh. per £ Fl., and 36/ Fl. per £ stg. Ans. £2586 14s. 8$\frac{1}{4}$d. [illegible].

SPAIN.

Spain exchanges with Britain the peso de plata for a varying number of pence. Par. 41.959 pence. Usance 2 months after sight, and 14 days of grace.

EXERCISES.

1. Reduce 89760 reals vellon, into reals old plate. Ans. 47685.
2. Reduce 18496 reals vellon into stg., exch. 38 pence per peso de plata. Ans. £194 9s. 5$\frac{1}{4}$d.
3. Reduce £485 stg. into reals vellon exch. 40 pence per peso de plata. £5477 reals 22 mar.
4. Purchased 100 quintals of Muscatel raisins, at 2 reals 17 mar. vellon per lb.; how much stg. do they come to, exch. 36 pence per peso de plata? Ans. £1992 3s. 9d.

5. In 375000 ducats of exch.; how much stg. exch. 39 pence per dollar of exch.? Ans. £84013 1s. 11¼d. $\frac{9}{17}$.
6. How many dollars, pistoles, and ducats of exchange, and of each an equal number, should I receive for £1000 stg. exch. 40 pence per peso? Ans. 940$\frac{220}{347}$.

PORTUGAL.

PORTUGAL exchanges with BRITAIN the milree for a variable number of pence sterling. Par. 61.842d. Usance 30 days sight, and 6 days of grace.

EXERCISES.

1. Reduce 3864 milrees 465 rees into stg., exch. 48 pence per milree. Ans. £772 17s. 10¼d. $\frac{7}{25}$.
2. Reduce £640 stg. into milrees, exch. 55 pence stg. per milree. Ans. 2792 milrees 727$\frac{3}{11}$ rees.
3. Reduce 5000 crusadoes of exch. into stg. exch. 50 pence per milree. Ans. £416 13s. 4d.
4. Purchased in Oporto 684 pipes of wine, at 3 milrees 525 rees per almud, and paid for freight, customs, &c. as much as it cost me at first purchase; for how much stg. must I sell it, to clear 12½ per cent., exch. 65 pence per milree? Ans. £38200 17s. 3¾d.

ITALY.

ROME exchanges with BRITAIN, the scudo of 15¼ paoli, for a variable number of pence. Par 84.59 pence. Usance 2 days after acceptance, and no days of grace.

VENICE exchanges with BRITAIN a variable number of lire piccoli for £1 stg. Par 48 lire. Usance 3 months after date, and 6 days of grace.

NAPLES exchanges with BRITAIN the ducato di regno for a variable number of pence. Par 44 pence. Usance 3 months after date, and 3 days of grace.

MILAN exchanges with BRITAIN a variable number of lire and soldi currency for £1 stg. Par 32 lire. Usance 3 months after date, no grace.

GENOA enchanges with BRITAIN the pezza of 5¾ lire for a variable number of pence. Par 48.858 pence. Usance 3 months after date, and 30 days of grace to the holder, but none to the acceptor.

LEGHORN and FLORENCE exchange with BRITAIN the

pezza for a variable number of pence. Par 49.455 pence. Usance 3 months after date, and no grace.

EXERCISES.

1. In £545 stg.; how many scudi at Rome. Exchange 81d, per soldo? Ans. 1614 so. 12 Pa. $6\frac{8}{27}$ baj.
2. In 4369 scudi 5 paoli 5 bajocci, how much stg. exch. 80 pence per scudo? Ans. £ꞁ1456 10s. 6d.
3. In 1 cwt. avoirdupois, how many lb. at Rome? Ans. 149 lb. 8 oz. 2 sc. $12\frac{8}{9}$ gr.
4. How much stg. should I receive for a bill of 86520 lire peccoli on Venice, exch. 48 lire peccoli per £ stg.? Ans. £1802 10s.
5. Reduce £375 stg. into Venice ducats, each 45 pence stg. per ducat. Ans. 2000.
6. Reduce £845 16s. into ducati di regno of Naples, exch. 42 pence per ducat. Ans. 4833 duc. 1 carlin. $4\frac{2}{7}$ gr.
7. Reduce 7640 ducati di regno into stg. exch. $40\frac{1}{2}$ pence per ducat. Ans. £ꞁ1289 5s.
8. In £676 stg., how many scudi at Milan, exch. 33 lire 10 soldi per £ stg.? Ans. 3774 scudi 2 lire.
9. In 16852 scudi at Milan, how much stg. exch. 36 lire per £ stg.? Ans. £2808 13s. 4d.
10. How many scudi of Genoa in £769, exch. 5/ stg. per pezza? Ans. 2210 scudi 7 lire.
11. How much stg. in 89520 lire of Genoa, exch. 5/3 per pezza? Ans. £4086 15s. $7\frac{3}{4}$d. $\frac{7}{23}$.
12. Required how much stg. in 985 pezza of Leghorn, exch. 45 pence per pezza. Ans. £184 13s. 9d.
13. Required for how many pezza a bill must be drawn on Florence, to be equal in value to £1650 stg., exch. 50 pence per pezza. Ans. 7920.
14. In ten pesi grossi and 12 pesi sottile at Venice, 20 cantari grossi and 10 cantari piccoli at Naples, 3(0 libri grossi and 50 pesi sottile at Milan, 12 pesi grossi 15 pesi sottile and 10 rotoli cantari at Genoa, and 100 lb. at Florence; how many cwt. avoirdupois? Ans. 53 cwt. 20 $\frac{87}{180}$ lb.
15. In 100 canne of silk and 100 canne of linen at Rome, 40 bracci of silks and 50 of flannel at Venice, 100 canne at Naples, 80 bracci at Milan, 10 canne grosse at Genoa, 4 canne of silk and 10 of flannel at Leghorn; how many yds. in London? Ans. 862.2418 +.
16. In 10 barile of wine at Rome, 20 amphore at Venice, 30 barile at Naples, 50 brante at Milan, 10 mazzarole at

Genoa, 12 barile at Leghorn and 10 at Florence; how many hhds. in London? Ans. 68 hhd. 31 gal. 2 qts. 1 pt.+.

17. In 6 rubbi of wheat at Rome, 10 stari at Venice, 12 carri at Naples, 10 mine at Milan, 20 mine at Genoa, 8 moggi at Leghorn and 12 moggi at Florence; how many Eng. quarters? Ans. 264 qrs. 5 bush. 2 gills +.

TURKEY.

Turkey exchanges with Britain the piaster for a variable number of pence. Par 13.972 pence. Usance 31 days after sight, and no established days of grace.

EXERCISES.

1. In £1000 stg., how many piasters at Constantinople, exch. $12\frac{1}{2}$ pence per piaster? Ans. 19200.
2. In 8750 piasters 20 paras at Aleppo, how much sterling, exchange $13\frac{1}{2}$ pence per piaster? Ans. £492 4s. $3\frac{3}{4}$d.

ARBITRATION OF EXCHANGES.

Arbitration of Exchanges is the method of comparing the course of exchange between several places, so as to ascertain the most profitable way of drawing and remitting money.

The calculations in arbitration of exchanges are performed, either by simple or compound proportion, according as the exchanges between three or more places are given.

EXERCISES.

1. When the exchange between London and Amsterdam is 34/6, and between London and Paris $10\frac{2}{3}$ pence per franc; how many grotes at Amsterdam are equal to one ecu of 3 fr. at Paris? Ans. $55\frac{1}{5}$.
2. When the exchange between London and Amsterdam is 34/4 Fl. per £ sterling, and between Amsterdam and Lisbon 52 pence Fl. per crusado; what is the par of exchange between London and Lisbon? Ans. $75\frac{75}{103}$d. per Milree.
3. When Leghorn exchanges with London at 50d. per pezza, and with Amsterdam at 95 grotes per pezza; what is the arbitrated par between London and Amsterdam? Ans. 38/ Fl. per £ sterling.

4. A of London is indebted to B of Paris £1000 sterling, which he can remit directly at 25 fr. 25 cents, per £, or through Amsterdam at 40/ Fl. banco per £, and $57\frac{1}{2}$ grotes Fl. banco per 3 francs; which is the most profitable remittance for London, and how much?

Ans. through Holland by $206\frac{1}{3}\frac{3}{8}$ fr.

5. Hamburg is indebted to London 6600 marks banco, for which London can draw directly at 40/ Fl. banco per £ sterling, or through Paris at 36sh. Hambro' banco for 3 fr. and 24 fr. per £ sterling; which is the most profitable way for London to draw, and how much?

Ans. directly by £73 6s. 8d.

6. A of Leith is indebted to B of Riga 10000 roubles; now if A can purchase a bill on Riga at 32 pence per rouble, and on Amsterdam at 38/6 Fl. banco per £ sterling, which of these is the most profitable remittance, supposing the exchange between Riga and Amsterdam be 30 stivers currency per rouble, agio $2\frac{1}{2}$ per cent?

Ans. through Amsterdam saves £66 6s. $1\frac{3}{4}$d. $\frac{118}{337}$.

7. N of London has credit in Leghorn for 27200 pezze, for which he can draw directly at 50d. per pezza, but wishing to try the circular exchange, he orders them to be sent first to Venice at 94 pezze per 100 ducats, thence to Cadis at 320 maravedies per ducat, thence to Lisbon at 630 rees per peso, thence to Amsterdam at 50 grotes per crusado of 400 rees, thence to Paris 56 grotes for 3 francs, and thence to London at $10\frac{3}{8}$d. per franc; whether did N gain or loss by this speculation, and how much, the expense of remittance being equal to $2\frac{1}{2}$ per cent. upon the proceeds?

Ans. gained £560 12s. $7\frac{223}{1527}$d.

8. A of Edinburgh has credit in Florence for 10000 pezze, for which he can draw directly at 58 pence per pezza, but wishing to try the circular exchange, they are sent to Venice at 91 pezze for 100 ducats, thence to Madrid at 340 maravedies per ducat, thence to Lisbon at 620 rees per peso de plata, thence to Amsterdam at 54d. Fl. per crusado of 400 rees, thence to Paris at 57d. Fl. per crown of 3 francs, and thence to London at $10\frac{1}{2}$d. sterling per franc; whether did A gain or loss by this speculation, and how much, each agent charging $\frac{1}{2}$ per cent. commission?

Ans. gained £111 10s. $8\frac{1}{4}$d +.

INVOLUTION.

INVOLUTION is the method of finding any proposed power of a given number.

The POWER of a number, is the product arising from the multiplication of the given number by itself the proposed number of times. The required power is either expressed in words, or by a small figure called the index or exponent, written at the upper and right hand corner of the proposed number. Thus 12^2, 12^3, 12^4, signify respectively, the second power or square, the third power or cube, the fourth power or biquaderate of 12, or $12\times12 =$ the square, $12\times12\times12 =$ the cube, $12\times12\times12\times12 =$ the biquadrate, &c.

EXAMPLES.—Required the cube of 16, 24.5, and $\frac{3}{7}$.

$16\times16 = 256\times16 = 4096.$

$24.5\times24.5 = 600.25\times24.5 = 14706.125.$

$\frac{3}{7}\times\frac{3}{7} = \frac{9}{49}\times\frac{3}{7} = \frac{27}{343}.$

In the first example 16×16 gives 256, which is the square of 16, and 256×16 gives 4096 which is the cube of 16. The second example is performed in the same manner. The third example is performed by multiplying $\frac{3}{7}$ by $\frac{3}{7}$, which gives $\frac{9}{49}$, and $\frac{9}{49}$ multiplied by $\frac{3}{7}$ gives $\frac{27}{343}$ the cube of $\frac{3}{7}$.

EXERCISES.

1. What is the square of 4, and cube of 8? Ans. 16, 512.
2. What is the cube of 15, and square of 24? Ans. 3375, 576.
3. What is the 36^3, and the 125^4? Ans. 46656, 244140625.
4. What is the 2.5^5, and the 16.4^6? Ans. 97.65625, 19456426.971136.
5. What is the 42^7, and the 84^8? Ans. 230539333248, 2478758911082496.
6. What is the cube of $\frac{1}{8}$, $\frac{631}{842}$ and .06? Ans. $\frac{1}{512}$, $\frac{251239591}{596947688}$, .000216.

EVOLUTION.

EVOLUTION is the method of finding any proposed root of a given number or power.

SQUARE ROOT.

To extract the square root of any power is to find such a number as multiplied by itself produces that power.

RULE.—Divide the given number into periods of two figures each, counting from right to left in integers, and from left to right in decimals. Find the greatest square number in the first period, write its square root in the quotient, and subtract the square number itself from the first period, and increase the remainder by next period. Double the quotient figure for a trial divisor, find how often it is contained in the increased remainder rejecting units place, write the result on the right both of the divisor and quotient, multiply the divisor by the quotient figure, and subtract the product as in division, and increase the remainder by next period. To the former divisor add its last figure for a new trial divisor, with which proceed as before, and so on, till all the periods are brought down, the quotient is the square root. If there is a remainder it may be increased and the root carried out decimally, by annexing two ciphers at each step of the process.

Reduce vulgar fractions to their least terms, then extract the root of the numerator and denominator separately, for the terms of the required root, but if this cannot be done without a remainder, reduce it to a decimal fraction, and extract its root.

EXAMPLE.—Extract the square root of 69576, and $\frac{16}{36}$.

	6,95,76(263.77
	4 Ans.
46	295
6	276
523	1976
3	1569
5267	40700
7	36869
52747	383100
	369229

$\sqrt{\frac{16}{36}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ Ans.

Having divided the given number into periods according to the rule, I seek the greatest square number contained in 6 the first period on the left, and find it to be 4, the square root of which is 2, I therefore write 2 in the quotient and 4 under 6, I then subtract, and increase the remainder by 95 the next period, and double the quotient figure for a divisor, I then try how often the increased remainder rejecting the units place, contains 4, and find it 7, but this 7 placed in the divisor gives 4 to carry which is too much, I therefore write 6 both in the quotient and divisor, and multiply and subtract as in simple division. I then increase the remainder by 76 the next period, and add 6 the last figure of the divisor to itself, gives 52 for a new trial divisor, with which proceed as before. Having brought down all the periods of the given number, the decimal of the root is carried out by annexing periods of ciphers, as in the example. The example in fractions is performed by reducing it to lowest terms, and

extracting the square root of the numerator and denominator as in the example.

EXERCISES.

FIND THE SQUARE ROOT OF

1. 3364	Ans. 58.	10. 7	Ans. 2·645751+.
2. 576	Ans. 24.	11. 5	Ans. 2·236068+.
3. 8328996	Ans. 2886.	12. 2	Ans. 1.41421356+.
4. 33016516	Ans. 5746.	13. $\frac{1728}{2700}$.	Ans. $\frac{4}{5}$.
5. 43623	Ans. 208·861+.	14. $\frac{792}{1548}$.	Ans. .71528+.
6. 50384985156	Ans. 224466.	15. $37\frac{36}{49}$.	Ans. $6\frac{1}{7}$.
7. 4712·81261	Ans. 68·649.	16. $76\frac{28}{34}$.	Ans. 8.7649.
8. ·07612816	Ans. ·2759+.	17. $2\frac{241}{333}$.	Ans. 1.65037+.
9. 10	Ans. 3·16227+.	18. $\frac{1}{3}$.	Ans. .5773502+.

APPLICATION OF THE SQUARE ROOT.

To find the side of a square equal to any given superficies.—Rule. The square root of the given superficies is the side of the required square.

To find the side of a square greater or less than a given square in any assigned proportion.—Rule. Multiply or divide the square of the side of the given square by the number expressing the proportion, the square root of the result is the side required.

To find the side of a square equal in area to several given squares.—Rule. The square root of the sum of the areas of the given squares is the side required.

To find the diameter of a circle greater or less in any assigned proportion than a circle whose diameter is given.—Rule. Multiply or divide the square of the diameter of the given circle by the number expressing the proportion, the square root of the result is the diameter of the required circle. The rule applies also to the circumferences.

When any two sides of a right angled triangle are given to find the other side.—Rule. The square root of the sum of the squares of the sides containing the right angle, is the length of the side subtending the right angle; and the square root of the excess of the square of the side subtending the right angle, above the square of either of the sides containing it, is equal to the other containing side.

The square root of the product of any two given numbers is a mean proportional between them.

EXERCISES.

1. A gentleman has a triangular field containing 164 poles

which he wishes to exchange for a square piece of land; what should the side of the square be? Ans. 12.8062+ poles.

2. A clergyman has a glebe of an irregular form, containing 10 acres, for which his heritors wish to give him a square glebe of the same extent; required the side of the square? Ans. 40 poles.

3. A gentleman has pleasure grounds in the form of a square containing 20 acres, but wishes to extend them to 40 acres; required the side of the square? Ans. 440 yds.

4. A maltster has 4 square kilns of the following dimensions, viz. 20, 25, 30, and 35 feet in the side, but wishes to pull them down and to build one equal to them all; required the side of the square? Ans. 56.124 + feet.

5. A distiller has a mash tun 15 feet in diameter, and wishes to erect another of the same depth that shall hold three times as much; required its diameter? Ans. 25.9807+ feet.

6. A cooper has a guage hoop, the interior circumference of which is 16 feet, and wishes to make one for a vessel of the same depth that will hold exactly ¼ of the quantity; required its interior circumference. Ans. 8 feet.

7. Required the length of a ladder that will reach to the top of a wall 81 feet high, its foot being placed 16 feet from the bottom of the wall. Ans. 82.565+ feet.

8. The wall of a castle is surrounded with a ditch 20 feet wide, and a line stretched from the top of the wall to the outer edge of the ditch measures 120 feet; required the height of the wall. Ans. 118.32+ feet.

9. A tower built on the centre of a circular island is 160 feet high, and a line stretched from its top to the inner edge of the water is 200 feet, and from its top to the outer edge of the water 500 feet; required the breadth of the water. Ans. 106.322+ yards.

10. Required the mean proportional between 4 and 9. Ans. 6.

11. Required the mean proportional between 9 & 16. Ans. 12.

12. One day wishing to purchase a cheese from a farmer, who, unfortunately, had no beam to weigh it, to supply this deficiency I suspended my staff by a string as near to the middle as I could guess, and found the cheese to weigh 49 lb. on one end, and 64 lb. on the other; what was its true weight? Ans. 56 lb.

CUBE ROOT.

The Cube Root of any number, is such a number as multiplied twice by itself produces that number; thus $3 \times 3 \times 3 = 27$, therefore 3 is the cube root of 27.

RULE.—Divide the given number into periods of three figures each, counting as directed in the square root. Find the cube root of the first period, write it in the quotient, and subtract its cube from the period, then increase the remainder by next period. Multiply the square of the quotient by 3, and to the product annex two ciphers for a trial divisor; find how often it is contained in the increased remainder, and write the result in the quotient. Multiply the former part of the quotient by this last figure and by 3, then annex a cipher to the product, and write it under the trial divisor, and under it write the square of the last figure in the quotient, the sum of these three numbers is the complete divisor, which multiply by the last quotient figure, subtract the product from the increased remainder, and increase this remainder by next period. To find a new trial divisor; annex two ciphers to the sum arising from adding the complete divisor to its second part, and double of its third part; with which find a quotient figure, complete the divisor as before, and proceed in the same manner till the periods are all brought down; the quotient is the cube root required. When the trial divisor is equal to or greater than the increased remainder, write a cipher in the quotient, increase the remainder by next period and the trial divisor by two ciphers, then proceed as before. If there is a remainder the root may be carried out decimally, by increasing the remainder with three ciphers at each step, and proceeding as before. When tho cube root of a vulgar fraction is required, prepare the fraction as directed for the square root.

EXAMPLES.—Required the cube root of 280111.385007, and $\frac{81}{192}$.

		280,111.385,007(65.43 root.	
		216	
Trial divisor = 6 × 6 × 3 =	10800	64111	Increased remainder.
2d part = 6 × 5 × 3 =	900	58625	
3d part = 5 × 5 =	25		
1st complete divisor =	11725		
2d trial divisor =	1267500	5486385	2d increased remainder.
	7800	5101264	
	16		
2d complete divisor =	1275316		
3d trial divisor =	1283148 0	385121007	3d increased remainder.
	58860		
	9	385121007	
3d complete divisor =	128373669		

$\sqrt[3]{\frac{81}{192}} = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$. Ans.

In the first example, having pointed the given number, I seek the cube root of the first period 280, and find it to be 6, I therefore write 6 in the quotient, and its cube 216 under 280, and having subtracted, I increase the remainder by next period. I then square 6 the quotient figure, and multiply the product by 3; to this last product I annex two ciphers, which gives 10800 for a trial divisor. I then try how often this divisor is contained in the increased remainder, and find it 5 times, which I write in the quotient. To complete the divisor I multiply 6 by 5, and that product by 3, gives 90, and annexing a cipher, gives 900, which I write under the trial divisor for its second part, under which I write 25, the square of the last figure in the quotient, for the third part, and the sum of these three is 11725, the first complete divisor. I then multiply the complete divisor by 5, the last quotient figure, and write the product 58625 under the increased remainder, from which I subtract it, and increase this remainder by next period 385. To find the next trial divisor, I add together 11725 the last complete divisor, 50 which is the double of 25 its third part, and 900 its second part; the sum is 12675, to which I annex two ciphers, gives 1267500 the new trial divisor, with which I find another quotient figure, and complete the divisor as before, and so on; the quotient is the cube root required. In the fractional example, I reduce the fraction to its lowest terms $\frac{27}{64}$, and the cube root of these terms is $\frac{3}{4}$ the answer required.

EXERCISES.

Required the Cube Root of

1. 46656.	Ans. 36.	13. .002.	Ans. .1259921+.
2. 21952.	Ans. 28.	14. .0001357.	Ans. .05138+.
3. 1331.	Ans. 11.	15. .00533.	Ans. 1746796+.
4. 2515456.	Ans. 136.	16. $5.\dot{7}6\dot{9}$.	Ans. 1.8040+.
5. 164566592.	Ans. 549.	17. [illegible]	Ans. [illegible]
6. 312908547069.	Ans. 6789.	18. [illegible]	Ans. [illegible]
7. 673373097125.	Ans. 8765.	19. $\frac{5}{8}$.	Ans. .85498+.
8. 27054036008.	Ans. 3002.	20. [illegible]	Ans. .92154+.
9. 11976.	Ans. 22.879+.	21. $15\frac{5}{8}$.	Ans. 2.5.
10. 9302348.	Ans. 210.312+.	22. 7985[illegible].	Ans. 19.9882+.
11. 2.	Ans. 1.259921+.	23. $36\frac{1}{3}$.	Ans. 3.31208+.
12. 5.	Ans. 1.709976+.	24. $1\frac{1}{2}$.	Ans. 1.1447+.

APPLICATION OF THE CUBE ROOT.

The solid content of a cube being given to find the length of its side.—RULE. The cube root of the solid content is the length of the side required.

To find the side of a cube which shall be equal to any given solid.—RULE. The cube root of the solid content of the given body is the side of the required cube.

To find the side of a cube equal to several given cubes.—RULE. The cube root of the sum of the solid contents of the given cubes is the side of the required cube.

The lineal dimensions of a solid body being given, to find the dimensions of a similar solid greater or less in any given proportion.—RULE. If the required solid is to be greater than that which is given, multiply; but if less, divide the cube of each dimension of the given solid by the number expressing the proportion, and the cube root of the several products, or quotients is the respective dimensions of the required solid.

The similar dimensions of two solids given and the weight, &c. of one of them, to find the weight, &c. of the other.—RULE. As the cube of the given dimension of the solid of which the weight, &c. is known, is to the cube of the other given dimension, so is the given weight, &c. to the required weight.

To find two mean proportionals between two given numbers.—RULE. Multiply the greater extreme by the square of the less, and the less extreme by the square of the greater, the cube root of the products is the means sought.

EXERCISES.

1. A cubic stone contains 180362.125 cubic feet; what is the length of its side? Ans. 56.5 feet.
2. The contents of a solid pentagon is 4775581.504 feet; required the side of an equal cube. Ans. 168.4 feet.
3. There are three cubic cisterns whose sides are 11.5, 17.25, and 20.2 feet respectively; required the side of another cistern that contains as much as the three. Ans. 24.6 feet.
4. There is a ship whose keel is 284 feet, midship beam 82 feet, and depth of hold 36 feet; what must be the dimensions of two similar ships, the one double, and the other half the burden? Ans. the greater 357.8+ feet keel, 103.3+ beam, 45.3+ hold; the less 225.4+ keel, 65.08+ beam, 28.5+ hold.
5. If a globe of brass, 4 inches diameter, weigh 9 lb., what is

the weight of another globe, the diameter of which is 8 inches? Ans. 72 lb.

6. The diameter of the bore of a cannon is 6 inches, which admits a ball of 27 lb. weight; what is the diameter of that cannon which admits a ball 3 lb. 6 oz. weight? Ans. 3 inches.

7. Required two mean proportionals between 6 and 162. Ans. 18 and 54.

8. Required two mean proportionals between 7 and 189. Ans. 21 and 63.

POSITION.

Position is the method of resolving a class of questions which does not fall directly under any of the other rules of arithmetic; and is called single or double position, according as one or two suppositions are required to bring out the answer.

SINGLE POSITION.

To single position belong such questions as have their results proportional to their suppositions, that is, which require the number sought to be multiplied or divided by any number; or to be increased or diminished either by itself, or by any part or parts of itself a proposed number of times.

Rule.—Suppose any number you please, and with it perform the same operations as the question directs to be performed with the number sought. Then say, as the result of this operation, is to the result in the question, so is the supposed number to the required number.

Example.—There is a certain number, which increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ of itself, and the sum divided by 3 the quotient is 24; what is the number?

$(60 + 30 + 20 + 10) \div 3 = 40$. As $40 : 24 :: 60 : 36$ Ans.

I here suppose 60 to be the number required, to which I add its $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$, and divide the sum by 3, the quotient is 40; I then say, as 40 is to 24, so is 60 to 36, which is the number sought. For proof, add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ of 36 to itself, and divide the sum by 3, the quotient is 24 as in the question.

EXERCISES.

1. A schoolmaster being asked how many scholars he had, said, if I had as many, $\frac{1}{6}$ and $\frac{1}{8}$ as many, I would have just 200; how many had he? Ans. 72.
2. A dashing young fellow was left a fortune, $\frac{1}{8}$ of which he spent in gambling, $\frac{1}{4}$ in purchasing an appointment in the army, and $\frac{2}{5}$ on dress and dissipation, he then finds he has only £2250 remaining; what was his fortune? Ans. £10000.
3. What number is that which multiplied by 5, and the product divided by 12, gives 30 in the quotient. Ans. 72.
4. A man being asked his age, and not choosing to give a direct answer, said, $\frac{1}{2} + \frac{1}{6}$ of my age multiplied by 4 makes 80; how old was he? Ans. 30.
5. A gentleman bought a chaise, horses, and harness for £260, the horses cost double of the chaise, and the chaise four times the harness; what did each cost? Ans. horses £160, chaise £80, harness £20.
6. Two boys having found a purse, disputed about dividing it, A said, when counting it at first B had pocketed $\frac{1}{3}$ of the whole, and when counting it a second time, he had kept $\frac{1}{3}$ of the remainder, and that there now only remained 30/; how much was in the purse, and how much should B give A to make them equal? Ans. In purse £3 7s. 6d., and B should return 3/9.
7. A, B, and C purchase a house for £360, of which A was to pay double of B, and B three times as much as C; what should each pay? Ans. A £216, B £108, C £36.
8. If a lion can eat a sheep in an hour, a wolf in $1\frac{1}{2}$ hour, and a dog in two hours; how long would they take when all eating together. Ans. $27\frac{9}{13}$ minutes.
9. Divide £216 into three parts, so that $\frac{1}{3}$ of the first, $\frac{1}{4}$ of the second, and $\frac{1}{2}$ of the third shall be equal to each other. Ans. £72, £96, £48.

DOUBLE POSITION.

Questions belonging to double position require two suppositions, and have not their results proportional to the supposed numbers, but the number sought is either increased or diminished by a given number which is not a known part of the required number.

RULE.—Suppose any two numbers, and perform with them the several operations which the question directs, and find the

difference between these results, and the result in the question, which call the ERRORS.—Then, if both the results be greater, or both less than the result in the question, multiply the first supposition by the second error, and the second supposition by the first error, the difference of these products divided by the difference of the errors gives the answer.—But when one of the results is greater, and the other less than the result in the question, then having multiplied as before, divide the sum of the products by the sum of the errors, and the quotient is the answer.

EXAMPLE.—Bought 100 gallons of spirits which cost me £119, part was brandy at 28/ a gallon, and the rest whisky at 7/ a gallon, required the quantity of each.

1st, Suppose
40 gal. brandy at 28/ = £56 | 40 × 21 = 840
60 gal. whisky at 7/ = £21 | 60 × 42 = 2520
1st result = £77 | 21)1680
£119—£77 = 42 first error. | Brandy = 80 gal.

2d, Suppose
60 gal. brandy at 28/ = £84 | 60 × 21 = 1260
40 gal. whisky at 7/ = £14 | 40 × 42 = 1680
2d result = 98 | 21) 420
£119—£98 = 21 second error. | Whisky = 20 gal.
or 100 gal.—80 gal. = 20 gal. of whisky.

In this example I first suppose there were 40 gallons of brandy and 60 gallons of whisky, and the price or result of this supposition is £77, which subtracted from £119 the result in the question, leaves 42 for the first error. Again I suppose there were 60 gallons brandy, and 40 gallons whisky, and the price or result of this supposition is £98, which subtracted from £119, leaves 21 for the second error. I then multiply 40 the quantity of brandy in the first supposition, by 21 the second error, and 60 the quantity in the second supposition by 42 the first error, the difference of these products is 1680, which divided by 21 the difference of the errors, gives 80, the gallons of brandy; then proceeding with the whisky in the same manner, we have 20, the gallons of whisky. In this example the quantity of whisky is obtained more simply, by subtracting the 80 gallons of brandy from 100 the whole quantity.

EXERCISES.

1. What number is that, which being multiplied by 3, and the product diminished by 6, one third of the remainder is 14? Ans. 16.

2. An angler caught a pike, whose head was 6 inches, the tail was as long as the head and half the body, and the body was the length of the head and tail; what was the length of the fish? Ans. 4 feet.

3. Three parties of soldiers sent out a foraging, came upon a shepherd making his escape with his flock, the first party took $\frac{1}{2}$ of his sheep and $\frac{1}{2}$ a sheep more, the second took $\frac{1}{2}$ of the remainder and $\frac{1}{2}$ a sheep more, and the third took $\frac{1}{2}$ of what remained and $\frac{1}{2}$ a sheep more, after which he had only 20 left; how many had he at first? Ans. 167.

4. An impertinent boy having asked a gentleman rudely the hour of the day, received for answer, the time since noon is equal to $\frac{2}{3}$ of $\frac{3}{10}$ of the time till midnight; required at what hour this reply was given. Ans. 2 o'clock.

5 A constable in a country parish when taking up the census, said to the schoolmaster, (who felt disappointed at not having been employed himself,) Sir, I wish to know the age of your family, who replied, the sum of our ages is 108 years; but a more particular answer being required, he said, my daughter's age added to my own, exceeds my wife's by 28 years, and my wife's age added to my daughter's exceeds my age by 12 years; how old was each of them?
Ans. man 48, wife 40, daughter 20.

6. A gentleman has a saddle worth £50, which, placed on one of his two riding horses, makes his value double that of the other, and when placed on the other, makes his value triple the former; what was each worth?
Ans. £30, and £40.

7. A farmer engaged a servant for 30 days, on condition that he should have 5/ for every day he wrought, and forfeit 2/6 every day he was absent, at the end of the time he received £4, 10s.; how many days was he absent? Ans. 8.

8. A grocer made a mixture of tea at 6/, 5/, and 4/7½ per lb., the mixture is worth £25, and for every lb. at 6/, he had 3 lb. at 5/, and for every 9 lb. at 5/ he had 8 lb. at 4/7½; how much of each kind was in the mixture?
Ans. 15 at 6/, 45 at 5/, 40 at 4/7½.

9. An honest woman driving her geese to the market, had the misfortune to get $\frac{1}{2}$ of them destroyed by the fox during her first night on the road, $\frac{1}{12}$ of the remainder plus 5 were

stolen the second night, in passing along the bank of a river $\frac{1}{8}$ of the remainder leaped in and made their escape, during her vain attempts to recover them, a thievish boy made off with other three, notwithstanding these losses she brought 60 to the market; how many had she at first?
Ans. 96.

10. A schoolmaster being asked how many pupils he had, replied, if, to $\frac{1}{5}$ of my pupils you add 5, and multiply the sum by 5, $\frac{3}{4}$ of $\frac{4}{3}$ of the product, increased by $\frac{1}{4}$ of itself is the number of my pupils; how many had he? Ans. 150.

ARITHMETICAL PROGRESSION.

When a rank of numbers increase, or decrease by the continual addition, or subtraction of any number, they are in ARITHMETICAL PROGRESSION. The first and last terms of an arithmetical series are called the EXTREMES, the intervening terms are called MEANS, and the number by which they increase or decrease is called the COMMON DIFFERENCE; thus, 1, 4, 7, 10, 13, 16, 19, is an increasing arithmetical series, of which 1 is the least extreme, and 19 the greatest extreme, 7 is the number of terms, 3 is the common difference, and $1+4+7+10+13+16+19 = 70$ is the sum of the series. Again, 16, 14, 12, 10, 8, 6, 4, 2, is a decreasing arithmetical series, of which the extremes are 16 and 2, the number of terms 8, the common difference 2, and the sum of the series 72. When any three of those five parts of an arithmetical series are given, the other two may be found.

CASE. 1.—When the extremes and the number of terms are given to find the sum of the series.

RULE.—Multiply the sum of the extremes by the number of terms, and half the product is the answer.

EXAMPLE.—If the least extreme be 3, the greatest 51, and the number of terms 17; what is the sum of the series?

$$(3+51) \times 17 = 918 \div 2 = 459 \text{ sum of the series.}$$

I here add the extremes 3 and 51, and multiply the sum by 17, the number of terms, the product is 918, which divided by 2, gives 459, the sum of the series.

EXERCISES.

1. The extremes of an arithmetical series, are 1 and 421, and the number of terms 23; required the sum of the series.
Ans. 4853.

2. The greatest extreme of an arithmetical series is 118, the common difference 5, and the number of terms 24, required the least extreme. Ans. 3.
3. The first extreme of a decreasing arithmetical series is 80, the common difference 6, and the number of terms 14; what is the least extreme? Ans. 2.

GEOMETRICAL PROGRESSION.

When any series of numbers increase, or decrease by a common multiplier, or divisor, they are in geometrical progression. Thus 1, 3, 9, 27, 81, 243, is an increasing geometrical progression, or series; and 243, 81, 27, 9, 3, 1, is a decreasing geometrical progression. In both these progressions, 1, and 243 are the extremes, the number of terms 6, the common ratio 3, and $1 + 3 + 9 + 27 + 81 + 243 = 364$ the sum of the series.

CASE 1.—When one of the extremes, the number of terms, and the common ratio are given, to find the other extreme.

RULE.—Involve the ratio to a power whose index is one less than the number of terms, by which multiply the least extreme to give the greatest, or divide the greatest extreme to find the least extreme.

EXAMPLE.—The least extreme of a geometrical progression is 2, the ratio 3, and the number of terms 9, required the greatest extreme.

$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2 = 13122$ greatest extreme.

I here find the eight power of the ratio, which multiplied by 2, the least extreme, gives 13122 the greatest extreme. If the greatest extreme 13122 had been given to find the least, I would have divided the greatest extreme by the eight power of 3, the quotient would have been 2 the least extreme.

EXERCISES.

1. The least extreme of a geometrical series is 6, the ratio 2, and the number of terms 8; required the greatest extreme. Ans. 768.
2. If the greatest extreme of a geometrical series be 524288, the ratio 2, and the number of terms 20; what is the least extreme? Ans. 1.

3. If the least extreme be 4, the ratio 3, and the number of terms 24; what is the greatest extreme?

Ans. 1506290861232.

Case 2.—When the extremes and the common ratio are given to find the number of terms.

Rule.—Divide the greatest extreme by the least, involve the ratio till it is equal to the quotient, the index of this power increased by 1 is the number of terms.

Example.—The extremes of a geometrical series are 7, and 3584, and the ratio 2; required the number of terms. $3584 \div 7 = 512$, and $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$ the ninth power of 2, and $9 + 1 = 10$ the number of terms of the series. I here divide 3584 the greatest extreme by 7 the least, the quotient is 512, I then involve 2 the ratio, and find that 512 is its ninth power, and 1 added to 9 gives 10 the number of terms.

EXERCISES.

1. The least extreme of a geometrical series is 2, the greatest 8192, and the ratio 2; required the number of terms.

Ans. 13.

2. The extremes of a geometrical series are 3, and 3221225472, and the ratio 4; required the number of terms. Ans. 16.
3. If the extremes of a geometrical series be 4, and 708588, and the ratio 3; what is the number of terms? Ans. 12.

Case 3.—The extremes and number of terms given to find the ratio.

Rule.—Divide the greatest extreme by the least, that root of the quotient whose index is 1 less than the number of terms is the ratio.

Example.—The extremes of a geometrical series are 2, and 8192, and the number of terms 13; required the ratio.

$8192 \div 2 = 4096$, and $13 - 1 = 12$, therefore $\sqrt[12]{4096} = 2$ the ratio. Or thus $\sqrt{4096} = 64$, and $\sqrt{64} = 8$, and $\sqrt[3]{8} = 2$ the ratio as before. I here divide 8192 the greatest extreme, by 2 the least, the quotient is 4096, and the number of terms is 13, from which take 1, leaves 12; I therefore extract the 12th root of 4096, which is 2 the ratio. The 12th root is obtained more expeditiously by the second method, namely, by extract-

ing the square root of the square root, and then the cube root of this last root is the 12th root.

EXERCISES.

1. The extremes of a geometrical series are 1, and 6561, and the number of terms 9; required the ratio. Ans. 3.
2. The extremes of a geometrical series are 4, and 67108864, and the number of terms 13; required the ratio. Ans. 4.
3. The extremes of a geometrical progression are 3, and 786432, and the number of terms 19; required the ratio. Ans. 2.

CASE 4.—To find the sum of the terms of an increasing geometrical series.

RULE.—Multiply the greatest extreme by the ratio, from the product subtract the least extreme, the remainder divided by the ratio minus 1, gives the sum of the series.

EXAMPLE.—The extremes of a geometrical series are 2, and 4374, and the ratio 3, required the sum of the series.

$4374 \times 3 = 13122 - 2 = 13120$, and $13120 \div 2 = 6560$ sum of the series. I here multiply 4374 the greatest extreme, by 3 the ratio, and from the product I subtract 2 the least extreme, and divide the remainder 13120 by 2, which is the ratio minus 1, the quotient is 6560, the sum of the series.

EXERCISES.

1. The extremes of a geometrical series are 3 and 729; what is the sum of the series? Ans. 1092.
2. A gentleman being disposed to purchase a lot of 10 fine sheep, was offered them for £2 a piece, but objecting to the price, he was offered the whole, provided he would give 7 pence for the first, 14 pence for the second, and so on, doubling it for each, to this proposal he agreed; what did they stand him a head? Ans. £2 19s. $8\frac{1}{10}$d.
3. A young silk-mercer being desirous of purchasing 16 fine India shawls from a crafty smuggler, was offered them at £50 each, but not choosing to risk so much on contraband goods, he was offered them on condition that he would give 3 farthings for the first, 12 for the second, and so on in geometrical progression for the 16, which was agreed to; required their price. Ans. £4473924 5s. $3\frac{3}{4}$d.
4. An old gentleman had his only daughter married on Monday the 1st of January, and gave her husband 1 farthing

in part of her portion, promising to double it in geometrical progression on the first day of every week for a year, but when he paid the thirtieth instalment, he found he had nothing remaining; what portion did the husband receive, and what remains due to make good the father's promise? Ans. Received £559240 10s. 7¾d. Due £2345624246681 12s.

5. A flesher desirous to purchase from a drover a lot of 12 fine bullocks, was offered them on condition that he would give 2 pence for the first, 6 pence for the second, and so on in a triple ratio, in geometrical progression for the 12; what would they cost at this rate? Ans. £6603 0s. 2d.

6. A young Cockney, with more command of purse than arithmetical knowledge, had a mind to purchase a favourite race-horse from a country jockey, and offered him £1000 for the animal; to which the jockey replied, Sir, if you wish to purchase him, you must give me a farthing for the first nail in his shoes, 3 for the second, 9 for the third, and so on, for the 32 nails; this proposal was readily agreed to; required the price of the horse?

Ans. £965114681693 13s. 4d.

PERMUTATIONS AND COMBINATIONS.

Permutation is the method of showing in how many different positions any given number of articles may be placed.

Combination is the method of showing how often a less number of articles can be taken out of a greater, and combined, without regarding either their position or order.

Case 1.—To find the number of permutations of which any given number of articles is susceptible.

Rule.—The continued product of the natural series of numbers expressing the given articles, is the permutations of which they are susceptible.

Example.—In how many positions can 8 persons be placed at table.

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$ the number of positions.

I here multiply together the digits in the natural series 1, 2, 3, 4, 5, 6, 7, 8, to the number of persons given, the continued product is 40320, the number of permutations of which they are susceptible.

EXERCISES.

1. In how many different positions may the 13 diamond cards in a pack be arranged? Ans. 6227020800.
2. In how many different positions can the 12 hour figures be placed on the dial of a watch? Ans. 479001600.
3. How many permutations or changes can be made with the letters in the word CHANGES? Ans. 5040.

CASE 2.—To ascertain how many permutations can be made out of any given numbers of articles, of which there are several, of several sorts.

RULE.—Find the continued product of the natural series to the number of articles as in last case. Find also the continued product of the several series representing the different sorts of articles. Divide the first of those products by each of the other products, the last quotient is the number of permutations.

EXAMPLE.—In how many different positions can 4 white men, 2 white women, a black man, and a black woman be placed at table.

$$\begin{array}{l} 1\times2\times3\times4\times5\times6\times7\times8 = 40320 \\ 1\times2\times3\times4 = 24\times2 \qquad = 48 \\ 1\times2=2 \end{array} \qquad \frac{40320}{48} = 840 \text{ Ans.}$$

In this example, the number of individuals is 8, I therefore find the continued product of the natural series 1, 2, 3, &c. to eight places, which is 40320, I next find the product of the series 1, 2, 3, 4, the number of white men, which is 24, then of 1, 2, the white women, which is 2, and then dividing 40320 by 24, and 2, or by their product 48, the quotient is 840, the number of positions.

EXERCISES.

1. How many changes may be made upon the letters in the word COMMANDMENT? Ans. 3326400.
2. Three soldiers, two sailors, a surgeon, a merchant, and a farmer met at dinner in a tavern; how often can they vary their position at table? Ans. 3360.

3. Five porters, four sweeps, 3 scavangers, two carters, a blind fiddler, a ballad singer, and a resurrection man, chanced to meet in a dram shop, and were so much delighted with the liquor, and each other's company, that they resolved not to part, so long as they could be placed in different positions when a new gill was called in; how much would they drink before parting? Ans. 1276275 tons.

CASE 3.—Any number of different articles being given, to find the various combinations which may be formed out of them, by taking any proposed number of them at a time.

RULE.—Find the continued product of the series, 1, 2, 3, &c. as far as the number of articles to be taken at a time, by which divide the continued product of a decreasing series of the same number of terms, having the given number of articles for its first term, the quotient is the number of combinations.

EXAMPLE.—How many combinations of 6 letters can be made out of the English alphabet?

$$\frac{\overset{13}{\cancel{26}}\times\overset{5}{\cancel{25}}\times\cancel{24}\times 23\times 22\times\overset{7}{\cancel{21}}}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}\times\cancel{5}\times\cancel{6}} = 230230 \text{ Ans.}$$

I here write 6 terms of the decreasing series, beginning with 26 the number of letters in the alphabet above a line, and 6 terms of the increasing series below the same line as in compound proportion, and having cancelled all the under numbers, I find the product of the numbers uncancelled in the upper number to be 230230, which is the answer.

EXERCISES.

1. How many combinations of 4 figures each, can be made out of the nine digits? Ans. 126.
2. Fourteen gentlemen enter into an agreement, that 6 of them should dine together at a tavern every day, so long as they could do so without the same 6 being present again at the same time, required how long this contract would bind them. Ans. 8 years 83 days.
3. How long may a gambler play at whist without holding the same hand twice, supposing he has a new hand every five minutes, there being 52 cards in a pack, and 13 in each hand? Ans. 6040844 years, 133 days, 1 hour, 20 minutes.

CASE 4.—To find how many permutations can be made out of any given number of different articles, by taking any given number of them at a time.

RULE.—Take a series decreasing by 1, having the given number of articles for its first term, and as many terms as there are articles to be taken at a time, the continued product of these terms is the number of permutations.

EXAMPLE.—How many ways can 4 cards be chosen out of 13?

$13 \times 12 \times 11 \times 10 = 17160$ number of different ways.

In this example, beginning with 13 the number of cards, I take 13, 12, 11, 10, the four first terms of the decreasing series, because there are only four cards to be taken at a time, the continued product is 17160, the number of permutations required.

EXERCISES.

1. How many different ways may 10 bullocks be drawn out of a score? Ans. 670442572800.
2. How many combinations can be made of 6 letters out of 12? Ans. 95040.

CASE 5.—To find how many compositions can be formed out of a given number of sets of different articles, having as many articles in each composition as there are sets.

RULE.—The continued product of the number of articles in each set, is the number of compositions.

EXAMPLE.—There are 5 companies in a tavern, in the first there are 9 persons, in the second 7, in the third 6, and in the fourth and fifth 3; how many compositions of 5 persons may be formed out of them?

$9 \times 7 \times 6 \times 3 \times 3 = 3402$ number of compositions.

I here multiply together the number of persons in each company, the product is the number of compositions.

EXERCISES.

1. How many different compositions can be made with 4 sovereigns, 4 guineas, 4 crowns, and 4 shillings, taking 4 at a time? Ans. 256.
2. There are 5 Englishmen, 6 Frenchmen, 3 Dutchmen, 4 Germans, 7 Italians, and 8 Russians; how many differ-

ent ways may a company of 6 men be chosen out of them, so as to have one of each nation? Ans. 20160.

CASE 6.—To find the number of combinations, which any number of different articles will admit of, taking them by twos, threes, &c. up to the given number.

RULE.—Raise 2 to the power whose index is the given number of articles, from which subtract the given number of articles increased by one, the remainder is the number of combinations.

EXAMPLE.—How many combinations can be made out of the nine digits, taking them two by two, three by three, &c. up to nine?

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512 - 10 = 502$ Ans.

I here involve 2 to the ninth power which gives 512, from which I subtract 10, which is 9 the number of digits increased by 1, the remainder is 502 the number of combinations required.

EXERCISES.

1. How many combinations can be made out of 13 cards, taking them by twos, threes, &c. up to 13? Ans. 8178.
2. How many combinations can be made out of the letters in the English alphabet, taking them two by two, three by three, &c. up to 26? Ans. 67108837.

CASE 7.—To find all the possible combinations, and permutations of which any proposed number of articles is susceptible, when taken by twos, threes, &c. to the whole number of articles.

RULE.—Involve the number of articles given, to a power whose index is one more than their number, from which subtract the number of articles, and the remainder divided by their number minus 1, is the answer.

EXAMPLE.—How many combinations and permutations can be made with 6 of the nine digits taking them by twos, threes, &c. up to 6?

$6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 - 6 = 279930 \div 5 = 55986$ Ans.

I here raise 6 to the seventh power, from which I subtract 6, and divide the remainder by 5, the quotient is the answer.

EXERCISES.

1. Required the number of combinations, and permutations which can be made with a cow and a calf, a ewe and a lamb, a goat and a kid, a sow and a pig, taking them by twos, threes, &c. up to eight? Ans. 19173960.
2. How many combinations and permutations can be made with the letters in the English alphabet, taking them by twos, threes, &c. up to 26?
Ans. 640980861716870437242497564154439927.

CALCULATION OF CHANCES.

The calculation of chances being of considerable importance in annuities, it shall be treated of briefly as introductory to that subject.

CASE 1.—When there is only one trial, and one chance of success.—Rule. The chance of success is expressed by a fraction having 1 for his numerator, and the chances both of success and failure for its denominator. The chances of failure are expressed by a fraction having the chances of failure for the numerator, and all the chances both of failure and success for the denominator.

EXAMPLE,—A gambler has a bag in which are one white, and six black balls; now, if drawing the white ball entitles to 14/, what is the chance of success, and its value?
$\frac{1}{7}$ = chance of success, $\frac{6}{7}$ of failure. Value $\frac{1}{7}$ of 14/ = 2/.

EXERCISES.

1. There is a lottery in which are 25 tickets, one of which is a prize of £500, and the rest blanks, what is the chance of success with one ticket, and what is it worth?
Ans. Chance $\frac{1}{25}$, value £20.

2. If the lot is to be cast on 9 prisoners, to take 1 for execution, and the rest for banishment; what is the chance in favour of banishment? Ans. $\frac{8}{9}$.

3. What is the chance of throwing 5 at first throw with a single die? Ans. $\frac{1}{6}$.

CASE 2.—When there are several trials, and only one point to be gained.

RULE.—Subtract the continued product of all the chances of failure from 1, the remainder is the chance of success.

Example.—What is the chance of throwing an ace at three throws with a single die?

$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$ chance of failure, and $1 - \frac{125}{216} = \frac{91}{216}$ chance of success. The chance of failure at one throw, by case first, is $\frac{5}{6}$, and in three throws it is $\frac{125}{216}$, and $\frac{125}{216}$ subtracted from 1 leaves $\frac{91}{216}$, the chance of success.

EXERCISES.

1. There is a bag in which are 8 balls, 1 black, and 7 red; what is the chance of drawing the black ball at 5 trials?

Ans. $\frac{15961}{32768}$, or 15961 to 32768.

2. What is the chance of throwing an ace at two throws with one die? Ans. $\frac{11}{36}$, or 11 to 36.

Case 3.—When there is but one trial, and several points to be gained.

Rule.—Involve the chance of gaining a single point to a power whose index is the number of points to be gained, for the chance of success; and the chance of success subtracted from 1, gives the chance of failure.

Example.—What is the chance of throwing three aces at one throw with three dice?

$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$ chance of success, and $1 - \frac{1}{216} = \frac{215}{216}$ chance of failure. The chance of success, with one die, by case first, is $\frac{1}{6}$, which involved to the third power, gives $\frac{1}{216}$, the chance of success with three dice, and $\frac{1}{216}$ subtracted from 1, leaves $\frac{215}{216}$ the chance of failure.

EXERCISES.

1. What is the chance of throwing 4 aces at one throw with four dice? Ans. $\frac{1}{1296}$.

2. What is the chance of cutting a pack of cards at the 4 aces, at the first four cuts, there being 52 cards in the pack?

Ans. $\frac{1}{270725}$.

COMPOUND INTEREST AND ANNUITIES.

The definition of Compound Interest has already been given.

An Annuity is a sum of money payable at stated equal intervals, to a person who possesses no other property in the capital from which it is derived.

TABLE I. Shows the amount of £1, Compound Interest, at

For Years	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.025	1.03	1.035	1.04	1.045	1.05
2	1.050625	1.0609	1.071225	1.0816	1.092025	1.1025
3	1.076890	1.092727	1.108717	1.124864	1.141166	1.157625
4	1.103813	1.125508	1.147523	1.169858	1.192518	1.215506
5	1.131408	1.159274	1.187686	1.216652	1.246181	1.276281
6	1.159693	1.194052	1.229255	1.265319	1.302260	1.340095
7	1.188686	1.229873	1.272279	1.315931	1.360861	1.407100
8	1.218403	1.266770	1.316809	1.368569	1.422100	1.477455
9	1.248863	1.304773	1.362897	1.423311	1.486095	1.551328
10	1.280084	1.343916	1.410598	1.480244	1.552969	1.628894
20	1.638615	1.806111	1.989788	2.191123	2.411714	2.653297
30	2.097566	2.427262	2.806793	3.243397	3.745318	4.321942
40	2.685062	3.262037	3.959259	4.801020	5.816364	7.039988
50	3.437108	4.383906	5.584926	7.106683	9.032636	11.467399

TABLE II. Shows the present value of £1, due at

Years hence.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	.975512	.970873	.966183	.961538	.956937	.952381
2	.951814	.942595	.933510	.924556	.915729	.907029
3	.928999	.915141	.901942	.888996	.876296	.863837
4	.905950	.88848	.871442	.854804	.838561	.822702
5	.883854	.862608	.841973	.821927	.802451	.783526
6	.862297	.837484	.813500	.790314	.767895	.746215
7	.841265	.813091	.785991	.759917	.734828	.710681
8	.820746	.789409	.759411	.730690	.703185	.676839
9	.800728	.766416	.733731	.702586	.672904	.644608
10	.781198	.744093	.708918	.675564	.643927	.613913
20	.610271	.553675	.502566	.456387	.413642	.376889
30	.476743	.411986	.356278	.308318	.267000	.231377
40	.372430	.306556	.252572	.208289	.171928	.142045
50	.290942	.228107	.179053	.140712	.110709	.087203

Table III. Shows the amount of £1 Annuity at

For Years.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.	1.	1.	1.	1.
2	2.03	2.035	2.04	2.045	2.05
3	3.0909	3.106225	3.1216	3.137025	3.1525
4	4.183627	4.214942	4.246464	4.278191	4.310125
5	5.309135	5.362465	5.416322	5.470709	5.525631
6	6.468409	6.550152	6.632975	6.716891	6.801912
7	7.662462	7.779407	7.898294	8.019151	8.142008
8	8.892336	9.051686	9.214226	9.380013	9.549108
9	10.159106	10.368495	10.582795	10.802114	11.026564
10	11.463879	11.731393	12.006107	12.288209	12.577892
20	26.870374	28.279681	29.778078	31.371422	33.065954
30	47.575415	51.622677	56.084937	61.007069	66.438847
40	75.401259	84.550277	95.025515	107.030323	120.799772
50	112.796867	130.997910	152.667084	178.503028	209.347995

Table IV. Shows the present Value of £1 Annuity at

For Years.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	.970873	.966183	.961538	.956937	.952380
2	1.913469	1.899694	1.886094	1.872667	1.859410
3	2.828611	2.801637	2.775091	2.748964	2.723248
4	3.717098	3.673079	3.629895	3.587525	3.545950
5	4.579707	4.515052	4.451822	4.389976	4.329476
6	5.417191	5.328553	5.242136	5.157872	5.075692
7	6.230283	6.114543	6.002054	5.892700	5.786373
8	7.019692	6.873955	6.732744	6.595886	6.463212
9	7.786109	7.607686	7.435331	7.268790	7.107821
10	8.530202	8.316605	8.110895	7.912718	7.721734
20	14.877474	14.212403	13.590326	13.007936	12.462210
30	19.600441	18.392045	17.292033	16.288888	15.372451
40	23.114771	21.355072	19.792774	18.401584	17.159086
50	25.729764	23.455617	21.482184	19.762007	18.255925

TABLE V. Shows the Annuity which £1 will purchase at

For Years.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.03	1.035	1.04	1.045	1.05
2	.5226108	.5264005	.5301961	.5339976	.5378049
3	.3535304	.3569342	.3603485	.3637734	.3672086
4	.2690271	.2722511	.2754901	.2787437	.2820118
5	.2183546	.2214814	.2246271	.2277916	.2309748
6	.1845975	.1876682	.1907619	.1938784	.1970175
7	.1605064	.1635445	.1666096	.1697015	.1728198
8	.1424564	.1454767	.1485279	.1516097	.1547218
9	.1284339	.1314460	.1344930	.1375745	.1406901
10	.1172305	.1202414	.1232909	.1263788	.1295046
20	.0672157	.0703611	.0735818	.0768761	.0802426
30	.0510193	.0543713	.0578301	.0613915	.0650514
40	.0432624	.0468273	.0505235	.0543431	.0582782
50	.0388649	.0426337	.0465502	.0506021	.0547767

TABLE VI. Shows the value of £1 Annuity on a Single Life.

Age.	4 per cent.	5 per cent.	Age.	4 per cent.	5 per cent.	Age.	4 per cent.	5 per cent.
1	13.465	11.563	21	15.912	13.917	41	13.018	11.695
2	15.633	13.420	22	15.797	13.833	42	12.838	11.551
3	16.462	14.135	23	15.680	13.746	43	12.657	11.407
4	17.010	14.613	24	15.560	13.658	44	12.472	11.258
5	17.248	14.827	25	15.438	13.567	45	12.283	11.105
6	17.482	15.041	26	15.312	13.473	46	12.089	10.947
7	17.611	15.166	27	15.184	13.377	47	11.890	10.784
8	17.662	15.226	28	15.053	13.278	48	11.685	10.616
9	17.625	15.210	29	14.918	13.177	49	11.475	10.443
10	17.523	15.139	30	14.781	13.072	50	11.264	10.269
11	17.393	15.043	31	14.639	12.965	51	11.057	10.097
12	17.251	14.937	32	14.495	12.854	52	10.849	9.925
13	17.103	14.826	33	14.347	12.740	53	10.637	9.748
14	16.950	14.710	34	14.195	12.623	54	10.421	9.567
15	16.791	14.588	35	14.039	12.502	55	10.201	9.382
16	16.625	14.460	36	13.880	12.377	56	9.977	9.193
17	16.462	14.334	37	13.716	12.249	57	9.749	8.999
18	16.309	14.217	38	13.548	12.116	58	9.516	8.801
19	16.167	14.108	39	13.375	11.979	59	9.280	8.599
20	16.033	14.007	40	13.197	11.837	60	9.039	8.392

Table VI—continued.

Age.	4 per cent.	5 per cent.	Age.	4 per cent.	5 per cent.	Age.	4 per cent.	5 per cent.
61	8.795	8.181	73	5.507	5.245	85	2.543	2.471
62	8.547	7.966	74	5.230	4.990	86	2.393	2.328
63	8.291	7.742	75	4.962	4.744	87	2.251	2.193
64	8.030	7.514	76	4.710	4.511	88	2.131	2.080
65	7.761	7.276	77	4.457	4.277	89	1.967	1.924
66	7.488	7.034	78	4.197	4.035	90	1.758	1.723
67	7.211	6.787	79	3.921	3.776	91	1.474	1.447
68	6.930	6.536	80	3.643	3.515	92	1.171	1.153
69	6.647	6.281	81	3.377	3.263	93	0.827	0.816
70	6.361	6.023	82	3.122	3.020	94	0.530	0.524
71	6.075	5.764	83	2.887	2.797	95	0.240	0.238
72	5.790	5.501	84	2.708	2.627	96	0.000	0.000

TABLE VII. Shows the value of £1 annuity on two Joint Lives.

Ages.		4 per cent.	5 per cent.	Ages.		4 per cent.	5 per cent.	Ages.		4 per cent.	5 per cent.
	5	13.591	11.984		25	12.229	10.989		40	9.820	9.016
	10	13.933	12.315		30	11.873	10.707		45	9.381	8.643
	15	13.479	11.954		35	11.445	10.363		50	8.834	8.177
	20	12.993	11.561		40	10.924	9.937		55	8.221	7.651
	25	12.633	11.281		45	10.330	9.448	40	60	7.490	7.015
	30	12.220	10.989		50	9.630	8.861		65	6.614	6.240
	35	11.732	10.572	20	55	8.869	8.216		70	5.571	5.298
5	40	11.150	10.102		60	7.995	7.463		75	4.457	4.272
	45	10.500	9.571		65	6.986	6.576		80	3.349	3.236
	50	9.742	8.941		70	5.826	5.532		45	8.990	8.312
	55	8.931	8.256		75	4.619	4.424		50	8.503	7.891
	60	8.011	7.466		80	3.443	3.325		55	7.948	7.441
	65	6.963	6.546		25	11.944	10.764	45	60	7.274	6.822
	70	5.768	5.472		30	11.618	10.499		65	6.453	6.094
	75	4.557	4.362		35	11.217	10.175		70	5.460	5.195
	10	14.277	12.665		40	10.725	9.771		75	4.386	4.206
	15	13.841	12.302		45	10.160	9.304		80	3.308	3.191
	20	13.355	11.906	25	50	9.488	8.739		50	8.081	7.522
	25	12.998	11.627		55	8.754	8.116		55	7.593	7.098
	30	12.586	11.304		60	7.906	7.383		60	6.989	6.568
	35	12.098	10.916		65	6.920	6.515	50	65	6.236	5.897
	40	11.513	10.442		70	5.780	5.489		70	5.306	5.054
10	45	10.851	9.900		75	4.589	4.396		75	4.285	4.112
	50	10.085	9.260		80	3.425	3.308		80	3.247	3.140
	55	9.256	8.560		30	11.313	10.255		55	7.179	6.735
	60	8.314	7.750		35	10.948	9.954		60	6.659	6.272

TABLE VII—continued.

Ages.		4 per cent.	5 per cent.	Ages.		4 per cent.	5 per cent.	Ages.		4 per cent.	5 per cent.
	65	7.236	6.803		40	10.490	9.576	55	65	5.986	5.671
	70	6.008	5.700		45	9.959	9.135		70	5.132	4.893
	75	4.725	4.522		50	9.321	8.596		75	4.171	4.006
	80	3.517	3.395	30	55	8.619	7.999		80	3.180	3.076
	15	13.411	11.960		60	7.802	7.292		60	6.226	5.888
	20	12.961	11.585		65	6.844	6.447		65	5.658	5.372
	25	12.630	11.324		70	5.729	5.442	60	70	4.900	4.680
	30	12.246	11.021		75	4.557	4.365		75	4.021	3.866
	35	11.787	10.655		80	3.406	3.290		80	3.092	2.992
15	40	11.234	10.205		35	10.612	9.680		65	5.201	4.960
	45	10.607	9.690		40	10.196	9.331	65	70	4.573	4.378
	50	9.872	9.076		45	9.706	8.921		75	3.806	3.665
	55	9.077	8.403		50	9.110	8.415		80	2.965	2.873
	60	8.170	7.622	35	55	8.448	7.849		70	4.087	3.930
	65	7.127	6.705		60	7.669	7.174	70	75	3.471	3.347
	70	5.933	5.631		65	6.747	6.360		80	2.757	2.675
	75	4.695	4.495		70	5.663	5.382	75	75	3.015	2.917
	80	3.492	3.372		75	4.516	4.327		80	2.448	2.381
20	20	12.535	11.232		80	3.383	3.268	80	80	2.068	2.018

CASE 1.—To find the amount, or interest of any sum, for any number of years at compound interest.

RULE.—To find the amount, multiply the number in table 1, which stands under the proposed rate, and opposite the given years, by the sum, the product is the amount. To find the interest, take the same tabular number as before, from which subtract a unit, multiply the remainder by the given sum, the product is the compound interest.

EXAMPLE.—What is the compound amount, and interest of £365 8s. for 5 years, at 4 per cent. ?

1.216652×365.4 = £444 11s. 3¼d. compound amount.
.216652×365.4 = £ 79 3s. 3¼d. compound interest.

In table 1, under 4 per cent. and opposite to 5 years, there is 1.216652, which multiplied by the principal, and the decimal valued mentally, gives £444 11s. 3¼d. the amount; again 1 taken from the tabular number, leaves .216652, which multiplied by the principal, and the decimal valued mentally, gives £79 3s. 3¼d. the compound interest.

EXERCISES.

1. What is the amount of £347 10s., in 4 years, at 4½ per cent., compound interest? Ans. £414 8s.
2. What will £1000 amount to in 10 years, improved at 3½ per cent. compound interest? Ans. £1410 11s. 11¾d.
3. What will £500 amount to in 15 years, at 5 per cent., interest payable half yearly? Ans. £1048 15s. 8d.
4. What is the compound interest of £630 18s., for 18 years, at 4½ per cent. per annum? Ans. £762 8s. 6¾d.
5. What will £100 amount to in 100 years, at 5 per cent. per annum, compound interest? Ans. £131491 4s. 9¾d.
6. What is the compound interest of £800 for 9 years, at 3 per cent. per annum? Ans. £243 16s. 4½d.

Case 2.—To find the present value, or the principal which will amount to a given sum, in a given number of years, at any proposed rate per cent.

Rule.—Multiply the given sum, by the present value of £1 for the given time and rate in table 2, the product is the principal or present value. Or thus, Divide the given sum by the amount of £1 for the given rate and time in table 1, and the quotient is the present value.

Example.—What is the present value of £876 12s. due 6 years hence, at 4 per cent. compound interest?

876.6 × .790314 = 692.789 = £692 15s. 9½d. Ans.
876.6 ÷ 1.265319 = 692.789 = £692 15s. 9½d. Ans.

By the first method, I multiply the given sum by the number in the second table under 4 per cent. and opposite to 6 years, the product is the present value; by the second method, I divide the given sum by the number in the first table under 4 per cent. and opposite to 6 years, the quotient is the present value. The product, and quotient are given only to the third place of decimals, and valued mentally.

EXERCISES.

1. What principal will amount to £324 18s. 6d., in 9 years at 5 per cent. compound interest? Ans. £209 8s. 11¾d.
2. What is the present value of £3000, due 30 years hence, allowing 3½ per cent. compound interest?
Ans. £1068 16s. 8¼d.

3. What is the present value of £4500, due 7 years hence, allowing compound interest at 4½ per cent. ?
Ans. £3306 14s. 6¼d.
4. Whether is £1000, due in 2 years, or £10000, due in 50 years, the most valuable, and how much, allowing compound interest at 5 per cent ? Ans. £1000 by £35.
5. What is the difference between the present value of £100, due 20 years hence, at 2½ per cent., and the same sum, for the same time, at 5 per cent. compound interest ?
Ans. £23 6s. 9¼d.
6. What is the difference between the present value of £1000, due 20 years hence, at 5 per cent. per annum, and the same sum for the same time and rate, the interest payable, or convertible into principal half yearly ?
Ans. £4 9s. 2¼d.

Case 3.—When the principal or present value, the amount, and rate per cent. are given, to find the time.

Rule.—Divide the amount by the principal, find the quotient, or number nearest to it, under the given rate in table 1, opposite to which is the time.

Example.—In what time will £209 8s. 11¾d., amount to £324 18s. 6d., at 5 per cent. compound interest ?

324.925 ÷ 209.4489583 = 1.551332, but under 5 per cent. and opposite to 9 years in table 1, is 1.551328, therefore 9 years is the time required.

EXERCISES.

1. In what time will £365 12s., amount to £481 2s. 1d., at 4 per cent. per annum, compound interest ? Ans. 7 years.
2. In what time will £800, amount to £2996 5s. 1d., at 4½ per cent. compound interest ? Ans. 30 years.
3. In what time will £1000, amount to £1410 11s. 11¾d, at 3½ per cent. compound interest ? Ans. 10 years.
4. In what time will £1107 1s. 9¾d, amount to £10000, at 4½ per cent. compound interest ? Ans. 50 years.
5. In what time will £555 10s., amount to £3910 14s. 3¼d. at 5 per cent. compound interest ? Ans. 40 years.

Case 4.—When the principal, amount, and time are given, to find the rate per cent.

Rule.—Divide the amount by the principal, and find the quotient or number nearest to it in table 1, opposite to the

number of years, and at the top of the column is the rate per cent.

EXAMPLE.—At what rate per cent. will £50, amount to £63 16s. 3½d., in 5 years, at compound interest?

63.8135416÷50=1.276270. The number nearest to this in table 1 opposite to 5 years, is found under 5 per cent., therefore 5 per cent. is the rate required.

EXERCISES.

1. At what rate per cent. compound interest, will £700, amount to £810, 6s. 9d., in three years? Ans. 5 per. cent.
2. At what rate per cent. will £376.8895, amount to £1000, in 20 years, compound interest? Ans. 5 per. cent.
3. If £616 12s. 9d. amount to £2000, in 30 years, compound interest; what is the rate per cent.? Ans. 4 per cent.
4. At what rate per cent. will £450 amount to £547 9s. 10½d. in 5 years, at compound interest? Ans. 4 per cent.

ANNUITIES FOR A LIMITED TIME.

CASE 1.—When the annuity, rate, and time are given, to find the amount.—RULE. Take from table 3, the amount of £1 for the given time, and rate, which multiply by the annuity, the product is the amount.

EXAMPLE.—What will an annuity of £60 per annum, amount to, in 9 years, at 4 per cent. compound interest?

10.582795 × 60 = 634.9657 = £634 19s. 3¾d. Ans.

I here take the number in table 3, under 4 per cent. and opposite to 9 years, which, multiplied by 60, and the decimal valued mentally, gives £634 19s. 3¾d. the amount.

EXERCISES.

1. What will an annuity of £50 per annum amount to in 10 years, improved at 3½ per cent. compound interest? Ans. £586 11s. 4¾d.
2. If an annuity of £100 per annum, be forborne 20 years, and improved at 5 per cent. compound interest, what will it amount to? Ans. £3306 11s. 11d.
3. What will an annual salary of £150 amount to in 30 years, at 4½ per cent. compound interest? Ans. £9151 1s. 2½d.
4. A clergyman was presented to a living of £350 per an-

num, but the presentation being disputed, he was kept out of it for 8 years, what does the stipend then amount to, supposing it improved at 5 per cent. compound interest? Ans. £3342 3s. 9d.

CASE 2.—When the amount, rate, and time are given, to find the annuity.—RULE. Divide the amount, by the amount of £1 annuity, for the given time, and rate, from table 3, the quotient is the annuity.

EXAMPLE.—What annual annuity, for 5 years, at 5 per cent. compound interest, will amount to £386 15s. 10¾d.?

386.794291 ÷ 5.525631 = £70 annuity required.

The application of the rule here is so simple that it requires no explanation. £70 is not the complete quotient, but it wants less than the hundredth part of a farthing of being complete.

EXERCISES.

1. What annual annuity will amount to £1000, in 10 years, at 4½ per cent. compound interest? Ans. £81 7s. 6¾d.
2. What annual annuity will amount to £215 10s. 1½d. in 4 years, at 5 per cent. compound interest? Ans. £50.
3. What is that annual annuity, which being forborne 7 years, at 4½ per cent. will amount to £801 18s. 3¾d? Ans. £100.
4. What annuity unpaid for 30 years, and improved at 3½ per cent. will amount to £3355 9s. 6d? Ans. £65

CASE 3.—When the annuity, amount, and rate are given, to find the time.—RULE. Divide the amount by the annuity, and find the number nearest to the quotient under the given rate, in table 3, opposite to which is the time required.

EXAMPLE.—In what time will £81 7s. 6¾d. amount to £1000, at 4½ per cent. compound interest?

1000 ÷ 81.378125 = 12.288314, the number nearest to which, stands opposite to 10 years, the time required.

EXERCISES.

1. In what time will a rental of £50 per annum, improved at 3½ per cent. amount to £586 11s. 4¾d.? Ans. 10 years.
2. In what time will a pension of £150 per annum, amount to £9151 1s. 2½d. at 4½ per cent.? Ans. 30 years.
3. In what time will an annual salary of £100, amount to £3306 11s. 11d. at 5 per cent.? Ans. 20 years.
4. In what time will £50 per annum, amount to £215 10s. 1½d., at 5 per cent. compound interest? Ans. 4 years.

CASE 4.—When the annuity, amount, and time are given, to find the rate.—RULE. Divide the amount by the annuity, find the quotient in table 3, opposite to the number of years, and the rate is found at the top of the column.

EXAMPLE.—At what per cent. will an annuity of £150, amount to £9151 1s. 2¼d. in 30 years?

9151.060416÷150=61·007069 which is found opposite to 30 years under 4½ per cent., the rate sought.

EXERCISES.

1. At what rate per cent. will an annuity of £50 per annum, amount to £6549 17s. 11d., in 50 years? Ans. 4.
2. If an annuity of £46 12s. 6d. per annum, amount to £514 2s. 3¼d., in 9 years, what is the rate? Ans. 5.
3. If an annuity of £50, in 7 years, amount to £407 2s., what is the rate per cent.? Ans. 5.
4. If an annual annuity of £250, amount to £6717 11s. 10½d. in 20 years, what is the rate? Ans. 3.

CASE 5.—When the annuity, time, and rate are given, to find the present value.—RULE. Multiply the present value of £1 for the given rate, and time, from table 4, by the annuity, the product is the present value.

EXAMPLE.—What is the present value of an annuity of £35, to continue 20 years, at 4½ per cent.?

13.007936 × 35=455.27776, or £455 5s. 6½d. present value.

The application of the rule here is so simple as to require no explanation.

EXERCISES.

1. What should be paid for an annuity of £120 per annum, to continue ten years, allowing interest at 3 per cent.? Ans. £1023 12s. 6d.
2. What is the present value of an annuity of £75, payable annually, to continue 40 years, at 3½ per cent.? Ans. £1601 12s. 7¼d.
3. What should I pay for an annual annuity of £375 10s. to continue 8 years, at 5 per cent.? Ans. £2426 18s. 8¾d.
4. What is the present value of an annuity of £80, to continue 9 years, allowing 4 per cent.? Ans. £594 16s. 6¼d.

CASE 6. When the present value of an annuity, with the time of its continuance, and rate per cent. are given, to find the annuity.—RULE. Divide the present value given, by

the present value of a £1, for the given rate, and time from table 4, the quotient is the answer.

EXAMPLE.—What annual annuity to continue 9 years, can I purchase for £1839 4s. 9d., allowing 4 per cent. compound interest?
$1839.2375 \div 7.435331 = 247.364$, or £247 7s. 3½d. annuity.

In this case the decimal seldom terminates, but the results obtained by the rules are sufficiently accurate for practice, being within less than a farthing of the truth.

EXERCISES.

1. What annuity to continue 10 years, can I purchase for £511 16s. 2¾d., when I am allowed three per cent. compound interest for the purchase money? Ans £60.
2. What annual annuity to continue 20 years can I purchase for £1246 4s. 5¼d., when I am allowed 5 per cent. compound interest? Ans. £100.
3. What annual annuity to continue 30 years, will £1466 purchase, allowing interest at 5 per cent.? Ans. £90.
4. What annual annuity to continue 8 years, may be purchased for £841 11s. 10¼d., at 4 per cent. compound interest? Ans. £125.

ANNUITIES IN PERPETUITY.

ANNUITIES IN PERPETUITY are those which continue for ever after they are purchased, such as freehold estates.

CASE 1. When the value of an annuity in perpetuity, and the rate per cent. are given, to discover the annuity.—RULE. Divide the rate by 100, multiply the value by the quotient, the product is the annuity.

EXAMPLE.—Having purchased an estate for £850, what should the yearly rent be, at 3 per cent.?

$3 \div 100 = .03$, and $£850 \times .03 = 25.5$, or £25 10s. rental.

This is nothing more than finding the simple interest of the purchase money, at the proposed rates.

EXERCISES.

1. What rental should an estate yield, which cost £25000, to give the purchaser 4½ per cent.? Ans. £1125.

2. Purchased a house for £2520, how should I let it to have 8 per cent. for my money? Ans. £201 12s.
3. If an estate cost £15355, what should the rental be to yield $3\frac{1}{2}$ per cent.? Ans. £537 8s. 6d.

CASE 2. When a perpetual annuity, and the rate are given, to find its value.—RULE. Divide the rate by 100, and the annuity divided by the quotient gives the value.

EXAMPLE.—What is the value of an estate which yields an annual rental of £75, allowing $3\frac{1}{2}$ per cent.?
$3.5 \div 100 = .035$, and $75 \div .035 =$ £2142 17s. $1\frac{3}{4}$d. value.

EXERCISES.

1. What must be paid for a perpetuity of £50 per annum, to yield 5 per cent.? Ans. £1000.
2. The rental of an estate is £4563 per annum, what is the estate worth at $4\frac{1}{2}$ per cent.? Ans. £101400.
3. The rental of a house is £48; what is it worth at $7\frac{1}{2}$ per cent.? Ans. £640.

CASE 3. When the annuity, and its value are given, to find the rate per cent.—RULE. Multiply the annuity by 100, and divide the product by the value.

EXAMPLE.—Bought a property for £850, the rental of which is £38 5s.; what is the rate?

$$38.25 \times 100 = 3825 \div 850 = 4\tfrac{1}{2} \text{ per cent.}$$

EXERCISES.

1. If an estate of £500 per annum is bought for £10000; what per cent. has the purchaser for his money? Ans. 5.
2. Gave £850 for a house, which lets for £68 per annum; what per cent. have I for my money? Ans. 8.
3. A freehold estate worth £7575 a year, was sold for £34087 10s.; what is the rate of interest? Ans. $4\frac{1}{2}$.

CASE 4.—To find how many years purchase a perpetuity is worth, the rate being given; or the rate, from the year's purchase being given.—RULE. 100 divided by the rate gives the year's purchase; and 100 divided by the year's purchase gives the rate.

EXAMPLE.—How many years purchase is a perpetuity of £350 worth, at 5 per cent.?
$100 \div 5 = 20$ years' purchase. $100 \div 20 = 5$ per cent.

EXERCISES.

1. How many years purchase is an estate worth, at 4 per cent.? Ans. 25.
2. How many years purchase is an estate of £750 worth, at $2\frac{1}{2}$ per cent.? Ans. 40.
3. An estate was bought at 30 years purchase; what per cent. does it yield? Ans. £3 6s. 8d.

ANNUITIES IN REVERSION.

ANNUITIES IN REVERSION are such as the purchaser does not possess, till the end of a specified time.

CASE 1. When the annuity, the rate of interest, the time before the reversion, and the time of continuance are given, to find the present value of the reversion.—RULE. Find the present value of the annuity for the whole time of its continuance, from which subtract its present value for the time before the reversion, the remainder is the present value of the reversion.

EXAMPLE.—What is the present value of the reversion of a perpetuity of £50, to commence 8 years hence, at 4 per cent.?

$50 \div .04 =$	£1250	= Value in perpetuity.
$6.732744 \times 50 =$	336 12s. 9d.	=Value for 8 years.
	£ 913 7s. 3d.	=Value of reversion.

I here find the present value of £50 annuity in perpetuity at 4 per cent., and the present value of the same annuity for 8 years, which, subtracted from the former, leaves £913 7s. 3d. value of reversion.

EXERCISES.

1. What is the present value of the reversion of a freehold of £80 per annum, to commence 10 years hence at 5 per cent.? Ans. £982 5s. $2\frac{3}{4}$d.
2. What is the present value of the reversion of an annuity of £100 per annum, to commence 20 years hence, and to continue for 20 years, interest at 4 per cent.? Ans. £620 4s. $10\frac{1}{4}$d.
3. What is the value of the reversion of a perpetuity of £360, to commence 50 years hence, at $4\frac{1}{2}$ per cent.? Ans. £885 13s. $6\frac{1}{4}$d.

4. A has the lease of a farm for 30 years, worth £1000 per annum, after which B has it in perpetuity; which is the most valuable possession, and how much, reckoning 3½ per cent? Ans. A's by £8212 13s. 3d.

CASE 2. When the value of a reversion, the time before it commences, the time of its continuance, and the rate of interest are given to find the annuity.—RULE. Find the amount of the value by Table 1, at the given rate, and for the time before the reversion; then the annuity which that amount will purchase for the time, and at the rate specified, is the answer.

EXAMPLE.—What annuity to commence 7 years hence, and to continue for twenty years, will £500 purchase, interest at 5 per cent?

1.4071 × 500 = £703.55 amount of £500 in 7 years.

703.55 ÷ 12.46221 = £56 9s. 1d. the annuity required.

I here find the amount of £500 for 7 years, at 5 per cent. by Table 1, which, divided by the present value of £1 annuity, for 20 years at 5 per cent. from Table 4, gives £56 9s. 1d. the annuity to continue 20 years, which that amount will purchase.

EXERCISES.

1. What annuity, to commence 10 years hence, and to continue 30 years, should I have for £600, interest 4 per cent.? Ans. £51 7s. 2¾d.
2. What perpetuity, to commence 20 years hence, should I have for £90, interest at 3 per cent.? Ans. £4 17s. 6½d.
3. Advanced £700 on a house, which I am to come into possession of 9 years hence; what should the free rental be to allow 5 per cent. interest? Ans. £54 5s. 11¼d.
4. A has an annual annuity of £60, to continue 10 years, which he intends to improve at 5 per cent.; what perpetuity will it then purchase, at 4 per cent.?
Ans. £30 3s. 8¾d.

LIFE ANNUITIES.

LIFE ANNUITIES are such as depend entirely on the life of one or more individuals, either for their commencement, or termination, or for both.

CASE 1. To find the present value of an annual annuity, at a given rate, to continue during the life of a person whose age is known.—RULE. Multiply the number in Table 6, corresponding with the given age, and rate, by the annuity, the product is the present value.

EXAMPLE.—What is the present value, at 5 per cent., of an annuity of £100, to continue during the life of a person aged 40 years?

11.837 × 100 = £1183 14s. present value.

I here take from Table 6, the number opposite to 40 years, and under 5 per cent., which is the present value of £1 annuity, during the remainder of the life, and this number multiplied by the annuity gives the present value.

EXERCISES.

1. What is the value of an annuity of £35, at 5 per cent., secured on the life of a person aged 35 years? Ans. £437 11s. 5d.
2. What is the value of the life rent of a farm, worth £150 per annum, at 4 per cent., the lessee being 79 years of age? Ans. £588 3s.
3. What sum should I give for the life rent of a house, worth £37 10s. per annum, secured on the life of a child aged 2 years, to have 4 per cent? Ans. £586 4s. 9d.
4. How much better is the life rent of an estate of £1000 per annum, secured on the life of a gentleman aged 20, than upon the life of his son aged 1 year, at 5 per cent.? Ans. £2444.
5. I am offered an annuity of £55 per annum, secured on the life of a person aged 40 years, at 4 per cent., or the same annuity secured on the life of his child, aged 2 years, at 5 per cent.; which is the best security, and how much? Ans. the son by £12 5s. 3¾d.

CASE 2. To find the present value of an annual annuity, secured on the joint continuance of two given lives.—RULE. Multiply the number in Table 7, common to both the given ages, by the annuity, the product is the present value.

EXAMPLE.—Required the present value at 4 per cent., of an annuity of £70, to continue during the joint lives of two persons whose ages are 35 and 40 years respectively?

10.196 × 70 = £713 14s. 5d. present value required.

I here take 10.196 from Table 7, being common to 35 and 40 years, which multiplied by the annuity gives the value.

EXERCISES.

1. What is the present value, at 5 per cent., of an annuity of £320 per annum, secured on the joint life of two persons whose ages are 45 and 60 years?
   Ans. £2183 0s. 9¾d.
2. There is an annuity of £130 per annum, to continue during the joint lives of two persons, whose ages are 5, and 30 years respectively; what is it worth at 4 per. cent.?
   Ans. £1588 12s.
3. What is the present value of an annuity of £75 12s. to continue during the joint lives of a man and his wife, whose ages are 70 and 55 years respectively, at 4 per cent.?
   Ans. £387 19s. 7d.

CASE 3. To find the present value of an annuity, to continue during the currency of two given lives.—RULE. From the sum of the values of the single lives, subtract the value of the joint lives, the remainder, multiplied by the annuity, gives the present value.

EXAMPLE.—What is the present value of an annuity of £25, to continue during two lives, aged 45 and 60 years, at 4 per cent.?

$12.283 + 9.039 - 7.274 = 14.048 \times 25 = £351$ 4s. Ans.

I here find the value of the single lives by Table 6, and from their sum subtract the value of the joint lives by Table 7, the remainder 14.048, multiplied by the annuity gives £351 4s. the present value.

EXERCISES.

1. What is the present value of an annuity of £100, at 5 per cent., to continue during the longest life of two persons, aged 35 and 40 years? Ans. £1500 16s.
2. What is the present value of an estate of £225, at 4 per cent., to terminate with the demise of a father and son, aged 60 and 25 years? Ans. £3728 9s. 6d.
3. What should a father pay at 4 per cent., to secure an annuity of £300 during his own, and his son's life, their ages being 45 and 5 years? Ans. £5709 6s.

CASE 4. To find the present value of an annuity during the joint continuance of three given lives.—RULE. Find the joint value of the two oldest by Case 2, and find in Table 6, what single life is equal to this value, then find the joint value of the life thus found, and the youngest life given by Case 2,

which, multiplied by the annuity, gives the present value required.

EXAMPLE.—Required the present value of an annuity of £120, at 4 per cent., secured on the joint continuance of three lives, of 60, 40 and 20 years respectively?

7.49 = joint value of two lives, of 60 and 40 years.
7.488 = value of a single life of 66 years.
6.754 = joint value of a life of 66 and 20 years.
6.754 × 120 = £810 9s. $7\frac{1}{4}$d. present value required.

I here find the joint value, at 60 and 40 years in Table 7, to be 7.49, and by Table 6, a single life, at 4 per cent. of 66 years, is very nearly equal to this value; I then find the joint value of the lives 20 and 66, but as 66 is not in the table, I take the value of 20 and 65, and also of 20 and 70, and $\frac{1}{5}$ of the difference of these values taken from the value at 20 and 65, leaves 6.754 the value at 20 and 66, which, multiplied by the annuity, gives the present value of the joint continuance of the three given lives.

EXERCISES.

1. Required the present value of an annuity of £80, at 5 per cent., secured on the joint continuance of the lives of a mother and her two daughters, whose ages are 40, 10, 5 years respectively? Ans. £725 7s. $2\frac{1}{2}$d.
2. What is the present value of an annuity of £95 10s. at 4 per cent., during the joint continuance of three lives aged 35, 30, 15 respectively? Ans. £920.

CASE 5.—To find the present value of an annuity to terminate with the longest survivor of three given lives.—RULE. Find the value of each single life, and also their joint value; from the sum of these values, subtract the sum of their values taken two by two, the remainder multiplied by the annuity is the present value required.

EXAMPLE.—Required the present value of an annuity of £120, at 4 per cent., to continue till three persons aged 60, 40, 20 years are all deceased.

(9.039+13.197+16.033+6.754) — (7.49+7.995+10.924)=
18.614, and 18.614 × 120 = £2233 13s. $7\frac{1}{4}$d. present value.

I here find the value of the three single lives 60, 40, 20 at 4 per cent., and also their joint value as in last case, from the sum of these values, I subtract the value of the lives taken in pairs, that is, the sum of the value of 60 and 40, of 60 and 20,

and of 40, and 20, the remainder is the present value of £1 on these lives, which multiplied by the annuity gives its present value.

EXERCISES.

1. Required the present value of an annuity of £80, at 5 per cent., to continue till the demise of a mother, and her two daughters, aged 40, 10, 5 years respectively?
Ans. £1440 17s. 7½d.
2. What is the present value of an annuity of £95 10s. at 4 per cent., to terminate with the demise of the oldest survivor of three persons, aged 35, 30, 15 years?
Ans. £1936 3s. 4½d.

CASE 6.—To find the value of the reversion of an annuity after the demise of 1, 2, or 3 persons whose ages are given.—RULE. Find the present value of the annuity for the whole time of its continuance, from which subtract its value for the time before the reversion, the remainder is the present value of the reversion.

EXAMPLE.—What is the present value of the reversion of an annuity of £60, at 5 per cent., after the death of the present possessor, and his son, whose ages are 65, and 35 years, the annuity to continue 30 years from this time?

$15.372451 \times 60 = 922.34706$ present value for 30 years.

$(7.761 + 14.039 - 6.747) \times 60 = 903.18$ present value before reversion.

$922.34706 - 903.18 =$ £19 3s. 4½d. present value of reversion.

I here find the present value of the annuity for 30 years to be £922.34706, and its value for the longest liver of the two ages 65 and 35 years, is £903.18, which is the present value before the reversion, and the difference of these two values, is the present value of the reversion.

EXERCISES.

1. A gentleman aged 68 years, holds an annuity of £125 per annum, which is to continue for 12 years; what is the present value of the reversion after his death, allowing 5 per cent. interest? Ans. £290 18s. 1½d.
2. What is the present value of the reversion of a perpetuity of £100 per annum, to commence at the death of the present possessor, aged 39 years, allowing 4 per cent. interest? Ans. £1162 10s.
3. What is the present value of the reversion of an annuity of £70, at 5 per cent., to continue 40 years from this

time, the reversion to commence with the demise of the last survivor of two ladies, aged 45, and 30 years? Ans. £275 2s. 1½d.

4. What is the present value, at 5 per cent., of the reversion of a perpetuity of £160, to commence with the extinction of two lives of 35 and 40? Ans. £798 14s. 5d.

5. What should I pay at present, at 5 per cent., for the reversion of £147 per annum, to terminate 50 years hence, the reversion to commence with the demise of the oldest survivor of three persons, whose ages are 40, 10, and 5 years respectively? Ans. £36 0s. 0¾d.

6. What ready money should I give, at 4 per cent., for the reversion of a perpetuity of £147, to commence when the first of three persons die, whose ages are 60, 40, and 20 years? Ans. £1947 3s. 3d.

CASE 7.—To find the present value of an annuity to be enjoyed for life by the purchaser or successor, after the demise of the present possessor.—RULE. From the single value of the successor's life, take the value of the joint lives of the possessor and successor, the remainder multiplied by the annuity is the present value required.

EXAMPLE.—What is the value of a wife's expectation of an annuity of £30 per annum, at 5 per cent., to continue during her life, after her husband's death, his age being 50, and hers 45 years?

$11.105 - 7.891 = 3.214 \times 30 =$ £96 8s. 5d. value of expectation.—I here find the value of the wife's life to be 11.105, from which I take 7.891, the value of their joint lives; the remainder 3.214, multiplied by the annuity gives £96 8s. 5d. the value of the wife's expectation.

EXERCISES.

1. If a clergyman is 65, and his wife 40 years of age, what is the value of her expectation, at 4 per cent. if she is entitled to an annuity of £45 per annum from the widow's fund, after her husband's death? Ans. £296 4s. 8¼d.

2. A gentleman aged 55, married a lady of 25, and settled upon her a jointure of £550 per annum; what is her expectation worth, at 5 per cent.? Ans. £2998 1s.

3. If the possessor of an entailed estate of £2000 per annum, be 48 years of age, and the next heir of entail 35 years of age; what is the value of his expectation, at 4 per cent.? Ans. £9381 4s.

CASE 8.—To find what sum must be paid at one payment, and also what annual payment must be made by a husband during his life, to secure any proposed annuity to his widow during the remainder of her life.—RULE. The widow's expectation, as found in last case, is the sum to be paid at one payment; which sum divided by the value of the husband's life, if the first payment be made one year after the insurance is effected, or by the value of his life increased by a unit, if the first payment be made immediately upon effecting the insurance, gives, in both cases, the annual payment during life.

EXAMPLE.—What must a husband, aged 50, pay at one payment, and also annually during life, if the first payment be made one year after the policy is extended, and when it is made immediately, to entitle his wife, aged 35, if she survive him, to an annuity of £80 per annum, for the remainder of her life, allowing interest at 4 per cent.?

$14.039 - 9.11 = 4.929 \times 80 =$ £394.32 at one payment.

$394.32 \div 11.264 =$ £35 0s. $1\frac{3}{4}$d. Annual payment at year's end.

$394.32 \div 12.264 =$ £32 6s. $0\frac{1}{2}$d. Annual payment made immediately.—I here find the amount at one payment by last case, which sum, divided by the value of the husband's life, gives the annual premium, when the first payment is made one year after the policy is extended, and this same sum at one payment, divided by the value of the husband's life increased by a unit, gives the annual premium when the first payment is made immediately upon extending the policy.

EXERCISES.

1. How much should a man, aged 40, pay at one payment, and also annually during life, at 5 per cent., to entitle his wife, aged 35, to an annuity of £45 per annum, after his death, provided she survives him, the first payment being made immediately?

   Ans. £142 13s. 11d. at once, £11 2s. $3\frac{3}{4}$d. Annually.

2. What annual payment must a husband, aged 65, make during life, that his wife, aged 30, if she survives him, may have an annuity of £60 per annum, interest 4 per cent., and the first payment made one year after the policy is extended? Ans. £61 7s. $2\frac{1}{2}$d.

CASE 9.—To find what sum must be paid annually during marriage, to entitle a widow to any proposed annuity.—RULE. The present value of the widow's expectation by Case 7, divid-

ed by the value of their lives jointly, or their value increased by a unit, according as the first payment is at a year's end or immediately, gives the annual payment during marriage.

Example.—What annual payment must be made during marriage, when the first payment is made at the end of a year, and also when made immediately, to entitle a wife, if she is the survivor, to an annuity of £130 per annum; the husband's age being 35 and the wife's 20 years, interest 5 per cent.?
$(14.007 - 10.363) \times 130 = £473.72$ Widow's expectation.
$473.72 \div 10.363 = £45$ 14s. 3d. Annual payment at year's end.
$473.72 \div 11.363 = £41$ 13s. 9½d. Annual payment immediately.—The operation here is the same as in last case, with the exception of using the value of the joint lives, instead of the husband's life for a divisor.

EXERCISES.

1. What should a man pay annually during marriage, to secure an annuity of £100 per annum to his wife, provided she survive him, his age being 45 and hers 40 years, interest at 4 per cent., and the first payment made immediately? Ans. £36 15s. 2¼d.
2. What annual payment must be made during marriage, to entitle a widow to an annuity of £60 per annum for the remainder of life, the husband's age being 55, and the wife's 35 years, interest 5 per cent., and the first payment to be made at a year's end. Ans. £35 11s. 4½d.

Case 10.—To find how much must be paid present, and also by annual instalments during life, to insure any proposed sum to heirs at death.—Rule. From the value of £1 in perpetuity, subtract the value of the given life, and divide the remainder by the perpetuity increased by a unit, the quotient, multiplied by the perpetuity, gives the value at one payment, which, divided by the value of the life, gives the annual instalment.

Example.—Required how much must be paid present, and also by annual instalments, at 5 per cent., by a man aged 40, to insure £1000 to his heirs at his death?
$20 - 11.837 = 8.163 \div 21 = .388714 \times 1000 = £388.714$ at one payment.
$388.714 \div 11.837 = £32$ 16s. 9¼d. Annual instalment during life.—I here find the value of £1 in perpetuity, at 5 per cent., to be £20, from which I subtract 11.837, the value of a life of

40, at 5 per cent., the remainder 8.163, divided by the perpetuity, plus 1 or 21, gives .388714, which, multiplied by £1000, the proposed sum, gives £388 14s. 3¼d., the value at one payment, which, divided by the value of the life, gives £32 16s. 9¼d., the annual instalment.

EXERCISES.

1. What should a man, aged 24, pay annually during life, at 5 per cent., to insure £3000 to his family at his death? Ans. £70.
2. What should a man, aged 62, pay annually during his life, at 5 per cent., to insure £500 to his heirs at his death; and what should he pay, at the same rate at one payment, to accomplish the same object? Ans. Annually £35 19s. 4½d.; at one payment £286 10s. 5¼d.

DUODECIMALS, OR CROSS MULTIPLICATION.

DUODECIMALS increase or diminish constantly by 12, that is, twelve of any inferior denomination make one of next superior denomination; and are generally used by artificers to cast up the contents of their work.—RULE. Write the multiplier under the multiplicand, with the same denominations directly under one another; multiply each denomination of the multiplicand, beginning with the lowest, by the feet in the multiplier, carrying at 12, and writing the remainders under their respective terms in the multiplicand; then multiply by the inches in like manner, carrying as before, and writing each remainder one place further to the right; proceed in the same way with every inferior denomination of the multiplier, writing each product a place further to the right than the one immediately preceding, the sum of these products is the answer.

EXAMPLE.—Multiply 7 feet, 6 inches, 3 seconds, by 5 f. 8 in. 10 seconds.

7	6′	3″		
5	8′	10″		
37	7	3		
5	0	2	0	
	6	3	2	6
43	1′	8″	2‴	6⁗

Having placed the factors, I first multiply by the 5 feet in the multiplier, beginning with the seconds in the multiplicand, carrying at 12, and writing the remainders directly under the denominations multiplied; I then multiply by the 8 inches, writing the remainders a

place further to the right than in last products; I then multiply by the 10 seconds, still writing the remainder one place further to the right than the immediate preceding remainders, the sum of these is the answer.

EXERCISES.

1. Multiply 4 f. 6′ by 2 f. 5′. Ans. 10 f. 10′ 6″.
2. Mutilply 5 f. 8′ by 4 f. 6′. Ans. 25 f. 6′.
3. Multiply 6 f. 3′ 6″ by 5 f, 3′. Ans. 33 f. 4″ 6‴.
4. Multiply 7 f. 10′ 5″ by 3 f. 4′ 3″. Ans. 26 f. 4′ 8″ 3‴ 3⁗.
5. Multiply 120 f. 6′ 2″ by 6′ 10″. Ans. 68 f. 7′ 6″ 1‴ 8⁗.
6. Multiply 53 f. 11′ 9″ by 14 f. 3′ 5″. Ans. 771 f. 11′ 1‴ 9⁗.
7. Multiply 36 f. 6′ by 48 f. 7′. Ans. 1773 f. 3′ 6″.
8. Multiply 20 f. 8′ 4″ by 8 f. 7′. Ans. 177 f. 7′ 6″.
9. Multiply 84 f. 11′ by 12 f. 4′. Ans. 1047 f. 3′ 8″.
10. Multiply 10 f. 11′ 6″ by 12 f. 4′ 6″. Ans. 135 f. 7′ 3″ 9‴.

The area of any rectangular surface is found by multiplying the length by the breadth, or mean breadth. The solid content of any rectangular body is found by multiplying the length, by the breadth, and depth, or by the mean breadth, and depth.

The solid content of round timber is found, by multiplying the length by the square of ¼ of the mean circumference.

EXERCISES.

1. What is the area of a board 10 f. 3′ long, and 1 f. 6′ broad? Ans. 15 f. 4′ 6″.
2. What is the area of a board 14 f. 6′ 7″ by 1 f. 11′ 9″? Ans. 28 f. 9′ 6″ 4‴ 3⁗.
3. How many superficial feet in a board 20 f. 6′ by 3 f. 5′? Ans. 69 f. 6″.
4. How many square yards are there in the ceiling, floor, and walls of a room 30 f. 4′ long, 24 f. 6′ wide, and 11 f. 6′ high? Ans. 305 yds. 2 f. 6′.
5. What is the solid content of a log of wood 30 f. 8′ long, and 2 f. 4′ the side of the square? Ans. 166 f. 11′ 6″ 8‴.
6. How many cubic feet in a stone, 15 f. 7′ long, 2 f. 8′ broad, and 1 f. 9′ deep; and what is its weight at 2520 oz. Avoirdupois per foot?
   Ans. 72 f. 8′ 8″. Weight, 5 t. 2 cwt. 1 qr. 1 lb. 12 oz.
7. How many cubic feet in a round tree 25 f. 3′ long, the girt in the middle being 46 inches? Ans. 23 f. 2′ 3″ 3‴ 9⁗.
8. How many feet in 6 deals, each 16 f. 5′ long, and 17 inches broad at one end, and 13 at the other? Ans. 123 f. 1′ 6″.

9. How many solid feet in a log of mahogany, 15 f. 9′ long, 4 f. 6′ broad, and 2 f. 9′ deep at one end, and 3 f. 10′ broad, and 2 f. 5′ deep at the other end? Ans. 169 f. 6′ 4″ 6‴.
10. How many cubic feet in a round tree, 40 f. 10′ long, the girts at four equidistant places being 5 f. 10′, 4 f. 4′, 3 f. 1′, and 2 f. 3′? Ans. 39 f. 6′ 8″ 3‴.

TONNAGE OF SHIPS.

To find the tonnage of ships, is the method of calculating what weight they will carry.—RULE. Multiply the length of the keel by the length, and by half the length of the midship beam, both taken within the vessel; this last product divided by 94 gives the tonnage.

EXAMPLE.—What is the tonnage of a ship, whose keel is 100 feet, and midship beam 40 feet?

$$\frac{100 \times 40 \times 20}{94} = \frac{40000}{47} = 851\tfrac{3}{47} \text{ tonnage required.}$$

I here multiply 100 the length of the keel, by 40 the length of the midship beam, and again by 20, half its length, and divide the product by 94, which gives $851\frac{3}{47}$, the number of tons the ship will carry.

EXERCISES.

1. What is the tonnage of a ship whose keel is 150 feet, and midship beam 56 feet? Ans. $2502\frac{6}{47}$ tons.
2. A ship's keel is 188 feet, and her midship beam 71 feet 6 inches, required her tonnage. Ans. $5112\frac{1}{2}$ tons.
3. If a ship's keel be 90 feet, and the midship beam 30 feet; what is her tonnage? Ans. $430\frac{40}{47}$ tons.
4. What is the tonnage of a ship whose keel is 130 feet, and the midship beam 47 feet? Ans. $1527\frac{1}{2}$ tons.

PROMISCUOUS QUESTIONS.

1. Write in figures, eight hundred and fifty-nine thousand nonillions sixty-three octillions one thousand and eight hundred quintillions seventy thousand billions three millions and nine hundred thousand.

2. A merchant left in his last will, £12856 to his old son, £4269 to each of his five younger sons, £2632 to each of his four daughters, £620 to each of 3 nephews, and £235 to each of 10 nieces; how much did he leave in all? Ans. £48938.

3. What is the difference between ten times three, and twenty; and ten times twenty three? Ans. 180.

4. Multiply 1842365, by 3841296 in three lines of products. Ans. 7077069305040.

5. Multiply 765436 by 8888 in two lines of products. Ans. 6803195168.

6. Find by subtraction, the product of 638954 by 9999; and also by multiplication in two lines. Ans. 6388901046.

7. The quotient of a division is 65, the remainder 8, and the dividend 853; what is the divisor? Ans. 13.

8. The product of two numbers is 1610, of which the less is 23; what is the square of their difference? Ans. 2209.

9. If $\frac{1}{2}$ of 6 is 2; what is the $\frac{1}{4}$ of 30? Ans. 5.

10. The sum of two numbers is 104, and their difference is 10; what are the numbers? Ans. 47, 57.

11. What number taken from 2 leaves $\frac{1}{3}$ of $\frac{9}{10}$? Ans. $1\frac{7}{10}$.

12. If 30 pence, and 40 groats, buy 50 pints of wine; What is the cost of 60 quarts in current English coin? Ans. £1 18s.

13. The globe of the earth under the equator is 360 degrees, each $69\frac{1}{2}$ miles; now, supposing two men to start at once, from the same place under the equator, and the one to travel directly east 30 miles a-day, the other directly west 35 miles a-day; at what distance of time will they meet? Ans. $384\frac{12}{13}$ days.

14. A silk mercer purchased 50 pieces silk, each 34 Fl. ells, and paid at the rate of 8/4 per Eng. ell; what was the price of the whole? Ans. £425.

15. A is indebted to B £240, payable 5 months hence, but if he pays £40 of it in hand, when should he pay the remainder? Ans. 6 months.

16. A can do a piece of work in an hour, B can do it in 3 hours, C in 5 hours, and D in 7 hours; what time will they require, when all working together, to do three times the quantity of work? Ans. 1 h. 47 m. $21\frac{2}{11}$ sec.

17. A cistern containing 300 gal. is supplied by a pipe which fills it in 2 hours, and discharged by another pipe, which empties it in $2\frac{1}{4}$ hours; how long would it require to fill it, if both pipes were set open at once? Ans. 18 hours.

18. A tradesman left $\frac{4}{5}$ of $\frac{5}{24}$ of his property to his old son, and $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{1}{3}$ of it to his second son, and the remainder to

their mother, which turned out £280 more than both their shares; what was the share of each?

Ans. Mother £440, old son £100, young son £60.

19. How much must be insured at $5\frac{1}{4}$ guineas per cent., to cover £813 11s. 3d. on a voyage from Jamaica to Leith?

Ans. £863 8s. 6d.

20. My agent in Oporto has purchased fruit on my account and risk, to the amount of £579 16s. 10d., commission and charges $7\frac{1}{2}$ per cent.; find the expense of insurance at $4\frac{3}{8}$ per cent. on the amount of the invoice. Ans. £27 5s. 5d.

21. Three merchants, A, B, and C, enter into partuership; A gains £12 in 6 months, B £15 in 5 mouths, and C, whose stock was £40, gained £21 in 9 months; required the stock of A and B. Ans. A £$34\frac{2}{7}$, B £$51\frac{3}{7}$.

22. A gentleman wishes to paper 5 rooms in his country-house, the first is 95 f. 7′, the second 84 f. 9′, the third 79 f. 11, the fourth 71 f. 6′, and the fifth 59 f.; the height of each is 10 f. 8′; how many yards of paper, 3 qrs. wide, will he require, deducting $129\frac{207}{1944}$ yds. for doors, windows, and fire places? Ans. 488 yds.

23. When the barometer stands at 30 inches, the weight of atmosphere on every square inch of surface is 14·75 lb.; what is the difference of weight sustained by a corpulent man, the surface of whose body is 20 square feet, and another whose surface is 16 feet? Ans. 75 cwt. 3 qrs. 12 lb.

24. Whereas a noble and a mark just 15 yards did buy,
How many ells of the same cloth for £50 had I?

Ans. 750

25. I lent three sums of money to three friends, A, B, C, and I have entirely forgotten both the sum lent to each, and the sum of the whole, but I remember distinctly that A and B together owe me £47, A and C together £71, and B and C together £88; what does each owe me?

Ans. A £15, B £32, C £56.

26. What present money will discharge a debt of £450, due 3 months hence, allowing true discount at 5 per cent.?

Ans. £444 8s. $10\frac{1}{2}$d. $\frac{2}{3}$.

27. A gentleman having purchased a horse from a jockey for £80, asked the fellow how much he had gained, who replied $\frac{1}{4}$ of what it cost me; what was his gain? Ans. £16.

28. A merchant has 4 cwt. 3 qrs. 17 lb. of tea, which he orders to be made up in parcels of 1, 2, 3, 4, 5, 6, 7 and 8 oz., and to have an equal number of each; how many of each should he have? Ans. 244.

29. A gentleman left 270 acres of land to be divided among his three nephews in proportion to their ages at his death, which was found to be 18, 12, 6 years respectively; required their shares. Ans. A 135, B 90, and C 45 acres.

30. A merchant wishes to mix teas at 8/, 7/6, 6/6 per lb. in such proportions that he may have a cwt. worth 7/4 per lb.; how much of each must he use?

Ans. 26⅔ lb. at 8/, 53⅓ lb. at 7/6, 32 lb. at 6/6.

31. Four merchants enter into copartnery, A put in £64 for 4½ months, B £78 15s. for 6 months, C £112 14s. for 8¾ months, D £125 5s. for 5½ months; they gain £108 18s. 4½d.; what should each have of the profits? Ans. A £13 2s. 9d. B £21 7s. 8½d. C £44 12s. 8d. D £29 15s. 3d.

32. A bankrupt owes A £313 7s. 3d. B £290 4s. 6d. C £700, D £486 13s. 8d. E £600, F £500, G £381 10s. and H £418; his effects are £1739 13s. 8½d., what will each of his creditors receive? Ans. A £147 14s. 11¼d., B £136 16s. 9¼d., C £330 0s. 10d., D £229 9s. 3¾d, E £282 17s. 10¼d., F £235 14s. 10½d., G £179 17s. 5½d., H £197 1s. 8d.

33. Sold £60 per annum in the long annuities, at 16½ years' purchase, brokerage ⅛ per cent. on the proceeds; what did I receive? Ans. £988 15s. 3d.

34. What rate of interest arises from vesting money in the 3 per cents. when selling at 89⅜? Ans. £3 7s. 0½d.

35. What sum must I insure to cover £1000 on a voyage to the East Indies, premium, policy, &c. 8½ per cent., and expense of recovering in case of loss 1½ per cent.?

Ans. £1111 2s. 2½d. ⅔.

36. As I was going to St. Ives, I met fifty old wives,
Every wife had fifty sacks, and every sack had fifty cats,
Every cat had fifty kittens; kittens, cats, sacks and wives,
How many in all were coming from St. Ives?

Ans. 2250000.

37. London remits to Amsterdam 5000 guilders when the exchange is at 36/6 Fl. banco per £ Sterling; what will London gain or loss by this transaction, if the amount is drawn for, when the exchange is 35/ Fl. banco per £ Sterling?

Ans. gain £19 11s. 4½d.

38. A merchant has wines at 14/, 15/, 19/, and 22/ per gallon, of which he wishes to make a mixture worth 18/ per gallon; find out for him all the different proportions of each sort which will answer his purpose.

	1st		2d		3d		4th		5th		6th		7th		
Ans.	1		4		5		4		5		1		5	gal. at	14/.
	4	or	1	or	1	or	5	or	4	or	5	or	5		15/.
	4		3		7		3		4		7		7		19/.
	3		4		4		7		7		3		7		22/.

or any other quantities, either more or less, *ad infinitum*, in the same proportions as either of the 7 quantities given.

39. A husband dying when his wife was with child, ordered, if she had a son, that he should heir $\frac{2}{3}$ of his father's fortune, and the widow the rest, but if a daughter, that she should heir $\frac{1}{3}$ of his fortune, and her mother the rest; but she had both a son and daughter, what should each have, supposing the son's share to be £1000 less than if he had not had a sister?

Ans. Son £6000, mother £3000, daughter £1500.

40. What number is that which, multiplied by 30, the product is $\frac{1}{8}$? Ans. $\frac{1}{240}$.

41. What is the difference between the simple and compound interest of £850, for 40 years, at 5 per cent.?

Ans. £3433 19s. $9\frac{1}{2}$d.

42. What is the difference between $\frac{1}{7}$ of 3 guineas, and $\frac{3}{7}$ of a £, performed decimally? Ans. $5.\dot{1}4285\dot{7}$d.

43. A merchant sold cloth for 11/6 per yard, by which he cleared 15 per cent.; what would he have cleared per cent. by selling for 12/ per yard? Ans. 20 per cent.

44. A gentleman was married to a young lady, and it so happened that their marriage day was the birth day of both, and the bridegroom's age was just double the bride's, but after they had lived together 30 years, his age was to hers as 2 to $1\frac{3}{7}$, and 30 years after this, his age was to her's as 2 to $1\frac{3}{5}$, what was their age on their marriage-day, and what should the husband have paid that day at 5 per cent. to secure an annuity of £500 a year to his wife if she survived him? Ans. Ages 40 and 20 years, and £2035 to secure the annuity.

45. A father destined his fortune to his three sons in the following manner, A $\frac{1}{2}$, B $\frac{3}{5}$, C $\frac{4}{7}$, now, if his fortune was £3000, what should each receive? Ans. A £897 8s. $8\frac{1}{2}$d. $\frac{6}{13}$, B £1076 18s. $5\frac{1}{2}$d. $\frac{2}{13}$, C £1025 12s. $9\frac{3}{4}$d. $\frac{5}{13}$.

46. Three merchants A B and C join in company, A gains £40 in 6 months, B £36 in 5 months, and C, whose stock was £72, gains £28 in 9 months; what was the stock of A and B? Ans. A £154 5s. $8\frac{1}{2}$d. $\frac{2}{7}$. B £166 12s. $6\frac{3}{4}$d. $\frac{3}{7}$.

47. Three merchants join in partnership, A's stock remained in company 12 months, B's 9 months, and C's 6 months,

now, A claimed $\frac{1}{3}$ of the gain, B $\frac{2}{5}$, and C, whose stock was £300, took up the remainder; what was the stock of A and B?

Ans. A £187 10s. B £300.

48. A young fellow having rudely asked a lady her age, received the following reply,

My age, if multiplied by three,
And two-sevenths of the product tripled be,
The square root of two-ninths of that is four;
Sir, tell my age, or never see me more!

What was her age? Ans. 28 years.

49. Three merchants enter into company for 6 months, and they found their gain to be £743 15s. now, if A's stock was £1800, B's £1500, and C's £1200; allowing 5 per cent. per annum on their several stocks, and the remainder of the gain to be equally divided among them, what should each receive of the profits?

Ans. A £255 8s. 4d., B. £247 18s. 4d., C £240 8s. 4d.

50. Paid £10$\frac{2}{3}$ for 27$\frac{1}{2}$ yds. of cloth $\frac{13}{12}$ yd. wide; what should I pay for 7$\frac{3}{4}$ yds. of the same quality which is $\frac{11}{12}$ yd. wide? Ans. £2 10s. 10$\frac{1}{4}$d. $\frac{11}{13}$.

51. A gentleman bestowing alms on some poor people found if he gave each 5d. he would be 4d. deficient, but if he gave each 4d. he would have 8d. remaining; how many beggars were there? Ans. 12.

52. Petersburgh is indebted to London 3718 rubles 54 cop. which may be drawn directly at 4/6$\frac{1}{2}$ per ruble, or through Holland at 52 stivers per ruble, if the exchange between Amsterdam and London be 33/10$\frac{1}{2}$ Fl. per £ Stg. and the expense of the circular remittance 2 per cent.; which is the most profitable way for London to draw? Ans. circular by £87 18s. 3$\frac{3}{4}$d.

53. Sessa the Indian who invented the game of chess, having showed it to his king, was requested to name his own reward, displeased his sovereign for asking no more than one grain of wheat for the first square on the chess board, two for the second, and so on in geometrical progression for each of the 64 squares; what did this modest demand amount to, supposing 7680 wheat-corns make a pint, and that it was worth 10/3$\frac{3}{5}$ per bushel? Ans. £19351404648857 11s. 10$\frac{1}{2}$d.

54. A father left his property to his three sons in the following manner, namely, to A $\frac{4}{7}\frac{4}{5}$, B $\frac{3}{7}\frac{3}{8}$ of the remainder, and the rest to C, now, if A's share was £500 more than B's; what was the sum left to each? Ans. A £772 2s. 0$\frac{1}{4}$d. $\frac{523}{2251}$. B £272 2s. 0$\frac{1}{4}$d. $\frac{523}{2251}$. C £342 1s. 4$\frac{3}{4}$d. $\frac{913}{2251}$.

55. Effected insurance on a cargo of sugar from Jamaica to Leith, to the amount of £3000 at 5 guineas per cent., to return £2 10s. per cent. if the vessel sailed with convoy and arrived safe, which she did, but the sugar was so much damaged that it sold for £2250, whereas, if sound, it would have brought £3400; what should I receive from the underwriters?

Ans. £1064 6s. 9d.

56. There is an island 73 miles in circumference, and A, B, and C all start together and travel the same way about it; A travels 5 miles a day, B 8, A 10, at what distance of time will they all meet again, and how many general meetings will they have in a year?

Ans. First meeting after 73 days, and 5 meetings in a year.

57. Calculate the interest on the following account till 21st February, the bank exacting 4½, and granting 3¾ per cent. on balances.

Dr.—A B. Esq. in account-current with the Royal Bank.—Cr.

June 8, To cash £107	Aug. 7, By cash £400
July 25, To cash £230	Oct. 3, By cash £220
Sept. 7, To cash £321	Dec. 27, By cash £180
Nov. 21, To cash £161	
Dec. 23, To cash £140	

Ans. £4 1s. 4d.

58. In an arithmetical series, the common difference is 4, the number of terms 20, and the greatest term 144; what is the sum of the series? Ans. 2120.

59. What is the value of $.\dot{3}$ of a moidore? Ans. 9/.

60. When first the marriage knot was tied betwixt my wife and me;
My age did her's as far exceed as three times three does three;
But after ten and half ten years, we man and wife had been,
Her age came up as near to mine as eight is to sixteen.
Now you skill'd in numbers tell me, I pray,
What money advanc'd on our wedding day,
Would yield a pension of fifty pounds sure
Every year's end while our joint lives endure?

Ans. £484 10s.

61. A is indebted to B £2000, due 3 months hence, but gave him £1200 in hand; when should he pay the balance?

Ans. at 7½ months.

62. What is the neat weight of a hhd. of tobacco, weighing gross 18 cwt. 3 qrs., tare 12 lb. per cwt., tret. 4 lb. per 104 lb.?

Ans. 16 cwt. 10$\frac{23}{26}$ lb.

63. What should a broker charge at 7/6 per cent. for transacting business to the amount of £216 10s. 6d. Ans. 16/2¾·49.

64. How many decimal places in the circle, which is the sum of $\cdot\dot{5}276\dot{3}$, $\cdot\dot{6}87124\dot{9}$, $\cdot\dot{1}2345678\dot{9}$? Ans. 315.

65. Bought a chest of tea, weighing gross 2 cwt. 1 qr. 16 lb. for £112; how must I sell it per lb. neat, to clear 15 per cent., tare 21 lb. per cwt., tret. 4 lb. per 104 lb ? Ans. $11/9\frac{1}{4}\frac{31}{121}$.

66. If a cubical piece of silver, whose side is 3 inches, is worth £$8\frac{7}{8}$; what is the side of a cube of the same silver worth £$26\frac{5}{8}$?
Ans. 4·326 + inches.

67. What is the value of a perpetuity of £500 a year, at 5 per cent. per annum ? Ans. £10000.

68. I have received the invoice of a cargo of fruit from Malaga, amounting to 9434 reals Vellon, exchange 3/4 per piaster; what is their value in sterling? Ans. £104 8s. $3\frac{1}{12}$d.

69. A of Amsterdam has order to remit to B of London, £1500 Fl. exchange $36/9\frac{1}{4}\frac{8}{9}$ Fl. per £ sterling, but sends them first to Paris at 56d. per crown, thence to Italy at 100 crowns for 60 ducats, thence to Hamburg at 100d. per ducat, thence to Portugal at 45d. per 400 rees, thence to London at 5/3 per milree; how much was gained by the circular remittance, ⅛ per cent. being charged at each place for commission and charges? Ans. £8 14s. 11¾d.

70. What is the square root of 50177996734720·41 ?
Ans. 7083642·9.

71. What is the cube root of 1371737997260631 ?
Ans. 111111.

72. A May pole there was, whose height I would know,
The sun shining bright, straight to work I did go;
The length of the shadow upon level ground,
Just sixty-five feet, when measured, I found;
My staff I had there, just five feet in length,
The length of its shadow was four feet one tenth;
How high was the May pole, I gladly would know,
And this is the thing you are desired to show ?
Ans. $79\frac{11}{41}$ feet.

73. A draper bought a quantity of cloth for £500, but ⅛ of the whole was so damaged that it sold for no more than 5/ a yard, by which he lost £50; at what rate per ell must he retail the remainder, in order to gain 10 per cent. on the whole?
Ans. 11/7¼d. ⅓.

74. Two shepherds driving home 8 pet lambs, 5 of them belonging to A, and 3 to B, were met by their masters in company with C, each of whom had an unruly mastiff, who set upon

the lambs, and tore them to pieces; C, in order to make good the damage done by his dog, paid 8/; how should it be divided between A and B? Ans. A, 7/ B, 1/.

75. A hare starts 40 yards before a grayhound, and is not observed till she has been up 40 seconds, running at the rate of 10 miles an hour; the dog on view makes after at the rate of 18 miles an hour; how long will the chase last after the dog started, and what is the distance run? Ans. $60\frac{5}{22}$ sec., 530 yards.

76. What principal will yield as much interest in 5 months as £729 17s. 6d. in 5 years? Ans. £8758 10s.

77. A jeweller having bought a diamond for 60 guineas, and after cutting it very neatly, which cost him £5 10s., it weighed 1·5 oz., which he sold for 3·25 shillings per grain; how much did he gain in all, and per cent.?

Ans. in all, £48 10s.; per cent. £70 16s. $0\frac{1}{4}$d. $\frac{1}{3}\frac{2}{7}$.

78. Suppose there is consumed annually in London 900000 barrels of porter at 1/8 per gallon; 10540 tons of cheese at /7½ per lb.; 50000 tons of butter at /10¾ per lb.; 5600000 lb. of tea, at 5/ per lb.; 12000 cwt. of tobacco at 3/4 per lb.; 6980000 gallons of milk at /2 per pint; and 630000 chaldrons of coals at 1/7 per bushel; how much does London expend annually on these articles; and suppose the dealers in them have 15 per cent. for their trouble, what money goes into their pockets?

Ans. Expends £12334300; dealers have £1608821 14s. $9\frac{1}{4}$d. $\frac{1}{2}\frac{2}{3}$.

79. Two ships sail from the same port at the same time, the one sails direct south 25 miles a day; the other direct west 36 miles a-day; how far are they distant at the end of 81 days?

Ans. 3550·116 + miles.

80. A debt is due ¼ every 4 months; what is the mean time for paying the whole at once? Ans. 10 months.

81. Purchased goods for £96 cash, and after keeping them 6 months, sold them for £112 cash; what did I gain, allowing interest at 5 per cent.? Ans. £13 12s.

82. Suppose the national debt to be eight hundred million sterling, as it nearly is, what revenue is required annually, to pay the interest and expense of management at 3½ per cent.; what would government save annually by reducing the interest ½ per cent.; if the whole amount is transferred once a year, what sum does it put into the hands of the stockbrokers; and if it were to be paid off in 50 years, what annual sum at 3½ per cent. would be required? Ans. Revenue £28000000, save £4000000, brokers £2000000, annual sum to pay the debt £37240161 8s. 6¾d.; this is a few shillings more than the truth, from the decimal in the table being extended only to 6 places.

83. What is the least sum divisable by each of the nine digits without a remainder? Ans. 2520.

84. How many more changes can be made of the letters in the word ROMANIS than of those in ROMANI? Ans. 4320.

85. What is the fifth root of 8349416423424? Ans. 384·3.

86. What is the twelfth root of 6127097573297673637 72416? Ans. 96.

87. A merchant in London remits to his correspondent in Amsterdam 1000 moidores, valued at 27/ sterling each; charges of remittance £5 19s. 6d. sterling; when weighed in Amsterdam, after deducting charges, they amounted to 14209 guil. 14 stivers currency; what did the London merchant gain or loss, exchange 33/6 Fl. banco per £ sterling, agio 5 per cent.?

Ans. Lost, £9 8s. 0½d. $\frac{6}{7}$.

88. How much Flemish currency is 798 guil. 17 st. banco, agio 3½ per cent.? Ans. 826 guil. 16 st. 3 penn.

89. If 7 calves cost as much as 9 lambs, and 9 lambs as much as 12 pigs, and a calf, a lamb, and a pig cost together £2 2s. 6d.; what were they a-head? Ans. Calf 18/, lamb 14/, pig 10/6.

90. A sold wheat which cost him £246 12s. to B, and B sold it to C, who disposed of it to D for £391 11s. 10d.; required for how much A and B sold it, each of the three merchants having gained at the same rate per cent.?

Ans. A for £287 14s., B for £335 13s.

91. What is the rebate of £795 11s. 2d. for 11 months, at the rate of 6 per cent. per annum? Ans. £41 9s. 5¾d.

92. A gentleman going to market dressed in a new coat and vest, on which were 12 dozen gold buttons, met with a corn man who fancied it, and having demanded the price, was answered, if you give me a wheat corn for the first button, two for the second, four for the third, and so on for the twelve dozen, the whole shall be yours—to which the baker readily assents; now supposing 544800 wheat corns make a bushel, how many bushels would pay for the cloths, and what would they cost, if wheat sell for 10/ per bushel?

Ans. Bushels 40933830114777208409573638532761309665 ¾+.
Cost £20466915057388604204786819266380654832 17s. 6d.

93. What is that number of which 9 is three fifths?

94. Place three 3's in such a manner as to make just eleven.

95. A snail in going up a May pole 36 feet high, ascended six feet every day, but came down three every night; how many days will he require to reach the top?

96. A poor man having a fox, a goose, and a peck of corn, came to a river, over which he can carry only one at a time; how is he to do this, that the fox may not be left with the goose, nor the goose with the corn?

97. Place the 9 digits in such order that any three figures in a right line shall make 15.

98. A gentleman's servant went to market with orders to buy 20 fowls for 20d., he fulfilled his orders by purchasing pigeons at 4d., larks at $\frac{1}{2}$, and sparrows at $\frac{1}{4}$ a piece, how many had he of each?

99. If 12 apples cost as much as 21 pears, and three pears cost a penny; what is the price of 70 apples?

100. Two merry companions are to have equal shares of 8 gallons of brandy, which are in a vessel containing exactly 8 gallons; now to divide it, they have only two other empty vessels, the one of which holds 5 and the other 3 gallons; how shall they divide the brandy equally by the help of these three vessels?

ADDITION TABLE.

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

SUBTRACTION TABLE.

	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	4	5	6	7	8	9
2	-	0	1	2	3	4	5	6	7	8
3	-	-	0	1	2	3	4	5	6	7
4	-	-	-	0	1	2	3	4	5	6
5	-	-	-	-	0	1	2	3	4	5
6	-	-	-	-	-	0	1	2	3	4
7	-	-	-	-	-	-	0	1	2	3
8	-	-	-	-	-	-	-	0	1	2
9	-	-	-	-	-	-	-	-	0	1

ADDITION TABLE is read thus, 1 and 1 are 2, 1 and 2 are 3, 1 and 3 are 4, &c.

SUBTRACTION TABLE is read thus, 1 from 1 leaves 0, 1 from 2 leaves 1, 1 from 3 leaves 2, 1 from 4 leaves 3, &c.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

MULTIPLICATION TABLE is read thus, twice 1 are 2, twice 2 are 4, twice 3 are 6, twice 4 are 8, twice 5 are 10, &c. Then three times 1 are 3, three times 2 are 6, three times 3 are 9, three times 4 are 12, &c.

TABLES OF MONEY, WEIGHTS AND MEASURES,

in which the new standards enforced by act of Parliament, 5 Geo. IV. cap. 74, and 6 Geo. IV. cap. 12, are carefully compared with the old standards of England, and Scotland.

MONEY TABLE.

qrs.	d.		
4	1	sh.	
48	12	1	£
960	240	20	1

2/6 = half-a-crown, 5/ = a crown, 6/8 = a noble, 13/4 = a mark sterling, 21/ = a guinea, 25/ = a jacobus, 27/ = a moidore, 36/ = a joannes.

SCOTCH MONEY. 2 pennies Scots = 1 bodle, 2 bodles = 1 plack, 3 placks, or 12 pennies = 1 sh., 13 sh and 1 plack = 1 mark, 20 sh. = 1 pound Scots, 12 pounds, or 18 marks Scots = £1 sterling. Scots money is the $\frac{1}{12}$ of sterling in value.

Troy Weight.

grains	dwt.	oz.	lb.
24	1		
480	20	1	
5760	240	12	1

Troy weight is used for gold, silver, jewels, and liquors.

Avoirdupois Weight.

Troy Grs.	Dram.	ounce.	pound.	stone.	qr.	cwt.	ton.
$27\frac{11}{32}$	1						
$437\frac{1}{2}$	16	1					
7000	256	16	1				
98000	3584	224	14	1			
196000	7168	448	28	2	1		
784000	28672	1792	112	8	4	1	
15680000	573440	35840	2240	160	80	20	1

Avoirdupois weight is used for every purpose, excepting weighing gold, silver, jewels, and liquors.

Apothecaries Weight.

T. gr.	scr. ℈	dr. ʒ	oz. ℥	lb.
20	1			
60	3	1		
408	24	8	1	
5760	288	96	12	1

Apothecaries use this weight for compounding their medicines, but buy and sell by avoirdupois.

Wool Weight.

Troy grs.	lb.	clove.	stone.	todd.	wey.	sack.	last.
49000	7	1					
98000	14	2	1				
196000	28	4	2	1			
1274000	182	26	13	$6\frac{1}{2}$	1		
2548 000	364	52	26	13	2	1	
30576000	4368	624	312	156	24	12	1

Hay and Straw Weight.

	T. grs.	lbs	truss.	load.
Straw =	252000	36	1	
Old Hay =	392000	56	1	
New Hay =	420000	60	1	
			36	1

In Edinburgh, straw is still sold by the winling, or bundle, and

Cubic or Solid Measure.

cub. in.	c. foot.		
1728	1		
46656	27	1	Cubic Yard.
69120	40	1	Load or Ton, Rough Timber.
86400	50	1	Do. Hewn do.
72576	42	1	Ton Shipping.

kemple; 40 winlings make a kemple. Standard kemple 15 tron stones, but weighs from 18 to 22 stones tron, average 20.

Lineal Measure.

inches.	foot.	yard.	pole.	furl.	mile.	league.	deg.
12	1						
36	3	1					
198	$16\frac{1}{2}$	$5\frac{1}{2}$	1				
7920	660	220	40	1			
63360	5280	1760	320	8	1		
190080	15840	5280	960	24	3	1	
4403520	366960	122320	22240	556	$69\frac{1}{2}$	$23\frac{1}{6}$	1

In lineal measure, 12 lines, or 3 barley-corns make 1 inch, 4 inches 1 hand for measuring horses, 6 feet 1 fathom in measuring depths; 60 miles 1 geographical or nautical degree.

Cloth Measure.

In.	nail.	qr.		
$2\frac{1}{4}$	1			
9	4	1		
27	12	3	1	Fl. Ell.
36	16	4	1	Imp. Yard.
45	20	5	1	Eng. Ell.
54	24	6	1	Fr. Ell.

Superficial Measure.

sq. inches.	sq. foot.	yard.²	pole.	rood.	acre.
144	1				
1296	9	1			
39204	$272\frac{1}{4}$	$30\frac{1}{4}$	1		
1568160	10890	1210	40	1	
6272640	43560	4840	160	4	1

$272\frac{1}{4}$ square feet = 1 rood of brick work, and 100 square feet = 1 square of flooring in England, and in Scotland 36 square yards = 1 rood of mason work.

Standard Wine Measure.

Old, cub. inches.	New, cubic inches.	gills or gl.	pint or pt.	quart or qt.	gallon.		
7.21875	8.6648125	1					
28.875	34.65925	4	1				
57.75	69.3185	8	2	1			
231	277.274	32	8	4	1		
2310	2772.74	320	80	40	10	1	Anker.
4158	4990.932	576	144	72	18	1	Rundlet.
9702	11645.508	1344	336	168	42	1	Tierce.
14553	17468.262	2016	504	252	63	1	Hogshead.
19404	23291.016	2688	672	336	84	1	Puncheon.
29106	34936.524	4032	1008	504	126	1	Pipe or Butt.
58212	69873.048	8064	2016	1008	252	1	Tun.

The imperial gallon may in practice be considered 20 per cent., or $\frac{1}{5}$ greater than the old wine gallon ; but if perfect accuracy is required, we must have recourse to the cubic inches : thus to find how many imperial gallons in 4 hogsheads, old measure, I multiply 14553 the cubic inches in a hogshead, old measure, by 4, and divide the product by 277.274 the cubic inches in an imperial gallon, and so for any other denomination.

Standard Ale and Beer Measure.

Old, cub. inches.	New, cub. inches.	gill.									
8.8125	8.6648125	1	pint.								
35.25	34.65925	4	1	quart.							
70.5	69.3185	8	2	1	gallon or gal.						
282.	277.274	32	8	4	1	firken or firk.					
2538	2495.466	288	72	36	9	1	kilderken or kil.				
5076	4990.932	576	144	72	18	2	1	barrel or bar.			
10152	9981.864	1152	288	144	36	4	2	1	hogshead or hhd.		
15228	14972.796	1728	432	216	54	6	3	1½	1	butt or bt.	
30456	29945.592	3456	864	432	108	12	6	3	2	1	tun or t.
60912	59891.184	6912	1728	864	216	24	12	6	4	2	1

In practice the imperial gallon may be considered $1\frac{12}{17}$ per cent. less than the old ale gallon; but for perfect accuracy recourse must be had to the cubic inches as shown in wine measure.

Standard Dry Stricken Measure.

Old, cubic inches.	New, cub. in.	pint.										
33.6003152	34.65925	1	quart.									
67.2006304	69.3185	2	1	gall.								
268.8025216	277.274	8	4	1	peck.							
537.6050432	554.548	16	8	2	1	bushel or bu.						
2150.4201728	2218.192	64	32	8	4	1	strike or str.					
4300.8403456	4436.384	128	64	16	8	2	1	coom.				
8601.6806912	8872.768	256	128	32	16	4	2	1	quarter or qr.			
17203.3613824	17745.536	512	256	64	32	8	4	2	1	chaldron.		
68813.4455296	70982.144	2048	1024	256	128	32	16	8	4	1	wey.	
86016.806912	88727.68	2560	1280	320	160	40	20	10	5	1¼	1	last
172033.613824	177455.36	5120	2560	640	320	80	40	20	10	2½	2	1

Also 4 gills = 1 pint, and 2 pottles = 1 gallon.

The imperial bushel may be considered in practice as $3\frac{1}{18}$ per cent. greater than the Winchester bushel, but for perfect accuracy they must be compared by the cubic inches in each.

Standard Dry Heaped Measure.

Old, Heaped. Cubic inches.	New, Legally Heaped. Cubic inches.	half quartern.	quartern.	hf. peck.	peck.	hf. bushel.	bu.	sack.	ch.
96.3094497	87.9839782	1							
192.6188995	175.9679564	2	1						
385.2377991	351.9359129	4	2	1					
770.4755983	703.8718258	8	4	2	1				
1540.9511966	1407.7436516	16	8	4	2	1			
3081.9023932	2815.4873032	32	16	8	4	2	1		
9245.7071796	8446.4619096	96	48	24	12	6	3	1	
110948.4861552	101357.5429152	1152	576	288	144	72	36	12	1

In practice, the imperial bushel legally heaped may be considered as $9\frac{6}{13}$ per cent. less than the old dry measure bushel similarly heaped, but for perfect accuracy recourse must always be had to the cubic inches.

Scotch Troy or Dutch Weight.

T. grs.	drop.	ounce.	pound.	stone.
29.75	1			
476.	16	1		
7616	256	16	1	
121856	4096	256	16	1

Scotch Superficial Measure.

Standard sq. in.	square ell.	square fall.	rood.	acre.
1374.110761	1			
49467.987396	36	1		
1978719.49584	1440	40	1	
7914877.98336	5760	160	4	1

The avoirdupois lb. is less than the Dutch lb. by $18\frac{4}{5}$ per cent. —The divisions of tron weight are the same as Scotch troy or Dutch weight, but the standard tron pound is 10216 troy grains and nearly $\frac{1}{2}$.—The length of the Scotch ell is here taken at 37.069 imperial inches.—The imperial acre is less than the Scotch by $26\frac{2}{11}$ per cent.

Scotch Standard Liquid Measure.

Any quantity of Scotch liquid measure may be readily converted into imperial, or the reverse, by means of the cubic inches in each. Three imperial gallons are $\frac{1}{2}$ a gill more than 1 Scotch gallon nearly.

Cub. inches.	gill.	mutchken.	choppin.	pint.	qt.	gal.	barrel.	hhd.
6.46275	1							
25.851	4	1						
51.702	8	2	1					
103.404	16	4	2	1				
206.808	32	8	4	2	1			
827.232	128	32	16	8	4	1		
6617.856	1024	256	128	64	32	8	1	
13235.712	2048	512	256	128	64	16	2	1

Scotch Standard Measure for Wheat, Rye, Peas, Beans, Salt, Grass-Seed, &c.

Cubic inches.	lippy.	hf. pk.	pks.	firlot.	boll.	chalder.
137.3334375	1					
274.666875	2	1				
549.33375	4	2	1			
2197.335	16	8	4	1		
8789.34	64	32	16	4	1	
140629.44	1024	572	256	64	16	1

SCOTCH STANDARD MEASURE FOR BARLEY, OATS, MALT, FRUIT, POTATOES, &c.

Cubic inches.	lippy.	hf. pk.	pk.	fir.	boll.	chal.
200.34525	1					
400.6905	2	1				
801.381	4	2	1			
3205.524	16	8	4	1		
12822.096	64	32	16	4	1	
205153.536	1024	512	256	64	16	1

The imperial bushel is 44½ per cent. less than the Linlithgow barley firlot. The imperial bushel legally heaped is 13⅞ per cent. less than the Linlithgow barley firlot.

YARN MEASURE.

Lint Yarn.

Feet.	thread.	cut.	heer.	slip.	spindle.
7½	1				
900	120	1			
1800	240	2	1		
10800	1440	12	6	1	
43200	5760	48	24	4	1

Cotton Yarn.

Feet.	thread.	skeen.	hank.	spindle.
4½	1			
360	80	1		
2520	560	7	1	
45360	10080	126	18	1

TIME TABLE.

60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 7 days = 1 week, 4 weeks = 1 month, 13 months and 1 day, or 52 weeks 1 day, or 365 days = 1 year, 366 days one leap year, 12 calendar months 1 year, 365¼ days = 1 common year, 365 days 5 hours 48 minutes 48 seconds = 1 solar year.

Thirty days hath September,
April, June and November;
All the rest have thirty-one,
Excepting February alone,
Which has only twenty-eight days clear,
But twenty-nine in each leap year.

Quarterly Terms.

Scotland.	*England.*
Candlemas, 2d February.	Lady-day, 25th March.
Whitsunday, 15th May.	Midsummer, 24th June.
Lammas, 1st August.	Michaelmas, 29th September.
Martinmas, 11th November.	Christmas, 25th December.

Astronomy.

60″ seconds = 1 minute, 60′ minutes = 1 degree, 30° degrees = 1 sign, 12 signs = 1 circle of the zodiac, 360 degrees the earth's circumference.

Memorandum.

12 sheets = 1 quire of long paper, 48 sheets = 1 quire of post paper, 20 quires = 1 ream, 2 reams = 1 bundle, 5 bundles = 1 bale. 60 skins = 1 roll of parchment.—12 articles = 1 dozen, 12 dozen = 1 gross, 12 gross = 1 great gross.—1 firken of butter = 56 lb., 1 firken of soap = 64 lb., 1 stone of glass = 5 lb., 1 gallon train-oil = 9 lb., 1 fother lead = 19½ cwt., 1 faggot of steel = 120 lb., 1 barrel gun-powder = 112 lb., 1 last of gun-powder = 2400 lb., Apothecaries fluid measure runs thus, 60 fluid minums F ♏ = 1 fluid dram F ʒ, 8 F ʒ = 1 fluid ounce F ℥. 16 F ℥ = 1 fluid pint O, 8 O = 1 gallon.

NOTES.

No branch of education is of more importance in a great commercial nation, than arithmetic. Without a knowledge of it, the statesman, the merchant, and the mechanic, as well as the philosopher, are left to grope their way in doubt and uncertainty. Too much attention, therefore, cannot be bestowed upon it in the education of youth, to fix upon their minds its principles as a science, and to render them confident, accurate, and expert in their application of it to the various departments of business. Under the influence of these impressions, this work has been prepared, with a view to render acquirements in arithmetic both more agreeable, and more substantial than can be expected from the school-books in general use.

The ancient nations of Europe were entirely unacquainted with our present admirable system of numbers, and had recourse to many different methods of calculation. The very best of these were clumsy and unwieldy, even in the common business of life, and presented an insurmountable barrier to the advancement of physical science. Arithmetic was first brought into Europe by the Arabs; but when, and by whom it was first invented, are still unknown, and will probably remain for ever a mystery.

CHARACTERS USED IN ARITHMETIC.—Parallel lines ($=$) placed between two numbers signify that the numbers between which they stand are equal; thus £1 $=$ 20s. and are called the sign of equality.

Addition ($+$) placed between two numbers signifies that they are to be added; thus $3 + 6 = 9$, and is called plus, or St. George's cross.

Subtraction (—) placed between two numbers signifies that the number on the right, is to be subtracted from the one on its left; thus $6 - 2 = 4$, and is called a straight line, or minus.

Multiplication ($\times$) placed between two numbers signifies that they are to be multiplied together; thus $6 \times 3 = 18$, and is called St. Andrew's cross.

Division ($\div$) placed between two numbers signifies that the number on the left of it, is to be divided by the one on its right; thus $12 \div 3 = 4$; division is also represented by) (, and $\frac{8}{2}$, or any such position of numbers.

Ratio (: :: :) denotes proportion; thus $2 : 4 :: 3 : 6$, and is read 2 is to 4 as 3 is to 6.

Root ($\surd$, or $\frac{1}{2}$) placed before a number, or $\frac{1}{2}$ on its right corner, signifies that the square root of the number is to be extracted; thus $\surd 16$, or $16^{\frac{1}{2}} = 4$, and is called the radical sign.

$\sqrt[3]{}, \sqrt[4]{}, \sqrt[5]{}, \sqrt[6]{}$ $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ placed before any number signify respectively, the cube, 4th, 5th, 6th root of the number before which they are placed, the same thing is signified by writing $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c. on the right corner of the number. These are all called radical signs.

A small figure placed on the right of another figure or number signifies the square, cube, &c. of that figure or number; thus 6^2, 8^3, 19^4, signify the square of 6, the cube of 8, and the fourth power of 19. $6^{\frac{2}{3}}$, $8^{\frac{4}{2}}$, $12^{\frac{5}{4}}$, signify the cube root of the square of 6, the square root of the 4th power of 8, the 4th root of the 5th power of 12, &c.

Vinculum ( ) or —— signifies that the numbers within the one, and under the other of these signs are all connected; thus $\surd (25 + 15 \times 3 \div 8) \times \surd (54 \times 3 \div 7)$ or $\surd \overline{25 + 15 \times 3 \div 8} \times \surd \overline{54 \times 3 \div 7}$ signifies that the sum of 25 and 15 is to be multiplied by 3 and the product divided by 8, and the square root of the quotient to be multiplied by the square root of 54 multiplied by 3 and divided by 7.

NUMERATION.—Before the pupil begins to learn the numeration scales, he should practice making the figures until he becomes familiar with them, and can make them neatly and expeditiously.

The numeration scales are extended to more places than what commonly occur in ordinary calculations. In the first

scale, the name of each place and its number in the scale are given; and in the second and third scales, the name of each period, and its number in the scale. By means of these scales the pupil may be trained to tell readily the place in the scale which any proposed figure occupies, from having its local value given; or its local value from its number in the scale. These exercises will be found useful to prepare the pupil for expressing large numbers accurately by figures.

The scales are divided into periods of six places each, according to the practice established in Britain, and invariably followed by our writers on arithmetic for more than 200 years, with the exception of a few late authors, who have divided the scale into periods of three places each, as it is on the continent of Europe. The reasons assigned for this deviation from the established custom are, that the division into periods of six places is neither so simple nor so uniform as into periods of three places. So far as uniformity is concerned, I can see no difference between uniformly dividing into periods of three, or of six places. But that the division into periods of six places is not so simple as into periods of three places would require, to support it, something more than bare assertion. It is evident that the first six places are learned in the same way, whether we divide into periods of three or six places; and when these are got off, the only remaining difficulty is that of learning the names of the higher periods, and surely it cannot be simpler to learn two names for every six succeeding figures, than to learn one.

It was in order to avoid the labour of inventing and learning names for every place in the scale, that numbers were first divided into periods of three figures, and a proper name given to the first two periods. These two periods and their names were found sufficient for the ordinary purposes of business for ages, and no appropriate names were invented for higher places. But when the sciences began to expand, and mathematical calculations to be introduced wherever they were applicable, great inconvenience was experienced in expressing the large numbers which then frequently occurred. This drew the attention of men of learning to the subject, who divided the scale into periods of six places, and assigned an appropriate name to each period, and from that time six figures have been considered the standard period in Britain, and three a semiperiod. But the greatest evil which results from changing at pleasure the division of the scale is, that it leads directly to all manner of confusion, and uncertainty regarding high numbers

expressed in words. Pupils taught in Britain, according to the ternary division of the scale, are utterly unqualified, either to express large numbers intelligibly in words, or to write them in figures when expressed in words. In proof of this, I shall take for example a remark in one of the works in question, where it is said "the number of seconds which have elapsed since the creation, does not amount to the fifth part of a trillion." Now this is perfectly unintelligible, (unless accompanied with the index of the scale), for by the established division of the scale, they are less than the five hundred thousandth part of a trillion. Again, suppose our national debt were to increase so as to give a unit in the next higher place in the scale, it would, according to this division, be read one billion ; and suppose a foreigner were to see this stated without explanation, and to calculate the interest upon it at 3½ per cent., he would find to his astonishment that it amounted to no less than 35 thousand millions annually, or more than 43 times our present national debt. But enough has been said to show the danger of innovations in the division of the numeration scale, upon which our very language regarding numbers is founded.

NOTATION.—Many rules have been given to assist the young pupil to express numbers in figures, but these are seldom understood, and therefore of little advantage. The best method of learning notation is, first to have a distinct knowledge of the local value of figures, and the names of the several places in the numeration scale, and then to have recourse to frequent exercises, without which no rules will be of any use. Instead of detaining pupils long upon these rules at first, as proficiency in them is not to be expected from mere beginners, without much loss of time, I would prefer causing them to read the answers to all their exercises, and occasionally to give them exercises in notation, and thus lead them gradually, and almost insensibly to read and write numbers accurately.

After the pupil is well acquainted with our own notation, and numeration, he may be exercised with advantage in Roman notation.

ROMAN NOTATION.—As the Roman notation is still in partial use, I shall here subjoin an epitome of it. Their numbers were expressed by the following letters of their alphabet, I. V. X. L. C. D. M. by the various combinations and markings of which all their intermediate and higher numbers were expressed.

Simple value of the characters.	I.	V.	X.	L.	C.	D or IↃ.	M or CIↃ or ∞.
	1.	5.	10.	50.	100.	500	1000.

Rule 1st.—As often as any character is repeated, so often is its value repeated, thus II = 2, XX = 20, CCC = 300, DD = 1000, MM = 2000.

Rule 2d.—A less character placed before a greater, signifies their difference, and placed after it, signifies their sum, thus IV = 4, IX = 9, XIX = 19, XL = 40, XC = 90, CM = 900, VI = 6, XI = 11, XXI = 21, LX = 60, CX = 110, MC = 1100.

Rule 3d.—Every inverted Ↄ placed after IↃ increases its value ten fold, but when not inverted, it has the same effect as a less character after a greater. Thus IↃↃ = 5000, IↃↃↃ = 50000, IↃↃↃↃ = 500000, IↃC = 600, IↃCC = 700.

Rule 4th.—Every pair of C's, the one annexed and the other prefixed to CIↃ increases its value ten fold. Thus CCIↃↃ = 10000, CCCIↃↃↃ = 100000, CCCCIↃↃↃↃ = 1000000.

Rule 5th.—A line or bar placed over a character, increases its value 1000 fold. Thus $\overline{\text{I}}$ = 1000, $\overline{\text{V}}$ = 5000, $\overline{\text{X}}$ = 10000, $\overline{\text{C}}$ = 100000, $\overline{\text{M}}$ = 1000000.

Rule 6th.—Points are sometimes used to divide the characters into periods, thereby materially altering their value. Thus XV.XVI.IↃCCLXVI = 1516766, and XIX.XXV. DCCCLXXXV = 1925885.

For a clear development of the various steps in the progress of numbers, I would refer the curious, to the Philosophy of Arithmetic by Professor Leslie.

SIMPLE ADDITION.—A simple number is that which consists of only one denomination, as £659. An abstract number is that which has no particular name or object assigned to it, and is considered purely as number. Simple addition is equally applicable to both these classes of numbers.

Addition is founded on the principle that "a whole is equal to the sum of all its parts." The reason why the same places must be written under each other is, that thereby all the units stand in one column, all the tens in one column, &c. and thus come to be added according to their local value. Example,—Suppose it is required to add 234, and 645 together, the sum

will be 879, because 9 in the place of units is equal to all the units in 5 and 4, and 7 in the place of tens is equal to the sum of 4 and 3 in tens place, and 8 in the place of hundreds is equal to the sum of 6 and 2 in hundreds place. The reason for placing the right-hand figure under the column added, and for carrying all on the left of it to next column, is manifest from the nature of the numeration scale. For when a sum consisting of two places is obtained by addition, it is evident that only one of them can occupy the same place in the scale as the column added, and that the left-hand figure belongs to the next higher place or column, and must be added to it. When the sum of any column consists of three places, the middle figure only belongs naturally to the next higher column, and the left-hand figure to the second higher column, but it is better to add the number expressed by both figures to next column, as it prevents the risk of forgetting to add it in its proper place. By this method the pupil is saved the trouble of finding mentally, how many tens are in the sum of any column, and how many units remain, which often lost much time, and discouraged him by the appearance of a mystery hanging over every step of the process. But the method given in the rule, and upon the same principle, renders the process so simple, that the youngest pupil will perform it without difficulty after an example or two has been explained. It is also much easier by this method to make the pupil comprehend the reason why he carries from one column to another. The method of formally taking the tens out of the several sums was not only perplexing to the pupil, but also injudicious, as it involved division at every step of the process, before any rule had been given for it. In the proof, I have preferred cutting off the under, instead of the upper number, as it brings the numbers to be added for the proof nearer each other, which is of advantage when the columns are long.

The reason of this method of proof is, that the difference of any two numbers added to the least, makes it equal to the greatest. But it is evident that the number cut off is the difference between the two sums when the work is right, and therefore being added to the least, must make it equal to the greatest.

There are other methods of proving addition besides that given in the text. 1 Begin at the top of the columns, and add them downward in the same manner as you added them up, and if the sum thus obtained be the same as the other, the work is considered to be correct. 2. If the columns are long divide

them into parcels of six or eight places each; find the sum of each of these parcels, then add these sums together, and if their sum is equal to the first the work is right. 3. Take the nines out of the uppermost number, or out of the sum of its digits, and set down the remainder, if any; do the same with all the numbers to be added; take the nines out of the sum of these remainders, or out of the sum of their digits, and set down the remainder; then take the nines out of the sum obtained by addition, and set down the remainder, and if it is equal to the preceding remainder the work is right. This property belongs to 9 and 3 exclusively, but it is too tedious for practice. The reason of this proof is, that any number divided by 9 or 3 leaves the same remainder as the sum of its digits divided by 9 or 3, and any figure in a number, excepting 9, taken in its name value, is the same that will remain after taking the nines out of its local value. Again, the sum of all the figures in any number is equal to the sum of the remainders after taking the 9's out of the local value of each figure, and the nines taken out of the sum of these remainders leaves the same remainder as the original number divided by 9. Also, if there are two or more numbers, and the nines be taken out of each of them, and again out of the sum of the remainders, this last remainder must be equal to the remainder after taking the nines out of the sum of all these numbers; which is the rule for the third method of proof. I shall illustrate the reasons here given by an example.

1st, 8765 $\div$ 9 leaves a remainder of 8, and $8+7+6+5=26$ $\div$ 9 leaves a remainder of 8.

Thus the excesses of nines in any number, and in the sum of its digits, are equal. 2d, If we take the nines out of the local value of any figure, suppose 8000, the remainder will be 8, for 8 in thousands place is equal to ten 8's in hundreds place, and taking away nine of these leaves 8 in hundreds place, which is equal to ten 8s in tens place, from which taking away nine of them leaves 8 in tens place, equal to ten 8s in units place, and nine of these taken away leaves 8, but 8 is the name value of the significant figure in 8000: the same is true of any other number and figure, excepting 9, which leaves no remainder. 3d, It is evident from the above, that the significant figures which compose any number must be the remainders, after taking the nines out of their local value. Thus in 746, taking the nines out of 700 leaves 7, and out of 40 leaves 4, and out of 6 leaves 6, or the same figures of which it was composed, but the nines taken out of 746, and out of 17,

the sum of its digits, leaves the same remainder 8, and this is true of every possible number. 4th, From this it is manifest, that if there are two or more numbers, and the nines be taken out of each of them, and again out of the sum of their remainders, the last remainder must be equal to the remainder after taking the nines out of the sum of these numbers, because the sum is equal to all its parts. From which the truth of the rule is evident. What is here proved of the nines is equally applicable to 3, for taking away 3 three times is the same as taking away 9.

EXAMPLE.—Add together 6875+3672+9704+3215+3639.

6875	Rem.	8
3672	—	0
9704	—	2
3215	—	2
3639	—	3
——		—
27105	—	6

We here find that the excess of nines in the sum of the digits in the first number is 8, and in the second 0, in the third 2, in the fourth 2, and in the fifth 3; and the sum of these is 15, which leaves an excess of 6, and 6 is also the excess in the total sum, from which we conclude that the work is right. Although the principles of this method of proof are perfectly correct, and will never fail to prove it right when it is so, yet it is possible that it may prove it right, when it is wrong; for the proof will come out correctly although 0 is written for 9, or 9 for 0; or the figures transposed, or when there are two errors, one too little, and the other as much too great. Besides, as this method involves division, it is not suited to the capacities of beginners.

SIMPLE SUBTRACTION.—There is no absolute necessity why the least number should be placed under the greater, as custom makes the operation equally simple, although the least number is uppermost, but whatever be the position of the numbers in this respect, we must always take the figure in the subtrahend, from the one in the minuend, which occupies the same place in the scale, that is, units from units, tens from tens, &c. The rule generally given for subtraction, when a figure in the under number is greater than its corresponding figure in the upper number, is "to add 10 to the upper figure, and take the under from the sum, or to take the under figure from ten, and add the remainder to the upper figure, and then add one to the next under figure before subtracting it." Although this method maintains the equilibrium, is sanctioned by custom, and rendered familiar by practice, yet, to most beginners it is perfectly unintelligible. Besides, the reason of the rule is seldom made so clear as to enable the young mind to

comprehend, why ten in preference to any other number is added to the upper figure, or how this ten is repayed by adding one to the next under figure. But the reason of the rule given in the text is evident from the very nature of numeration, and one or two examples explained at full length, is, in most cases, sufficient to remove every difficulty, and to make the pupil comprehend it perfectly. I am aware the rule given in the text is not new, it has had its advocates as well as its opponents, and it is only when we reflect on the almost invincible power of established customs, that we can account for its not having been universally preferred to the common method, which has evidently been invented with great care, to make a mystery of what is naturally simple. Molineux, in taking notice of this method, maintains that the established method is more eligible, particularly when the next digit in the upper line happens to be a cipher. But I can see no difficulty in this case, as ciphers in the upper line invariably become nines when one is borrowed from the significant figure on their left. I shall illustrate this by an example.

5.9.9.9.13.5 9.12
60003602
36574254
23429348

Suppose it were required to take 36574254 from 60003602. Having placed the numbers in the usual manner, proceed thus, 4 from 2, I cannot, borrow one from 6, the first significant figure on the left, leaves 5, and the 1 being placed on the left of the cipher makes it 10, from which take one, leaves it 9, and the 1 being placed on the left of the 2 makes it 12, we have then 5, 9, 12 as represented by the small figures, and 4 from 12 leaves 8, 5 from 9 leaves 4, 2 from 5 leaves 3; again 4 from 3 I cannot, borrow 1 from 6 leaves 5, and makes the first 0 ten, from which take 1, leaves 9, and makes next 0 ten, from which take 1 leaves 9 and makes next 0 ten, from which take 1, leaves 9, and makes the 3 thirteen, as represented by the small figures, then 4 from 13 leaves 9, 7 from 9 leaves 2, 5 from 9 leaves 4, 6 from 9 leaves 3, and 3 from 5 leaves 2.—From this it is evident that borrowing 1 from a significant figure on the left of one or more ciphers, leaves them all 9's, and the figure on the left is diminished by one; and the under figures never change their value. The reason of the rule is this. When any under figure is greater than the one above, it is evident that one must be taken from the next higher place, which is equal to ten in the next inferior place, and any figure added to 10 is the same as placing 1 to the left of that figure; and where there are one or more ciphers, the 1 taken from the next significant figure on the left, makes them 10,

100, 1000, &c. from which, taking away 1 leaves 9, 99, 999, &c.· thus invariably leaving a 9 for every cipher. I prefer the method given in the above example when it is to be explained to young pupils.

The reason of the first method of proof is, that, the difference of any two numbers, added to the less, gives a number equal to the greater.—The reason of the second method is, that the difference of any two numbers taken from the greater, leaves a number equal to the less.

The number to be subtracted is called the subtrahend, the number from which it is to be taken, is called the minuend, and the number remaining is called the difference, or remainder.

When two or more numbers are given to be subtracted from two or more numbers; it is generally best to add together all the numbers belonging to the minuend, and then all the numbers belonging to the subtrahend, and to take the sum of the one from the sum of the other.

The pupil should be accustomed to subtract with the less number placed above the greater, as this is often found convenient in business.

$$\begin{array}{r} 756 \\ 756 \\ 756 \\ 756 \\ 756 \\ 756 \\ 756 \\ \hline 5292 \\ \hline \end{array} \qquad \begin{array}{r} 756 \\ 7 \\ \hline 5292 \end{array}$$

MULTIPLICATION.—Multiplication, as I have already observed, is a short method of performing addition, when the same sum is to be repeated, any given number of times. Suppose it were required to find the sum of 7 times 756. To do this by addition, we must repeat 756 seven times as shown in the margin, and then proceed as in addition, which gives the sum 5292. Again by multiplication, we write down 756, and multiply it by 7, which gives us 5292 the same as before. If the sum given were to be repeated two or three thousand times, it is evident that the labour by addition would be quite intolerable, and therefore is never employed in such cases.

DEMONSTRATION OF THE GENERAL RULE.—When the multiplier is a single figure, it is evident the product will be correctly obtained by the method given in the rule, for the figures in every place of the multiplicand are taken as often as if the operation had been performed by addition, as we see from last example. There the number 756 being repeated seven times, we have in the column of units, seven sixes, in the column of tens, seven fives, and in the column of hundreds seven sevens, which is evidently the same as multiplying each figure in the given number by seven, and the rule for writing down,

and adding to next column, or product is the same in both, therefore the total sum or product must be the same. When the multiplier consists of several places, having found the product of the multiplicand by the figure in units place as before, we proceed in the same manner with the figure in tens place, but as this figure, from the place it occupies, is ten times its name value, its product must also be ten times its simple value, which is obtained by writing the right hand figure in tens place, or directly under the figure by which we are multiplying. For the same reason, the product by the figure in hundreds place, is a hundred times its simple value, therefore its right hand figure must stand in hundreds place, or under the figure multiplied by, as before. By proceeding in this manner with each figure of the multiplier, and writing the products according to the local value of the figures by which they are produced, it is evident that we obtain the true value of each product, and consequently the sum of these products must give the total product required. The following example will render the reason of the rule as obvious as possible. Multiply 3426 by 6234.

3426	
6234	
13704	= 4 times multiplicand.
10278	= 30 times do.
6852	= 200 times do.
20556	= 6000 times do.
21357684	= 6234 times do.

We first multiply by 4, which gives us 4 times the multiplicand, then by 3 which gives three times the multiplicand, but the 3 stands in tens place, which makes it 30, we therefore write the whole product by 3, one place farther to the left, which increases it ten fold, or makes it 30 times the multiplicand; we then multiply by 2 which gives us twice the multiplicand, but it should have been two hundred times, because the 2 stands in hundreds place, we therefore remove the whole product two places to the left, which makes it 200 times the multiplicand; again we multiply by 6 which gives 6 times the multiplicand, but it should be 6000 times, because the 6 is in thousands place, we therefore remove the whole product three places to the left, which makes it 6000 times the multiplicand. We have now 4 times, 30 times, 200 times, and 6000 times the multiplicand, which, added together, gives 6234 times the multiplicand. This shows clearly the reason why the right hand figure of each product is placed under the figure by which it is produced.—Proof. The method of proof given in the text, depends entirely for its accuracy, upon the probability of

not bringing out the same result incorrectly, after making the factors change places. There are other two methods of proof besides that which is already given, neither of which can be practised by the pupil at the present stage, as they both presuppose a knowledge of division.

1st METHOD.—Divide the product by either of the factors, and the quotient will be the other factor, without remainder, when the work is right.

This is perhaps the most natural method of proving multiplication. The reason is this, the product of two numbers, divided by any one of them must give the other.

2d METHOD.—Take the nines out of the two factors, and if there are any remainders, multiply them together, and if the excess of nines in their product, and the excess of nines in the total product be the same, the work is right.

This rule is equally applicable either to 9 or 3, and depends on the peculiar property of these two figures explained in addition. This method is subject to the same objections here as in addition, and also when the products are wrong placed.

EXAMPLE for illustrating the process.

$$\begin{array}{ll} 987 = 6 = \text{excess of nines in the multiplicand.} & \\ \underline{872} = 8 = \text{excess of nines in the multiplier.} & \\ 7895 & \\ \underline{71064} \quad 6 \times 8 = 48 \div 9 = 3 \text{ excess of nines in the factors.} & \\ 860664 = \qquad 3 \text{ excess of nines in the total pro-} & \end{array}$$

duct, and being the same as the excess in the factors, the work is right.

CONTRACTION 1.—The reason of this contraction is sufficiently manifest from the nature of notation; for writing a cipher on the right of any number, removes each figure in it one place further to the left in the scale, and thereby increases its value ten fold, and writing two ciphers on the right of any number, removes each figure in it, two places to the left in the scale, and thereby increases its value a hundred fold. The same reasoning is applicable to any number of ciphers.

CONTRACTION 2.—This contraction depends upon the same principle as the preceding one, as the following example will illustrate. Multiply 240 by 300. We place the significant figures, and multiply by them as if no ciphers were present, which gives 72; but this number must be removed as far to the left as there are ciphers in both factors, that is, three places; we therefore write 3 ciphers on the right of 72, which makes it 72000.

$$\begin{array}{r} 240 \\ \underline{300} \\ 72000 \end{array}$$

This contraction prevents the useless repetition of ciphers.

5

CONTRACTION 3.—Two reasons may be given for this contraction. 1st, Any number multiplied by the component parts of another number, must give the same product as if it were multiplied by that number at once. If any number were to be multiplied by 54, it is equally the same to multiply first by 9, and that product by 6, because 9 times any number mulplied by 6, is 54 times that number, as evidently as 9 times 6 are 54. 2d, If it were required to add any number to itself 54 times, instead of writing the given number 54 times, we might write it nine times, and add it up, and write the sum 6 times, which being added, would give a sum equal to 54 times the original number, as before.

CONTRACTION 4.—This contraction is often very convenient in large sums, and depends on the same principle as last contraction, together with a mature knowledge of the scale of notation. The following example will serve to make the reasons of the rule as intelligible as possible. Multiply 7534243 by 4484856.

```
        7534243
        4484856
   ------------
    30136972
     60273944
      421917608
  3375340864
 --------------
  33789994924008
```

By looking over the multiplier we find that 4 in thousands place multiplied by 2, will give 8 in hundreds place, and 8 multiplied by 7, gives 56 on the right, and that 56 multiplied by 8 gives 448. Having discovered this connection among the figures, we proceed to multiply by 4, and write the first figure of the product under the 4, for reasons already explained. From last contraction, we know that the product of any number by 4, when multiplied by 2, is equal to its product by 8, we therefore multiply last product by 2 to give 8 in hundreds place, writing the first figure of the product under 8 in hundreds place, in conformity to the general rule; again we multiply this last product by 7 to give the product by 56, the two figures on the right, and write the first figure of the product under units place; again 8 times 56 is 448, the 3 figures on the left, we therefore multiply last product by 8, and write the first figure of the product under 8, the right hand place of 448; we then add these products in the order in which they stand, and their sum is the total product required. A very little practice will render this contraction as familiar as the general rule. It may also be seen from the above example and explanation, that the first figure of each product, in whatever manner it may be produced, must stand directly under that figure in the multiplier to which it belongs; and when

the product of two or more figures in the multiplier is produced at once, the first figure of the product must stand under the right hand place of those figures.

CONTRACTION 5.—The reason of this contraction is obvious, for the several figures in the multiplicand are added in the same place, and order, as if they had been written under the product by the figure in units place, and then added.

The principle of this contraction may be extended to a variety of cases.—1st, If the multiplier consists of any figure in units place, and two or more 1's on the left.—RULE. Multiply by the figure in units place, and add to the product as it arises, the sum of two, three, four, &c. places of the multiplicand immediately to the right of the figure multiplied, according as there are two, three, four &c. 1's in the multiplier.

EXAMPLE.

```
21342        21342
 1116         1116
--------    --------
23817672     128052
             21342
            21342
           21342
           --------
           23817672
```

By comparing the work at large with the contraction, it is easy to see the reason of this method, and also what figures to take in at the different steps of the process; and when the multiplication by the figure in units place is finished, add the figure which you would have carried to next place to the sum of the right hand figures which you would have taken in at next step, and when you have taken in as many figures on the right as there are 1's in the multiplier, reject one on the right at each subsequent step, until the multiplicand is exhausted. The teacher can easily supply exercises.

2d, When the multiplier consists of any two figures.—

EXAMPLE.

```
 324     324
  23      23
----    ----
7452     972
        648
        ----
        7452
```

RULE. Multiply by the figure in units place, and add to the product as it arises 2, 3, 4, 5, &c. times the figure immediately on the right of the one last multiplied. This method is of little use when the left hand figure of the multiplier exceeds two, or three.

3d, When the multiplier is all 1's, to obtain the product in one line.—Rule. Write the figure on the right of the multiplicand for the first of the product, then the sum of the first two for the second, then the sum of first three for the third of the product, &c. till you have as many figures in the product as there are 1's in the multiplier, after this reject one place on the right for every subsequent figure of the product, taking in one farther

to the left; and after they are all taken in to the left, continue as before rejecting one on the right at every step, till the last.

EXAMPLE.

$$\begin{array}{r} 342134 \\ 111 \\ \hline 37976874 \end{array} \qquad \begin{array}{r} 342134 \\ 111 \\ \hline 342134 \\ 342134 \\ 342134 \\ \hline 37976874 \end{array}$$

This affords an easy method of multiplying when the same figure is in every place of the multiplier, whatever that figure may be; for we can find the product for as many 1's, and then multiply this product by the figure in the multiplier.

To multiply when the multiplier consists of 9's.—RULE. Annex as many ciphers to the multiplicand as there are nines in the multiplier, then subtract the original multiplicand from itself thus increased, the remainder is the answer.

EXAMPLE.—Multiply 142464 by 9999.

$$\begin{array}{r} 7424640000 \\ 742464 \\ \hline 7423897536 \end{array}$$

The reason of the rule is this, by annexing one, two, or three, &c. ciphers to any number, we in effect repeat that number 10, 100, 1000, &c. times, instead of 9, 99, 999, &c. times which was required, but $10 - 1 = 9$, $100 - 1 = 99$, $1000 - 1 = 999$, &c. it is thus evident, that annexing a cipher to the multiplicand for every 9 in the multiplier, we repeat the multiplicand once too often, and must therefore subtract the multiplicand from this product, to give the true product. On this principle we can multiply any number consisting of nines, either by itself or by any other number consisting of nines without the trouble of writing down either the multiplicand or multiplier, or annexing the ciphers, for the product will invariably consist of as many 9's on the left, wanting one, as there are places in the multiplicand; then 8, then as many ciphers, wanting one, as there are places in the multiplier, and lastly, 1 in units place.

To multiply any number by 5.—RULE. Annex a cipher to the multiplicand, and divide by 2.

The reason of the rule is this: annexing a cipher to the multiplicand makes it 10 times its original value, and dividing by 2 gives half this, or 5 times the multiplicand.

EXAMPLE.—$68974 \times 5 = 689740 \div 2 = 344870$.

This principle may be extended to a variety of cases, thus

$68754 \times 25 = 6875400 \div 4$, because 25 is $\frac{1}{4}$ of 100.

$34672 \times 50 = 3467200 \div 2$, because 50 is $\frac{1}{2}$ of 100.

$21324 \times 125 = 21324000 \div 8$, because 125 is $\frac{1}{8}$ of 1000, &c.

From an attentive consideration of the principles on

which the several contractions given in the text and notes depend, many other curious, if not useful contractions may be discovered.

The following method, although not of itself a contraction, is extremely simple, and convenient when the factors consist of many places, and frequently saves the labour of repeating the work two or three times.

Make a table consisting of the product of the multiplicand by each of the nine digits, or of the digits in the multiplier; the work is then performed by transferring the numbers in the table, into their proper places under the multiplier, the sum of these numbers will be the total product.

The table is constructed very simply and without much risk of error in the following manner:—Write down the multiplicand for the 1st. number in the table, double the 1st. gives the 2d., add the 1st. and 2d. gives the 3d., double the 2d. gives the 4th., add the 2d. and 3d. gives the 5th., double the 3d. gives the 6th., add the 3d. and 4th. gives the 7th., double the 4th. gives the 8th., add the 4th. and 5th. gives the 9th.

EXAMPLE.—Multiply 2768954837 by 74829536.

Table.

1	2768954837	$=1$
2	5537909674	$=1\times 2$
3	8306864511	$=2+1$
4	11075819348	$=2\times 2$
5	13844774185	$=2+3$
6	16613729022	$=2\times 3$
7	19382683859	$=3+4$
8	22151638696	$=4\times 2$
9	24920593533	$=4+5$

```
        2768954837
          74829536
      16613729022
      8306864511
    13844774185
   24920593533
   5537909674
 22151638696
11075819348
19382683859
207199605657665632
```

It will easily be observed by the above process that the labour is increased in some respects, by the whole operation of constructing the table; but we are more than remunerated for this labour, by the ease with which the table is constructed, and the certainty which it affords; compared with the difficulty in long operations, where the eye has to travel continually from the multiplier, to the multiplicand, and product, which are at some distance, and requires very considerable mental exertion, and the risk of errors which the work is subject to under such circumstances.

Lord Napier of Merchiston, inventor of logarithms, and one of the brightest stars in the history of science, considering the great advantage of tabling the multiplicand in the preceding manner, contrived a mechanical method still more certain and expeditious, called after their inventor, Napier's rods. This method is derived from a mature consideration of the multiplication table, as may be seen from the following diagram.

NAPIER'S RODS FOR MULTIPLICATION.

Index Rod										
1	1	2	3	4	5	6	7	8	9	0
2	2	4	6	8	1/0	1/2	1/4	1/6	1/8	0
3	3	6	9	1/2	1/5	1/8	2/1	2/4	2/7	0
4	4	8	1/2	1/6	2/0	2/4	2/8	3/2	3/6	0
5	5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5	0
6	6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4	0
7	7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3	0
8	8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2	0
9	9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1	0

It will be easy to see the construction of the rods from thi diagram. The rod on the left is called the index rod. The little squares in the other rods, with the exception of the top square, are all divided by diagonal lines. When there is only one figure in the square it is written below the diagonal, but when there are two figures in the square, the left hand one is written above, and the other below the diagonal.

To perform operations of any considerable extent by this method, it will be necessary to have about 60 rods, that is 6 for each digit, and 6 for ciphers.

To multiply by the Rods.—Lay down the index rod, and on the right of it, lay the rods which have the same figures on their top as in the multiplicand from left to right, and in the same order, then is the multiplicand tabled for all the 9 digits. To take out the figures, begin with the right hand rod, which

stands opposite to that figure in the index which is the same as the one on the right of the multiplier, take the figure under the diagonal for the first of the product; then the sum of the figure above the diagonal in the first and under it in the second rod, for the second figure of the product; and so on with all the rods.

EXAMPLE.—Multiply 85906 by 69548, by Napier's rods.

1	8	5	9	0	6
2	1/6	1/0	1/8	/0	1/2
3	2/4	1/5	2/7	/0	1/8
4	3/2	2/0	3/6	/0	2/4
5	4/0	2/5	4/5	/0	3/0
6	4/8	3/0	5/4	/0	3/6
7	5/6	3/5	6/3	/0	4/2
8	6/4	4/0	7/2	/0	4/8
9	7/2	4/5	8/1	/0	5/4

	85906
	69548
Against 8 in index is =	687248
Against 4 in do. is =	343624
Against 5 is =	429530
Against 9 is =	773154
Against 5 is =	515436
Total product =	5974590488

Product by 8 is found thus, opposite to 8 in the index, is 8 on right, then $0+4=4$, then 2, then $0+7=7$, then $4+4=8$, then 6, and so on with all the figures of the multiplier.

Either of the factors may be made the multiplicand, and the other the multiplier without altering the product.

$7\times5=7$ repeated so often as there are units in 5, thus $7+7+7+7+7=35$; and $5\times7=5$ repeated so often as there are units in 7, thus $5+5+5+5+5+5+5=35$.

If three, or more numbers are to be multiplied together, the last product will be the same in whatever order they are taken, and this product is called the continued product of the factors.

$$5\times7\times4=35\times4=140, \text{ and } 7\times5\times4=35\times4=140$$
$$5\times4\times7=20\times7=140, \text{ and } 4\times5\times7=20\times7=140$$
$$7\times4\times5=28\times5=140, \text{ and } 4\times7\times5=28\times5=140$$

We have here 140 in whatever manner these factors are taken, and 140 is the continued product of $4\times5\times7$, and this is true with any number of factors whatever.

If either or both factors be divided into any number of parts, and each part of the one, be multiplied by the whole, or each

part of the other, the sum of these products is equal to the product of the given factors, thus,

$3+4+5=12$, and $12\times9=108$, but $3\times9+4\times9+5\times9$ are also $=108$

$3+4+7=14$, and $2+3+6=11$, and $14\times11=154$, but

$3\times2+4\times2+7\times2+3\times3+4\times3+7\times3+3\times6+4\times6+7\times6=154$.

The reason is simply this, the whole is equal to the sum of all the parts, therefore if one number, or every part of it, be multiplied into every part of another number, then all the parts of the one are multiplied by the whole, or all the parts of the other, consequently the sum of the products of all the parts must be equal to the total product.

To multiply, when the multiplier contains a fraction. 1st. When the upper figure of the fraction is one.—Rule. Divide the multiplicand by the under figure of the fraction for the first line of the product, and write the first figure of the product by the figure in units place, under the right hand figure of last product; then proceed by the general rule.

Example.

24624
$23\frac{1}{6}$
4104
73872
49248
570456

I here divide the multiplicand by 6, the under figure of the fraction which gives 4104, I then multiply by 3 and write 2 the first figure of the product under 4, the right hand place of the product by $\frac{1}{6}$, &c. The reason of not shifting a place after finding the product by the fraction is, by shifting a place we would in effect have multiplied by a number 10 times as great as the integral part of the given multiplier.

EXERCISES.

1. Multiply 87651 by $26\frac{1}{3}$.
2. Multiply 69735 by $55\frac{1}{8}$.
3. Multiply 7637504 by $74\frac{1}{7}$.
4. Multiply 7569432 by $578\frac{1}{5}$.

When the upper figure of the fraction is greater than one.—Rule. Multiply the multiplicand by the upper number of the fraction, and divide the product by the under number; then proceed as above.

Example.

4767
$42\frac{3}{7}$
7) 14301
2043
9534
19068
202257

I here multiply the multiplicand by 3 the upper figure of the fraction, and divide the product by 7, its under figure. Reason. It is evident that $\frac{3}{7}$ of any number is equal to $\frac{1}{7}$ of a number 3 times as great. The same reason applies to any other fraction of this class.

EXERCISES.

1. Multiply 73467 by $37\frac{2}{7}$.	3. Multiply 43524 by $\frac{8}{9}$.
2. Multiply 37408 by $65\frac{7}{8}$.	4. Multiply 68783 by $732\frac{2}{11}$.

DIVISION.—I have already mentioned that division is a compendious method of performing subtraction, and that multiplication is a compendious method of performing addition, but as subtraction is the very reverse of addition, so division is the reverse of multiplication. It is shown in the notes to multiplication, that it can be performed by addition; in like manner division can be performed by subtraction, for $24 \div 6 = 4$; but 6 continually subtracted from 24 till 0 remains, the number of these subtractions will be 4, which is the same as the quotient of $24 \div 6$. This method, although simple in itself, yet, in general, the labour attending such subtractions would be intolerable, and division was invented to abridge the process.

According to division, the dividend is the number from which the subtractions are made, the divisor is the number to be subtracted, the quotient is the number of times the divisor can be subtracted from the dividend, and if any thing remain after division, it is the same as would remain after subtracting the divisor from the dividend so often as the number in the quotient represents. But it is evident we may diminish the number of subtractions; for, if we multiply the number to be subtracted, by any other number which gives a product equal to, or less than the number from which the subtractions are to be made, and subtract this product, the remainder will be the same as if we had subtracted the given number as often as the number by which it was multiplied represents. If the remainder is still greater than the number to be subtracted, we may again multiply and subtract the product as before, and so on till 0, or a number less than the number to be subtracted remains.

This is the basis on which division is founded, and may serve as a reason for the general rule. I shall illustrate this principle by an example. Let it be required to divide 4298689 by 576.

Method by subtraction.

	576)4298689(7000	
576 × 7000 =	4032000	
1st Remainder =	266689	400
576 × 400 =	230400	
2d Remainder =	36289	60
576 × 60 =	34560	
3d Remainder =	1729	3
576 × 3 =	1728	7463 sum of quotients.
Sum of 7463 multipliers.	1 = Remainder.	

By the general rule.

Divisor. Dividend. Quotient.

576)4298689(7463
4032
2666
2304
3628
3456
1729
1728
1

But 576 × 7463 + 1 = 4298689 the number to be divided.

We here see that 576 can be taken 7000 times from 4298689, because multiplying 576 by 7000 gives a product less than it, but by 8000 gives 4608000 which is greater, we therefore multiply 576 by 7000, which gives the first number to be subtracted; this leaves a remainder from which 576 can be subtracted 400 times, we therefore multiply 576 by 400, and subtract the product from 1st remainder, this leaves a remainder from which 576 can be taken 60 times, and proceeding as before, we have a third remainder, from which 576 can be taken 3 times, and 1 of a remainder, and being less than 576 the work is finished. It is therefore evident that from the dividend we have subtracted 7463 times 576, because we have subtracted the product of 576 by 7000 + 400 + 60 + 3 = 7463.—The general rule for division is only a farther contraction of this process; for instead of taking in the whole dividend at once, we take only the fewest possible figures on the left of it that will contain the divisor, in which it goes 7 times, we write 7 in the quotient, multiply the divisor by it, and subtract the product from 4298, to the remainder we annex next figure of the dividend, which makes it 2666, in this the divisor goes 4 times, and proceeding by the general rule we have at last a remainder of 1 the same as before; and by placing each subsequent figure of the quotient to the right of the one immediately preceding we have the same quotient 7463. It may be remarked, that the first figure of the quotient must be a thousand times its single value, for there are still 3 places on the right of the dividend not taken in, and it acquires this value from its position in the quotient, the 4 is also, from its position, a hundred times its single value, and the

6 is ten times its single value, each figure in the quotient being of the same value as by the other method.

Again, if we compare the two methods attentively, we will find that they must invariably give the same result, for by the method of subtraction, there are ciphers under three figures on the right at first step, then under two, then under one, thus taking in one figure more at every step of the process, the same as by the general rule. The rule is therefore true in every respect.

The greatest difficulty the pupil finds when the divisor is large, is to discover how often it is contained in the several steps of the process. To lessen this difficulty, consider how often the first figure, or two first of the dividend contains the first of the divisor, for the first figure of the quotient, and if upon trial this is found to be too much, diminish it till it answer. In all cases where you require two places of the dividend to contain the first figure of the divisor, you must write the first figure of the product under that figure of the dividend which is one place farther to the right than the whole number of places in the divisor.

How to ascertain whether the quotient figure is right or not. —Rule. When the product of the divisor, by the quotient figure, is greater than the part of the dividend under which it stands, the quotient figure is too great. But when the remainder, after subtracting, is equal to, or greater than the divisor, the quotient figure is too small. When, therefore, the product of the divisor by the quotient figure is not greater than the figures under which it stands, and when the remainder is less than the divisor, then the quotient figure is correctly taken; this holds true in every case.

If the divisor and dividend have the same number of places, the divisor is not contained in the dividend above 9 times.—Reason. Suppose two numbers having the same number of places, one of them the least, and the other the greatest that can be expressed by the same number of places, still if we multiply the least by 10 it will be greater than the other, thus, 10 is not contained 10 times in 99, for $10 \times 10 = 100$, and therefore greater than 99.

If the divisor, and dividend, have the same number of places, and the divisor is not contained in the dividend; by annexing another figure to the dividend, it cannot contain the divisor above 9 times.—Reason. 9 is the greatest figure that can be annexed to any number, and by doing so, we in effect multiply the number by 10 and add 9, because we thereby remove all its

figures one place higher, and write 9 in units place. But the divisor was greater than the dividend before 9 was annexed, suppose by 1, therefore ten times the divisor must still exceed the dividend by 1. Thus 99 is not contained in 98, and annexing 9 gives 989, but 10 times 99 is 990, which is still greater than 989 by 1. Proceeding therefore according to the general rule, the dividend can never contain the divisor above 9 times. —PROOF. The reason of the proof given in the text is this; the quotient represents the number of times the divisor is contained in the dividend, therefore the dividend must be equal to the quotient repeated as often as there are units in the divisor, and the remainder added to the sum. But any number repeated as often as there are units in another number, is equal to the product of those numbers; consequently the quotient multiplied by the divisor, and the remainder added to the product, must in all cases be equal to the dividend.

There are other methods of proof besides that given in the text.—1st. Subtract the remainder from the dividend, divide this remainder by the quotient, and the quotient thus arising is equal to the former divisor.—REASON. Subtracting the remainder from the dividend, leaves a number equal to the product of the divisor and quotient, and dividing the product of two factors by either of them gives the other.

2d. Add the remainder to the product of the divisor by the several quotient figures, in the order in which they stand in the work, and their sum will give the dividend.—REASON. The product of the divisor by the several quotient figures in the order which they stand in the work, is the product of the divisor by the whole of the quotient, to which if the remainder be added, the sum must be the dividend.

3d. Subtract the remainder from the dividend, their difference is a product, of which the divisor and quotient are the factors, then cast out the nines as in multiplication.—REASON. The same as was given in the notes to addition.

CONTRACTION 1st. This contraction depends on the same principle as the general rule, and is called short division, from the circumstance of the multiplication, and subtraction being performed mentally, and the quotient only written down.

CONTRACTION 2d. The reason of this contraction is evident from the nature of notation; because cutting off one, two, three, &c., places from the right of any number, we make it 10, 100, 1000, &c., times less than before, which is in effect dividing by these numbers.

The figures cut off are the same as would remain after per-

forming the operation by the general rule. This contraction is important in calculating interest.

CONTRACTION 3d. The reason of this contraction is the same as the 2d. But as it supposes the actual operation of division after cutting off the figures from the divisor and dividend, it may be necessary here to remark, that like parts of two numbers give the same quotient as the numbers themselves. Thus $24 \div 8 = 3$, and $12 \div 4 = 3$, and $6 \div 2 = 3$, and $3 \div 1 = 3$, but 12, 6, 3, are respectively the half, fourth, and eight parts of 24, and 4, 2, 1, are the half, fourth, and eight parts of 8, and this is true in all cases. The remainders in this contraction are the 10, 100, &c., part of the true remainder, according as 1, 2, &c., ciphers were cut off from the divisor. To find the true remainder; multiply by 10, 100, &c., and if significant figures were cut off from the dividend, they must be added to the product, or, which is the same thing, annex the figures cut off to the remainder without multiplying, as in the rule.

CONTRACTION 4th. I have already shown in the notes to multiplication, that the continued product of one number by the parts of another, taken in any order, is equal to the product of these numbers. But division being only the reverse of multiplication, it follows, that the continued quotient of one number by the component parts of another, must be the same as the quotient of the one number divided by the other.

The reason of the rules for finding the true remainder is, every unit in each subsequent remainder is equal to the product of all the preceding divisors. This I shall illustrate by an example.

Divide 21791 by 24. We may take 4 and 6 as the component parts of 24, because $4 \times 6 = 24$.

$$24=\left\{\begin{array}{r|l l} 4 & 21791 & \text{Remainders.}\\ 6 & \underline{5447} & 3\\ & \underline{907} & 5\end{array}\right.$$

Dividing by 4 we have 3 of a remainder which is simply 3 units, and 5447 of a quotient; but every unit in any quotient, is equal to the divisor which produced it, therefore every unit in 5447 is equal to 4; again when dividing by 6 we have 5 units of the dividend for a remainder, but every unit in last dividend was equal to 4, therefore we must multiply the last remainder 5, by 4 the first divisor, and to the product add 3 the first remainder, the sum is 23, which would have been the remainder by the general rule. The same reasoning may be extended to any number of remainders; the rule is therefore true.

ANOTHER METHOD.—RULE. Multiply the last quotient by the whole of the divisor, and subtract the product from the

dividend, leaves the true remainder. The reason of this is evident, for multiplying the quotient by the divisor, and adding the remainder to the product gives the dividend; consequently subtracting the product of the divisor and quotient from the dividend, must leave the remainder.

ANOTHER METHOD.—RULE. Multiply each remainder after the first, by all the preceding divisors for the true remainder at that part of the process, and write the product, add these remainders to give the true remainder.

EXAMPLE.—Divide 53171 by 84. We may here take 3, 4, 7, as the component parts of 84, because $3\times4\times7=84$.

84 =	3	53171	
	4	17723	2 = 1st. Remainder.
	7	4430	9 = 2d. Remainder = 3×3.
		632	72 = 3d. Remainder = 6×4×3.
			83 = True Remainder.

This is only a simplification of the method given in the text, the principle is the same.

After the pupil is expert in performing division by the general rule, and understands the different contractions, the work may still be considerably abridged by the following method.—RULE. Subtract from the dividend, the product of the divisor by the quotient figure as it arises, without writing it down, thus,

```
     47)685732(14590
          215
           277
            423
Remainder =   2
```

This is called the Italian method, and becomes very simple by a little practice.

The principle is the same as the general rule, and may be applied to any of the exercises given under it.

Division may be rendered more certain, and much easier, by making a table of the divisor in the same way as we did of the multiplicand, explained in the notes to multiplication. Having formed the table, the quotient figure and its product by the divisor are found by inspection; and by the help of the preceding rule, the work may be performed very expeditiously. This method will be found extremely useful in many cases, but particularly in calculating accounts in partnership, &c. where the same divisor is frequently used. Napier's rods may also be used for tabling the divisor.

Example.—Divide 987043787 by 352467.

Table of the Divisor.

1	352467
2	704934
3	1057401
4	1409868
5	1762335
6	2114802
7	2467269
8	2819736
9	3172203

352467)987973787(2803
704934
2830397
2819736
1066187
1057401
8786

In comparing the tabled divisor with the first six places of the dividend, we find that the number standing opposite to two is the greatest in the table which we can subtract from it, we therefore write 2 in the quotient and subtract the corresponding number in the table from the dividend, and the number nearest to the increased remainder stands in the table opposite to 8, we write 8 in the quotient and subtract its tabular number, and so on till the work is finished. I have given the work here at large, so that the whole process may be seen, but it may be shortened by the preceding rule. The principle is the same as the general rule.

When the divisor contains a fraction.—Rule. Multiply both dividend and divisor by the under figure of the fraction, and add the upper figure to the product of the divisor, then divide.

Example.—Divide 476 by $5\frac{3}{7}$.

$5\frac{3}{7}$)476
7 7
38)3332($87\frac{26}{38}$
304
292
266
Remainder=26

By multiplying according to the rule we take away the fraction from the divisor; and I have already shown that like aliquot parts of two numbers have the same quotient as the numbers themselves. For the same reason like multiples of two numbers have the same quotients, thus $12\div3=4$, but if we multiply both by any number, suppose 5, we have $60\div15=4$ which is the same quotient as before. This is true in all cases.

EXERCISES.

1. Divide 3742 by $7\frac{5}{7}$.
2. Divide 3241 by $37\frac{4}{11}$.
3. Divide 73954 by $376\frac{6}{13}$.
4. Divide 56432 by $1653\frac{105}{108}$.

When the units place of the divisor is 5.—Rule. Multiply the divisor and dividend, by any digit which gives most

ciphers on the right of the divisor, then divide by 2d. or 3d. contraction.

75)63426
4 4
3,00)2537,04
845 $\frac{204}{300}=\frac{17}{25}$

The reason for multiplying is to make it subject to 2d. or 3d. contraction, and the principle is the same as these contractions together with that of last case.

To divide by a number having the same digit in all its places.

Rule. Divide by as many 1's as there are figures in the divisor, then divide the quotient by the figure repeated in the divisor for the true quotient. The true remainder is found as in contraction 4.

Example.—Divide 865342 by 7777.

1111)765342(688÷7 = 98 and 2 remainder.
2d. Rem. 2)6666
true rem. 3190 9874
8888
9862
8888
1st. remainder 974

This is merely an extension of the 4th contraction, the principle is the same. The true remainder is found by multiplying the 1st. divisor by the 2d. remainder and taking in the 1st. remainder.

EXERCISES.

1. Divide 734632 by 444.
2. Divide 897698 by 666.
3. Divide 98975943 by 88888.
4. Divide 89789576 by 999999.

REDUCTION.—The only apparent difficulty in reduction is to understand why every new product assumes a new name, or becomes a new denomination. To illustrate this, let it be required to reduce £24 to farthings. We here observe an effect different from what took place in simple multiplication, where the product was always an abstract number, or an applicate number of the same species as the multiplicand. But the reason of this change will appear obvious when we consider the nature of the operation, for if £1 is equal to 20 shillings, £24 must be equal to 20 shillings repeated 24 times, or multiplied by 24. For this reason we must not consider the operation as the multiplication of £24 by 20, the direct product of which would certainly be £480, but as the multiplication of 20 shillings by 24 which evidently gives 480 shillings. Again, because 1 shilling is equal to 12 pence, 480 shillings are equal to 12 pence repeated 480

24
20 = sh.
480 = sh.
12 = d.
5760 = d.
4 = qrs.
23040 = qrs.

times, or to 5760 pence, and because 1 penny is equal to 4 farthings, 5760 pence are equal to 4 qrs. repeated 5760 times, or to 23040 qrs. as in the example, and these qrs. are equal in value to £24. We have thus multiplied a number by 20, 12, and 4, without altering its value in the least degree, the whole effect consisting in the changing of one denomination into another. Besides from what has been taught in the notes to multiplication, we can make either of the factors the multiplicand, and in this way we may suppose 20s. 12d. and 4 farthings to be the several multiplicands in the above example, which takes away the seeming inconsistency of applying to the product, at each step of the process, a name different from that of the multiplicand. From what has now been said, the reason of the rule and the nature of the process must both appear evident, as far as reduction descending is concerned.

CASE 2. To illustrate reduction ascending, I shall take the foregoing example; namely, to reduce 23040 qrs. to pounds. The reason of the operation is this, since 4 qrs. are equal to 1 penny, as often as 4 is contained in any number of farthings, so many pence are they equal to in value, therefore qrs. divided by 4, give pence. And since 12d. are equal to 1 shilling, as often as any number of pence contains 12 so many shillings are they equal to in value, and therefore pence divided by 12, give shillings. Again, because 20 shillings are equal to £1, as often as 20 is contained in any number of shillings so many £'s are they equal to in value, therefore shillings divided by 20 give pounds.

```
   qrs.
4)24040
 12)5760
  20)480
     £24
```

In this last example, we see the proof of the previous example, and in all cases reduction ascending and descending mutually prove each other.

CASE 3. This case requires the application of both multiplication and division; and the reason of the rule will appear evident from the following example. Reduce 135 guineas to crowns. In this example we cannot convert guineas into crowns by multiplication, for a guinea is not a multiple of a crown, or does not contain any even number of crowns.

```
Guineas.
  135
   21
  135
 270
5)2835
  567 crowns.
```

We therefore multiply the guineas by 21, which reduces them into shillings, because 21s. make a guinea, and then dividing the shillings by 5, gives crowns, because 5 shillings makes a crown, and as often as 5 is

contained in any number of shillings, so many crowns are they equal to in value. Although the above is the neatest and shortest method of reducing guineas into crowns, yet it might have been done otherwise, and quite consistent with the rule. For, if we reduce guineas to sixpences, and divide by ten, the sixpences in a crown, we have the same result; or if we reduce guineas into pence, and divided by 12 and 5, or by 60, the pence in a crown, the result would still have been the same.

Guineas are reduced to £'s, by dividing them by 20, and adding the quotient to the dividend; because every 20 guineas make £21. £s are reduced to guineas, if you divide them by 21, and subtract the quotient from the dividend; because £21 make 20 guineas. Yards are reduced to English ells, if you divide by 5, and subtract the quotient from the dividend; because 5 yards make 4 English ells. English ells are reduced into yards, if you divide by 4 and add the quotient to the dividend; because 4 English ells make 5 yards. The same thing is practicable in all cases where the multiplier and divisor differ from each other by unity.

Several other contractions might be pointed out in particular cases, but these I shall leave to exercise the ingenuity of the pupil.

COMPOUND ADDITION differs from simple addition only in the method of finding how many units of the next higher denomination are contained in the immediate inferior denomination, which is easily discovered by simple division. The principle on which the rule is founded is in all respects the same as in simple addition. I shall illustrate the process by an example, which will place the reason of the rule, and the method of operation in the clearest point of view. In writing down accounts in compound addition, the same denominations must be placed orderly under each other, as if they were separate and distinct accounts in simple addition. This is performed with greater accuracy by writing down, first the name of the several denominations, and then placing the number of each denomination directly under their respective names, and carefully seperating them by comas as in the example.

£	s.	d.	qrs.	
37 ,,	16 ,,	4 ,,	2	
537 ,,	10 ,,	8 ,,	1	
398 ,,	19 ,,	9 ,,	3	
156 ,,	14 ,,	7 ,,	0	
574 ,,	12 ,,	5 ,,	2	
745 ,,	13 ,,	9 ,,	3	
2447 ,,	84 ,,	42 ,,	11	First method.
2451 ,,	7 ,,	8 ,,	3	Second method.

According to the first method, the sum is £2447 ,, 84s. ,, 42d. ,, 11qrs., the sum of each denomination being written under itself, and this is all that properly belongs to addition. But it is evident that we may simplify the first sum, for having found the number of farthings as before, we may divide them by 4, because four farthings make a penny, and 11 $\div$ 4 = 2d, and 3qrs. of a remainder, we write the 3 under the column of farthings, and add the 2d. to the column of pence, which gives 44, and this divided by 12, the pence in a shilling, gives 3s. and 8d. over, we write the 8d. under the column of pence, and add the 3 to the column of shillings, which gives 87sh., and divided by 20, the shillings in a pound, gives £4, and 7sh. over, we therefore write 7s. under the column of shillings, and add the £4 to the first column of pounds, which gives the answer as in the second method. This is the same value as before, but more concisely expressed, and the only method used in business. By this method there can never be so many of any inferior denomination under any column, as make a unit of the next higher denomination; neither can the sum of all the inferior, ever equal a unit in the highest denomination.

The reason of dividing by 4 is this, 4 farthings make a penny, and as often as 4 is contained in any number of farthings so many pence are they equal to in value, and these evidently belong to the column of pence; for the same reason, as often as 12 is contained in any number of pence, so many shillings are they equal to in value, and these belong to the column of shillings, and as often as 20 is contained in any number of shillings, so many £s are they equal to in value. The same reasoning applies to weights and measures as well as money. In every possible case, therefore, we divide the sum of the column of an inferior denomination, by the number of units of that denomination which makes 1, or a unit of next higher denomination, the remainder is always written under the column added, and the quotient added to the next higher denomination, until you come to the denomination in which you

wish your answer to be. It is not always necessary to bring the left hand column to the highest denomination of which it can admit. For example, if the highest column is in cwts., and the answer required in cwts., it is not necessary to bring them into tons, and if the answer be required in miles, it is not necessary to reduce them into leagues or degrees. From what has been said, it is evident that the whole operation of compound addition, is nothing more than a combination of simple addition, and simple division, for every denomination is added as a separate and independent account, in simple addition, and then by simple division, we find how many units of the next higher denomination the sum contains. The pupil should be accustomed as soon as possible to find out mentally, how many units of next superior denomination are contained in the sum of the column of any immediate inferior denomination, without performing the operation on the slate.

To perform compound addition without division.—RULE. Begin with the lowest denomination, and if either that, or any other denomination to the highest consist of two places of figures, add them at once and make a point or dot when you come to a sum equal to, or greater than a unit of next higher denomination, but less than two such units, adding to the next number the excess above the unit where you make the point, proceed in the same manner through all that denomination, and write under the column the excess after last point; add one to the next higher denomination for every point in the preceding column, do the same with every denomination to the highest, which add as in simple addition.

The following example will illustrate the rule.

£	s.	d.	qrs.
37 ,,	14· ,,	6· ,,	2·
29 ,,	18· ,,	9 ,,	3
96 ,,	10 ,,	8· ,,	1·
82 ,,	9· ,,	10· ,,	2
58 ,,	15· ,,	9· ,,	2·
47 ,,	13 ,,	8 ,,	3
354 ,,	3 ,,	5 ,,	1

Beginning with the column of farthings, I say 2 and 3 are 5, which is 1 penny, and 1 farthing, I point 2· and add 1 to 2 are 3 and 1 are 4, I point 1·, 3 and 2 are 5, I point 2·, and write the 1 which was over under the column, there are three points in the farthings column. I therefore add 3 to the denomination of pence, thus 3 and 8 are 11 and 9 are 20, which is 1s. and 8d., I point the 9· and add 8 to 10 are 18, which is 1s. and 6d., I point 10· and add 6 to 8 are 14, 1s. and 2d., I point 8· and add 2 to 9 are 11 and 6 are 17, 1s. and 5d., I point 6·, and write 5 under the column, there are 4 points in the column, I there-

fore add 4 to the denomination of shillings, and proceed with them as before.

The denomination of shillings may be conveniently added, without pointing, thus, add the figures in units place, write the right hand figure of the sum under the column, and add the left hand figure, or figures along with the figures in tens place, if the sum is an even number, add the half of it to the £s, but if it is an odd number, write 1 in tens place under shillings, and add half of the even number to the right hand column of £s. This is always a convenient method of adding shillings.

The method of pointing is objected to, because it blots the accounts in which it is used, but it is only designed for those who have not attained a sufficient knowledge of addition and division, to enable them to perform this simple but useful process with accuracy, and pointing can scarcely be a fault in any business which such accountants can be intrusted with.

There is another method of performing compound addition by tables, but like that of pointing, is designed for those only, whose knowledge of arithmetic is very limited, and being more complex, shall not be given here.

COMPOUND SUBTRACTION.—The principle of the rule for compound subtraction, is the same as for simple subtraction, and therefore requires no farther demonstration. When the several denominations of the subtrahend, can be taken from those of the minuend immediately above them, the process is equally easy as in simple subtraction. But when any denomination of the subtrahend is greater than the corresponding denomination of the minuend, the unit taken from next superior denomination must be reduced into the same denomination as the subtrahend, then subtract the under number from the number above it, increased by the unit of next higher denomination. In compound subtraction it is often more convenient to subtract the under number from the unit of the superior denomination, and add their difference to the upper number, for the remainder. Suppose it were required to subtract £3 12s. 8d. from £6 4s. 3d., as 8d. cannot be taken out of 3d., a unit or 1 is taken from the 4s., which being reduced into pence is 12d., take 8d. from 12d. and 4d. remain, to which add the 3d. of the minuend, gives 7d.; again 12s. from 3s. (because 1s. was taken from the 4s.) I cannot, I therefore take £1 from £6, which is 20s., take 12s. from 20s. leaves 8s., to which add 3s. makes 11s., and £3 from £5, (because £1 was taken from £6,) leaves £2.—There is another variety which I shall here ex-

plain, namely, when some of the denominations are wanting in the minuend. To make this process perfectly evident I shall place the numbers in proper position for subtraction.

	9	19	11	4
£	10	0	0	0
	3	12	4	$\frac{1}{4}$
£	6	7	7	$\frac{3}{4}$

I cannot take one farthing from 0, and there is nothing in either of the next two higher denominations, £1 must therefore be taken from the £10, which leaves £9, and the £1 being changed into shillings makes 20s., from this take 1s. leaves 19s., and 1s. changed into pence makes 12d., from this take 1d., leaves 11d., and the 1d. changed into farthings makes 4 farthings, the £10 is thus subdivided into £9 19 11 4, as represented by the small figures above each denomination. The operation is then performed thus, 1 from 4 and 3 remain, 4 from 11 and 7 remain, 12 from 19 and 7 remain, £3 from £9 and £6 remain. This method is the most natural, and perhaps the simplest which can be proposed. For supposing you had £10 and were obliged to pay out of it a debt of £3 12 4 $\frac{1}{4}$, you could not do this without changing £1 into shillings, then one of these shillings into pence, and one of these pence into farthings, your £10 would then evidently be converted into the following denominations, £9 19s. 11d. 4 qrs., you would then pay the 1 farthing out of 4 farthings, 4d. out of 11d., 12s. out of 19s., and £3 out of £9, which is exactly the process in the preceding example. I have already in simple subtraction adverted to the mystery which beginners see hanging over every step of the process by the common method, and the difficulty of making them clearly comprehend it, but the difficulty of explaining the reason of this practice in compound subtraction becomes still more difficult. Indeed it is seldom attempted, and when it is, the teacher is often obliged to multiply words and reasons to explain that, which to him, through long custom, has become like a self-evident truth, and the pupil is generally left as ignorant as he was before. The difficulty of explaining this process, and of making the operation clear and intelligible to the young pupil, induced me to depart from a practice which has obtained for ages. Nor do I see any good reason for adopting an artificial method in any case whatever, when the natural method is as short, and much more simple. The truth is, the operation by the common practice is generally performed by memory without the assistance of the judgment, which should never be the case in so very useful and important a part of education.

The following is the reason of the process by the common

method, a person demands payment of £3 12s. 4¼d., you have £10, and therefore more than value to pay him, but you have no change, you therefore borrow from him 4 farthings, out of which you pay him ¼, you then borrow from him 12d., and out of this you pay him the 4d. which was due and the penny which you borrowed in farthings; you then borrow from him 20s., and out of this you pay him the 12s. which were due and the 1 shilling which you borrowed in pence; you then pay him the £3 which were due and the £1 which you borrowed in shillings. This is the principle on which the common rule is founded, and I think the most rational method of explaining it. But this is not the ordinary process in the affairs of life, and is not therefore so easily comprehended by the pupil, and in many cases cannot be explained so satisfactorily as in examples of money which I have here selected to show the principle of the rule.

Suppose it were required to take 3 miles, 4 furlongs, 12 poles, from 10 miles, I am persuaded it would not be easy to make the young pupil comprehend it perfectly. There is no doubt that taking 12 poles from 40 poles, 5 furlongs from 8 furlongs, and 4 miles from 10 miles, gives the same result as taking 1 mile from the 10 and reducing it into 8 furlongs, and 1 furlong from the 8, and reducing it into 40 poles, then subtracting the 12 poles from the 40, the 4 furlongs from the remaining 7, and the 3 miles from the remaining 9 miles. But any person who understands the first principles of arithmetic will easily perceive which of these methods is the most natural, and any teacher who shall make the experiment must discover which of them is the most intelligible to the pupil.

When both the minuend and subtrahend consist of two or more numbers, as is the case in most of the promiscuous exercises given in compound subtraction, it is generally best to take the sum of all the numbers belonging to the minuend, for the complete minuend; and the sum of all the numbers belonging to the subtrahend, for the complete subtrahend, and having found these sums by compound addition, the subtraction is performed at once as if the minuend and subtrahend had only consisted of one number.

It may also be observed that there can never be a number under any inferior denomination equal to a unit of next higher denomination, unless the answer is required in that denomination, nor can the sum of all the inferior denominations ever equal a unit of the highest denomination when performed according to the rule.

COMPOUND MULTIPLICATION.—The operation of compound multiplication is performed by a combination of simple multiplication, and simple division, as the following example will illustrate. Multiply £ 6, 7 4 ½ by 7.

£ 6 ,, 7 ,, 4 ,, 2
7
£42 ,, 49 ,, 28 ,, 14 = Product without division.
£44 11 7 2 = Product by division.

I here multiply the several denominations of the multiplicand by the multiplier 7, which gives £42 49s. 28d. 14 qrs. for the product, and this is all that multiplication can effect. But something more is necessary, for it would be extremely clumsy, and inconvenient in business to leave the lower denominations involving one or more units of the highest denomination. To obviate this inconvenience, division is applied to each product as it arises, the same as in compound addition. If we take the first product £42 49s. 28d. 14 qrs. and apply division to it, thus, divide the farthings by 4, the pence by 12, and the shillings by 20, carrying the quotients to next higher denomination before dividing, and writing the remainders, the product then stands £44 11 7 2 as in the example. The process of division should be performed at each denomination as directed in the general rule. I have given the product in the annexed example in both ways, in order to show more clearly what part belongs to multiplication, and what to division.

The principle of the general rule is the same as that of the general rule for simple multiplication, for although the number of units of a lower denomination required to make a unit of next higher varies, yet this does not alter the principle, nor affect the truth of the rule.

CASE 2.—The principle of this rule is the same as the principle of the third contraction in simple multiplication.

It may be remarked that this rule is of little practical use when the multiplier is large, or such that its factors are not easily discovered, for then more time is often lost in finding the proper factors than what might be necessary to perform the whole operation by rule 4th or 5th.

CASE 3.—The principle of this rule is the same as the preceding, for example, suppose it were required to multiply any number by 67, we might multiply it by 11, and that product again by 6, which would give us 66 times the multiplicand, which wants 1 time, or the multiplicand to make 67 times. It is evident, therefore, if we multiply the multiplicand again by

1, and add this product to the former, that their sum will be 67 times the multiplicand as required. Again, if we had multiplied first by 10, and that product by 7, it would have given 70 times the multiplicand, which is too much by 3 times the multiplicand, we would then have multiplied the multiplicand by 3, and subtracted this last product from the former, which would have left 67 times the multiplicand, because taking three times any number, from 70 times the same number, evidently leaves 67 times that number. From what has been said, it is manifest that the rule is correct, and that the result will be the same whether the composite number taken be greater or less than the given multiplier.

CASE 4.—The principle of this rule is evidently the same as the preceding, but as the operation may appear considerably involved, I shall take an example, and point out the reason of the several steps in the process. Let it be required to multiply £2 3s. 5¼ by 2406.

£2	3	5¼	× 6
		10	
21	14	4½	
		10	
217	3	9	× 4
		10	
2171	17	6	× 2
4343	15	0	
868	15	0	
13	0	7½	
£5225	10	7½	

The multiplicand in this example is £2 3s. 5¼d, which multiplied by 10, gives a product equal to ten times the multiplicand, and this product multiplied again by 10 gives a second product equal to ten times the first, or a hundred times the multiplicand. I multiply this last product again by 10, gives a third product equal to ten times the preceding or a thousand times the multiplicand; but we require 2406 times the multiplicand. We therefore multiply the last product by 2, which gives 2000 times the multiplicand, and the second by 4, which gives 400 times the multiplicand, and as there is a cipher in ten's place of the multiplier, we pass over the first product, and multiply the multiplicand by 6, which gives 6 times the multiplicand; and these products written under each other as in the example, and added give £5225 10s. 7½d. equal to 2406 times the multiplicand; but 2406 was the multiplier, therefore £5225 10s. 7½d. is the answer.

It may be remarked here, that when we multiply by 10 as directed in the rule, we have a line of products for every place in the multiplier above units place, which place corresponds with the multiplicand itself. We may therefore with propriety call the multiplicand the units place, the first product the tens place, the second product the hundreds place, &c. of the multiplicand, and if we multiply each place of the multiplicand by

the figure in the corresponding place of the multiplier, and add the products, their sum must in all cases give the answer of the question. From this explanation, a new, and perhaps a more simple rule may be given, namely,—" Multiply the given price or quantity successively by 10, so often wanting one as there are places in the multiplier, then multiply each place of the multiplicand by the figure in the corresponding place of the multiplier, and the sum of these products is the answer." From this rule, the reason for not repeating any place of the multiplicand, when there is a cipher in the corresponding place of the multiplier must appear evident. A little practice makes this rule extremely simple, and as it admits of general application it should be thoroughly understood.

CASE 5.—REASON. By reducing the multiplicand to the lowest name mentioned in it, and then multiplying it by the given multiplier, it is evident we thus multiply the whole of the multiplicand by the whole of the multiplier, and then by division we obtain the answer sought. When there is a fraction annexed to the multiplier, it is performed in the same way as directed for the fraction in the notes to simple multiplication.

USE OF MULTIPLICATION.—If the value, weight, or measure, &c. of one article, is given to find the value, &c. of any given number of articles, multiply the value, weight, &c. of one, by the number of articles. Also to find the amount of wages earned, interest of money gained, quantity of provisions consumed, space travelled by uniform motion, or any thing that increases regularly according to the time; multiply the given rate per year, month, week, or day, &c. by the number of years, or months, &c. Scarcely any general directions can be given, so as to ascertain in all mixed questions, when multiplication is required, excepting to consider well the nature of the question, and the effect of multiplication, which is to repeat a number or to take it a certain number of times. If, therefore, the sense of the question indicates that a given number of things is to be repeated, or that a number is to be found, equal in value or quantity to a certain given number of things repeated as often as some other number in the question contains unity. In all such cases multiplication is required.

COMPOUND DIVISION.—There are several ways in which questions in compound division may be expressed, such as, to find how often the dividend contains the divisor, or to find what part of the dividend the divisor is equal to, or to find

a number which is contained in the dividend as often as the divisor expresses, or to find a number which is the same part of the dividend, that the divisor expresses.

Before proceeding to the explanation of the several rules of compound division, it may be of advantage clearly to understand the nature of a compound or mixed number.

It has been found necessary, for the accommodation of society, that all great quantites, both in money, weight, and measure, should be subdivided into lesser quantities, and these subdivisions into others still lower, and so on, each subdivision being of itself a distinct denomination; but in such a manner that every inferior subdivision, or denomination should be a known and determined part of the next superior denomination. For example, £1 sterling is divided into 20 shillings, a shilling into 12 pence, and a penny into 4 farthings. Each of these lesser divisions of money, is a distinct denomination, and therefore when unconnected with a higher denomination, is as much an integer as £s are. Thus, we may say that a certain article is worth £42, or it is worth 49 shillings, or it is worth 28 pence, or it is worth 14 farthings, without any impropriety. But if we say a certain article is worth £42 49s. 28d. 14qrs. we use an unintelligible, or at least a very clumsy expression, and violate the established subdivisions of money. In order therefore to reduce the above expression into its proper form, we would take the pence out of the farthings, the shillings out of the pence, and the £s out of the shillings, as taught in compound addition and multiplication, which would give us a new expression the same in value as the former, viz. £44, 11s. 7½d. This last expression is what is properly called a compound, or mixed number, each of the inferior denominations being less than a unit of the next superior denomination, and consequently all the inferior denominations taken together, less than a unit of the highest denomination. What is here explained of money, and its subdivisions, máy supersede the necessity of giving examples of weights and measures, as the principle is precisely the same in them all. If what is here premised be perfectly understood, the following explanation of the principle of the general rule will be easily comprehended.

To explain the general rule.—We shall take the previous example, and divide it by 7. Dividing £44 by 7, we have 6 for the quotient and £2 over, which reduced into shillings, and the 11 shillings added, is 51 shillings, and divided by 7, gives 7 in the quotient, and 2 shillings over, which reduced into pence and the 7d. added, is 31d. and divided by 7, gives

4d. in the quotient, and 3d. over, which reduced into farthings and the 2 farthings added, gives 14 farthings, and divided by 7, gives 2 farthings and nothing over. Now, instead of £44, 11s. 7½d. if we take the equivalent expression £42, 49s. 28d. 14qrs., and divide it by 7, we will have the same quotient as before, each of the subdivisions at the same time being a multiple of the divisor. Consequently, to divide a compound or mixed number, by any number, is the same as dividing all the parts of which it is composed by that number. And although some of the parts of the number to be divided are not an exact multiple of the divisor, yet by placing the proper value of the number by which it exceeds that multiple, in the next lower denomination and dividing it, the dividend will thus be divided by the divisor and the true quotient found according to the rule. The reason of the proof is the same as given in simple division.

CASE 2.—To show the principle on which this rule is founded, it is only necessary to state, that by bringing the divisor and dividend into the same name, we do not affect the value of either. Take the example in the text under this rule, viz. to divide £69 16s. by 14s. 6½d., and it must appear evident that as often £69 16s. contains 14s. 6½d. so often precisely will the halfpence in £69 16s. contain the halfpence in 14s. 6½d. But 33504 are the halfpence in £69 16s.; and 349 are the halfpence in 14s. 6½d., consequently as often as £69 16s. contains 14s. 6½d. so often will 33504 contain 349. From this it appears clearly that the rule is correct, and must hold good in all cases.

CASE 3.—The reason of this rule has already been given in the notes to simple division, and is this, like multiples of any two numbers, give the same quotient as the numbers themselves.

CONTRACTION 1st.—This contraction depends on the same principle as the general rule already explained.

CONTRACTION 2d.—The reason of this contraction is the same as the 4th contraction in simple division.

CONTRACTION 3d.—The reason of this contraction is given under the second contraction in simple division.

CASE 4.—I have not seen a rule directly applicable to this case in any treatise on arithmetic which I have met with; and although it is possible to reduce it under the head of the second rule, the principle being strictly the same, yet as there always appears in these questions to be several divisors, and an equal quotient required by them all, the young arithmetician

no sooner meets with a question belonging to this class than he stops short, unable to proceed a single step. As there are many important practical questions of this nature, I have considered it not superfluous to annex a particular rule for them.

REASON.—When we consider that division is only an abreviation of subtraction, as already explained, the reason of this rule will appear evident. For it is plain, that as often as we can subtract several numbers from any given sum, so often can we subtract their sum. Therefore if we take one of each of the parts of which there are to be an equal number, and add them together for a divisor, the number to be divided will contain it as often as it would do all the parts of which it is composed, and the quotient must be the equal number of each. The reason for reducing the sum of the parts, and the dividend into the same name is given in the notes to the second rule.

USE OF COMPOUND DIVISION.—When the value, weight, or measure, &c. of any number of articles is given to find the value, &c. of one. Divide the whole value, &c. by the number of articles, &c.

When the value, &c. of the whole is given, and also the value, &c. of one, to find the number of articles, &c. Divide the value, &c. of the whole, by the value, &c. of one.

When the wages earned, the work performed, the provision consumed, the interest gained, the space passed over, &c. in any given number of years, months, weeks, days, &c. is given to find how much it is in a year, or month, &c., divide the whole amount by the number of years, or months, &c.

When it is required to find the time in which a given amount of wages may be earned, work performed, provisions consumed, interest gained, or space passed over by uniform motion, &c. divide the whole amount, by the amount in one year, or one month, &c.

The only general directions which can be given for ascertaining when division is required in mixed questions, is to consider well the nature of division as already explained. When the nature of the question requires you to find how often one number is contained in another; or to find such a part of any number, as another number indicates; or such a part, as a unit of any species of things is to a certain given number of the same things, then division is required.

PRACTICE.—Practice consists of contractions in the Rule of Three, when either the 1st or 3d term is one, and derives its name from the almost universal application of its rules to the

solution of questions which occur in trade or mercantile transactions. It's position here has been determined by its immediate connection with compound multiplication and division.

CASE 1.—The exercises under this case belong to compound multiplication. The rule is general, and the exercises under it may often be calculated more concisely by some of the subsequent rules.

CASE 2.—The exercises under this case are all performed by the rules of compound division.

CASE 3.—As aliquot parts are first introduced in this case, I shall explain what is meant by them. An aliquot part of any number, is such a part as being repeated a certain number of times will exactly make that number, or such as will divide it without leaving a remainder. Thus, 1, 2, 3, 4, 6, are aliquot parts of 12, and 1, 2, 4, 5, 10, are aliquot parts of 20, also $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, &c. are aliquot parts of 1.

The reason of the rule is this, if one article cost 4s. or $\frac{1}{5}$ of a pound, it is evident that 265 will cost 265 fifth parts of a £, or which is the same thing, the fifth part of £265. Again, if one article cost 4d. or $\frac{1}{3}$ of a shilling, 265 will cost 265 third parts of a shilling, or the third part of 255 shillings. If one article cost 1 farthing, or the $\frac{1}{4}$ part of a penny, 265 will cost 265 fourth parts of a penny, or the 4th part of 265 pence. The same species of reasoning is applicable to every question belonging to this case.

The above being only a particular case of a general rule, I shall subjoin the rule itself with an example for illustration.

GENERAL RULE.—When the price of one is less than a penny, suppose it to be a penny, when more than a penny but less than a shilling, suppose it to be a shilling, and when more than a shilling but less than a £, suppose it to be a £, the given number of articles will then be the answer at the supposed price, and in the same denomination. Divide the given price into aliquot parts of the supposed price, or of each other, with which take part of the answer at the supposed price, and the sum of the quotients thus arising will be the answer at the given price.

EXAMPLE.—What is the value of 687 gallons of oil, at 8s. 3$\frac{1}{2}$d per gal.?

sh.		£	s.	d.			£	s.	d.
		£687			=	value at £1, the supposed price.			
5	$=\frac{1}{4}$	171	15	0	=	value at	£0	5	0
2 6	$=\frac{1}{2}$	85	17	6	=	— —	0	2	6
6	$=\frac{1}{5}$	17	3	6	=	— —	0	0	6
3	$=\frac{1}{2}$	8	11	9	=	— —	0	0	3
$\frac{1}{2}$	$=\frac{1}{6}$	1	8	$7\frac{1}{2}$	=	— —	0	0	$0\frac{1}{2}$
		£284	16	$4\frac{1}{2}$			£0	8	$3\frac{1}{2}$ the given price.

In this example, because the price is more than a shilling, I suppose it to be a £ according to the rule; and it is evident that the number of articles 687, is the answer in £s at that price. But as 5 shillings is the $\frac{1}{4}$ part of a £, so $\frac{1}{4}$ part of £687 or £171 15s. is the value at 5 shillings, and as 2s. 6d. is the $\frac{1}{2}$ of 5 sh. so $\frac{1}{2}$ of £171 15s. or £85 17s. 6d. is the value at 2s. 6d. Also as 6d. is the $\frac{1}{5}$ of 2s. 6d. so $\frac{1}{5}$ part of £85 17s. 6d. or £17 3s. 6d. is the value at 6d. Again as 3d. is the $\frac{1}{2}$ of 6d. so $\frac{1}{2}$ of £17 3s. 6d. or £8 11s. 9d. is the value at 3d. Farther as $\frac{1}{2}$ is the $\frac{1}{6}$ of 3d. so $\frac{1}{6}$ of £8 11s. 9d. or £1 8s. $7\frac{1}{2}$d. is the value at $\frac{1}{2}$. Now the sum of all these parts is 8s. $3\frac{1}{2}$d., consequently the sum of the values belonging to these parts, is the value at 8s. $3\frac{1}{2}$d. the given price, which was required. By the same manner of reasoning the truth of any example whatever may be proved.

In the above example the supposed price is £1, therefore the value comes out in £s, but if the supposed price had been a shilling, the value would have come out in shillings, and if it had been a penny, the value would have come out in pence, in both of which cases it must be reduced to £s.

Case 4.—The reason of the operation in this case, must appear evident, from the general rule, and the explanation of the example under it, in the note to last case.

Case 5.—The rule for this case diverges a little from the general rule under case 3, when the price exceeds one shilling, for by that rule we would have considered the price £1, and then have taken parts with the whole of the given price.

But by the rule for this case, we first calculate for so much of the price as is under a shilling, by the general rule, the value of which comes out in shillings, and as the given price is one shilling for each article more than this price, we add the number of articles considered as shillings to the former value, and the sum divided by 20 is the answer in £s.

Case 6.—The reason of the rule for this case will appear evident from the note to contraction 3 in simple division, where

it is shown that like parts of any two numbers give the same quotient as the numbers themselves. It is quite clear if we multiply the shillings in the given price by the number of articles and divide the product by 20, the quotient is the answer in £s, and the remainder is shillings. Again, if we multiply half the number of shillings by the number of articles, we have only half of the former product, which, divided by half the divisor, or 10, evidently gives the same quotient as before, and the remainder is in tenth parts of a £, which, being doubled, gives twentieth parts, or shillings. It is also evident from contraction 2d in simple division, that the right hand figure is invariably the remainder when the divisor is 10, and that the other figures remain unchanged. But when there is an odd shilling, its value is found by case 3d and added to the former, which gives the full answer; thus every part of the rule is plain.

When there are £s in the price they may be prefixed to half the shillings, and proceeded with as shillings, thus, for £3 14s. to 7, half of the shillings, prefix the £3, gives 37 for the multiplier, which is simpler than reducing the £s and shillings into shillings, and then taking half of them, which gives the same number for the multiplier.

Case 7.—It is evident if we multiply the number of articles by the shillings in the price, and add to this product their value at the lower denominations, that the sum is the answer in shillings, which, divided by 20, gives £s. The 2d. rule for this case is the same as the general rule under case 3d.

Case 8.—If we multiply the given number of articles by the £s in the price, and to the product add their value at the lower denominations, it is evident the sum must be the value in £s. The second part of the rule is the same as the second part of the rule for case 7th.

Case 9.—When we suppose the price of one article to be a £, the number of articles is the answer in £s. If the given price is less than a £, this answer is evidently too much by the same part of the whole number of articles considered as £s, that the difference between the given price and 20 shillings is of a £, we must therefore subtract this part from the whole number of articles, the remainder is the answer at the given price. But if the given price is above a £, we must add to the number of articles considered as £s, the same part of them, that the excess of the given price above 20 shillings, is of a £. The principle on which this rule is founded will appear evi-

dent from the general rule in the note to case 3d. compared with the note to rule 3d. in compound multiplication.

Case 10.—The reason of the rule is this; when we multiply the price by the under figure of the fraction, it is evident that we make it 2, 3, 4, &c. times greater than the given price, and consequently the answer is as many times greater than the true answer as the under number of the fraction contains unity, and must therefore be divided by the under number to give the true answer.

Case 11.—If we add to the value of the whole number, the value of the fractional part, it is evident that the sum must be the answer. The value of the fraction may be found either by dividing the upper number of the fraction, into aliquot parts of the under number, or by multiplying the price by the upper number, and dividing the product by the under number of the fraction. The principle of the second part of the rule is the same as case 10.

The general rule in the note to case 3d. may very properly be applied to this case, by substituting the value of the fraction at the supposed price instead of the fraction itself, and then take parts with the given price, as the following example will illustrate.

Calculate the value of 267⅞ cwt. at 18/4 per cwt.

		£	s.	d.	
10/	½	267	17	6	
5/	½	133	18	9	
2/6	½	66	19	4½	
/10	⅓	33	9	8¼	
		11	8	2¾	
		£245	11	0½	Ans.

or thus

		267	17	6	= Price at £1.
1/8	$\frac{1}{12}$	22	6	5½	Subtract.
		£245	11	0½	Ans.

The 17/6 affixed to the 267 is the value of ⅞ of the supposed price, £1.

Case 12.—According to the limitation of this case, if we multiply the given price by the number of the highest denomination, the product is their value, and if to this product we add the value of the lower denominations according to the rule, it is evident the sum will be the complete answer. This case may also be performed by the general rule in the note to case 3d. by substituting the value of the lower denominations at the supposed price, and taking parts with the given price as illustrated by the example in the note to case 11. It often happens that the price of the highest denomination mentioned in the quantity is not given, but must be found from knowing the price

of some lower denomination. In all such cases we reduce the higher denominations into that of which the price is given, and then proceed as before. For example, if the price of a cwt. were given, and the price of any number of tons, cwts., &c. required; reduce the tons, &c. into cwts. and then proceed according to the rule.

Observations to facilitate Mental Calculations.—1. When the price of one dozen is any number of shillings, the price of one is as many pence.

2. When the price of one is any number of pence, the price of a dozen is as many shillings.

3. When the price of a gross is so many shillings, that of a dozen is as many pence.

4. When the price of a dozen is so many pence, that of a gross is as many shillings.

5. When the price of a ton, or any twenty articles is so many £s, that of a cwt. or of one article is as many shillings.

6. When the price of a cwt. or of one article is so many shillings, that of a ton or twenty articles is as many £s.

7. When the price of a cwt. is given, to find the rate per lb., multiply the price in shillings by 3, and divide the product by 7, the quotient is the price of a lb. in farthings.

8. When the price of a lb. is given, to find the rate per cwt., multiply the farthings in the price of a lb. by 2⅓, the product is the answer in shillings.

9. When the rate per day is given, to find the rate per annum, multiply £1 10s. 5d. by the pence in the rate per day, the product is the rate per annum.

10. When the rate per day is given, to find the rate for all the work days in a year, multiply £1 6s. 1d. by the pence in the rate per day, the product is the rate per annum.

Many other concise methods might be pointed out in particular cases, but the ingenuity of the practical arithmetician will readily discover those which are applicable to his own purpose.

COMMERCIAL ALLOWANCES.—In the greatest part of wholesale mercantile transactions, the goods, bought and sold, are not unpacked in order to be weighed; but a certain, fixed, ascertained or stipulated allowance is made for the weight of that which contains them, this allowance is called tare; and being deducted from the gross or whole weight, leaves the weight of the goods.

REAL TARE is the weight of whatever contains the goods, ascertained by weighing it after the goods are unpacked.

CUSTOMARY TARE is a certain established allowance, instead of the real tare.

PROPORTIONATE TARE is a certain stipulated allowance per cent. or per cwt., instead of the real tare.

AVERAGE TARE is an allowance of a certain number of lbs. on each parcel or package, ascertained from the mean weight of a few.

TRET and CLOFF, although introduced in some of the examples in the text, are nearly out of use in the commercial world.

DRAFT is allowed on cotton wool, at the rate of 1 lb. per bag, and in London, on madder at 4 lb. per cask.

In calculating commercial allowances, remainders are generally rejected when less than half a lb., but when half a lb. or more they are considered as 1 lb. The reason of the rule for the several cases of commercial allowances, must appear so plain from what has been said on the rules of practice, as scarcely to require any further illustration.

CASE 1st.—It is evident when draft only is allowed, that by subtracting it from the whole leaves the neat weight.

CASE 2d.—The same reason applies to the tare in this case.

CASE 3d.—It is evident if the tare of one cask, &c. be multiplied by the number of casks, &c., the product is the whole tare, which, subtracted from the gross, must leave the neat weight.

CASE 4th.—If the tare be at so much per cent. or per cwt., if we take the same parts of the gross weight which the tare is of 100 lb. or 112 lb., it is evident that we have the whole tare, and subtracted from the gross leaves the neat weight as before.

CASE 5th.—When tret is allowed at the rate of 4 lb. per 104 lb., or $\frac{1}{26}$; it is evident, that by subtracting the $\frac{1}{26}$ part to the tare suttle from itself the remainder is the neat weight.

CASE 6th.—When cloff is allowed at the rate of 2 lb. per 3 cwt. or $\frac{1}{168}$, by subtracting the 168 part of the tret suttle, (or of the tare suttle when tret is not allowed) from itself, the remainder is the neat weight.

There are other cases in which allowances are made, and perfectly distinct from those of which we have been treating, and not depending on the weight of that which contains the goods.

The following rule applies generally to all such cases.—RULE.

Multiply the given quantity by the rate, and divide the product by the number on which the rate depends, the quotient added to, or subtracted from, the given quantity, according to the nature of the allowance, gives the answer.

EXERCISES.

1. Gave 450 yards raw plaiden to be scoured and prepared for blankets, by which process it was contracted at the rate of 2 yards in 9, how much did it measure out? Ans. 350 yds.

2. Malted 474 bolls of barley, by which it increased at the rate of 3 bolls in 8; what quantity of malt was there?
Ans. 651$\frac{3}{4}$ bolls.

3. Bought 680 yards green linen, and was allowed 1 yard gratis to each score; how many yds. did I pay for? Ans. 646.

4. Bought 123700 Lochfine herrings, and was allowed 20 gratis for every hundred I paid for; how many was paid for?
Ans. 98960.

The reason of the rule for this case is clear, for if the quantity is to be diminished in any given proportion, suppose at the rate of 2 in 9 as in the first example, it is evident the answer must be less than the given quantity by $\frac{2}{9}$ of itself, and the 2 ninths is found by multiplying the quantity by 2 and dividing the product by 9, and the quotient subtracted from the given quantity leaves the answer. But when the quantity is to be increased the quotient is added.

SIMPLE PROPORTION.—This rule is called Simple Proportion, because by it we find a number, which has to a given number, the same proportion which subsists between other two given numbers, without being limited by any other condition.

It is called the RULE OF THREE, because in all the questions which it solves, three terms are given to find a fourth. It is also called the GOLDEN RULE, from its excellent and extensive application.

I shall here point out a few of the fundamental principles upon which the rule is founded, without entering minutely into the doctrine of proportion, and the arithmetic of ratios, which may be seen fully, and ably handled by the ingenious Alexander Malcolm of Aberdeen, in the fourth book of his system of arithmetic.

PRINCIPLES.—The ratio of one number to another, is the quotient which arises from dividing the one by the other, and is expressed thus, 8 : 2, or $\frac{8}{2}$, both expressing the ratio of 8 to 2. But 8 contains 2 four times, therefore 4 is the ratio of 8 to 2,

and for the same reason, the ratio of 2 : 8 is $\frac{1}{4}$, because 2 is the fourth part of 8.

The terms of any ratio are the numbers compared, of which the first is called the antecedent, and the second the consequent; thus, in the ratio 8 : 2; the terms are 8 and 2, of which 8 is the antecedent, and 2 the consequent.

Two ratios are equal when the antecedents contain their consequents, or the consequents contain their antecedents the same number of times; thus, the ratio of 8 : 2 is equal to the ratio of 16 : 4, because each of the antecedents contains their consequents the same number of times.

When two ratios are equal their terms are proportional; thus, because the ratio of 8 : 2 is equal to the ratio of 16 : 4, their terms are proportional, that is, as 8 is to 2, so is 16 to 4, and is written thus 8 : 2 : : 16 : 4.

In proportion, the first and last terms are called the extremes, and the middle terms the means, and the product of the extremes is always equal to the product of the means, thus, in the proportion 8 : 2 : : 16 : 4, the product of 8 × 4 is equal to the product of 2 × 16.

When the product of any two numbers is equal to the product of any other two numbers, these four numbers must be proportionals by making the factors of the one product the extremes, and the factors of the other product the means, thus, since 8 × 4 is equal to 2 × 16, 8 : 2 : : 16 : 4, or 4 : 16 : : 2 : 8, or 2 : 8 : : 4 : 16, or 16 : 4 : : 8 : 2, &c.

From these principles the method of finding a fourth proportional to three given numbers is manifest; for the second and third terms of a proportion are invariably the means, and their product is always equal to the product of the extremes; but the product of any two factors divided by one of them gives the other, therefore the product of the second and third terms, which is equal to the product of the first and fourth, being divided by the first term, must give the fourth term.—The Rule given in the text is founded upon these principles.—The Proofs are founded upon the same principles; for it is evident, if we make the former extremes the means, and one of the former means the first extreme, that the operation must give the other extreme equal to the former mean.—The second method of proof depends on this principle, the product of the extremes is invariably equal to the product of the means.

My reason for placing the term upon which the demand lies in the third place is, that ratio can exist only between similar

quantities. For example, there is a certain ratio between 3 miles and 12 miles, and also between £4 and £16, because the one either contains, or is contained by the other, a certain number of times; but there is no ratio between 3 miles and £4, for three miles are no part whatever of £4, neither are £4 any part whatever of 3 miles.

It may be observed still farther respecting ratio, that it is not altered by multiplying, or dividing its terms by the same number; thus, let 8 : 2 be the terms of a ratio, and multiplying both by 2, the terms become 16 : 4, but 16 contains 4 as often as 8 contains 2. Again, let 8 : 2 be divided by 2, we have 4 : 1, but 4 contains 1 as often as 8 contains 2, therefore the ratio of 8 : 2 is equal to the ratio of 4 : 1. Therefore the terms of a ratio being divided by the same number, does not alter the ratio; and it has been shown that multiplying by the same number does not alter it, consequently ratio is not altered by multiplying or dividing its terms by any number. And if the ratio of two numbers be equal to the ratio of other two numbers, it is not altered by dividing or multiplying a corresponding term of each by the same number. Thus 4 : 12 : : 10 : 30, and dividing 4 and 10 by 2, we have 2 : 12 : : 5 : 30, or dividing 12 and 30 by 6, we have 4 : 2 : : 10 : 5, &c. The same holds good if we multiply. Upon these principles depend several useful contractions in the Rule of Three.

General Contraction.—If the first term and either the second or third is a multiple of the same number, or the one a multiple of the other, divide both by that number, and use the quotients instead of them.

Particular Contractions.—Divide either the second or third term by the first, and multiply the other term by the quotient, the product thus arising is the answer.—2d. Divide the first term by either the second or third, and divide the remaining term by the quotient, the last quotient is the answer.

Example.—1st. If 18 yards of broad cloth cost £24, what will 27 yards cost?

	yds.	yds.	£
As	18 :	27 : :	24
or	2 :	3 : :	24
or	1 :	3 : :	12
			3
		Ans.	£36

By 1st. contraction. I see that 18 and 27 are both multiples of 9. I therefore divide them by 9, and the stating becomes as
yds. yds. £
2 : 3 : : 24. This stating may be still reduced by the same process, because 24 in the third term is a multiple of 2 in the 1st. I therefore divide the 1st. and 3d. terms by

yd. yds. £
2, and the stating becomes, as 1 : 3 : : 12, and the first term being 1, the product of the 2d. and 3d. terms is the answer in the same name as the third term. Or the work may be performed shorter, thus, as $\cancel{18} : \underset{3}{\cancel{27}} : : \underset{12}{\cancel{24}}$ and $3 \times 12 = 36$ Ans.
I here divide the first term by 18, and 9, and 2 being the factors of 18, I divide the 2d. by 9 and the 3d. by 2, which gives 3 and 12, and these multiplied together gives £36 the answer. When the first term is 1, the answer is found by multiplying the other two, and when either the 2d. or 3d. term is 1, the answer is found by division.

EXAMPLE 2d.—Paid for 56 firkins of butter at the rate of £18 12s. 6½d. for 8 firkins, what did they come to?

F.	F.	£	s.	d.
As $\cancel{8}$:	$\underset{7}{\cancel{56}}$: :	18	12	6½
				7
		£130	7	9½ Ans.

1st. Particular Contraction. I here divide the second term by the first, the quotient is 7, by which I multiply the third term, gives £130 7s. 9½d. Ans. This process is a saving of much labour, and is easily comprehended. The process is the same as the first contraction, as it is in reality dividing the first and second terms
F. F. £ s. d.
by 8, which gives a new stating, as 1 : 7 : : 18 12 6½, but the 1st. term being 1, the product of the second and third terms is the answer.

EXAMPLE 3d.—Bought 36 yds. of cloth for £25 6s. 8d., how must I sell 9 yards of it to gain at the rate of £4 10s. 4d. upon the whole?

yds.	yds.	£	s.	
As $\underset{4}{\cancel{36}}$:	9 : :	29	17	
		£7	9	3 Ans.

2d. Particular Contraction. I here divide the first term by the second, the quotient is 4, by which I divide the third term, and the quotient is the answer. It is evident this is only a case of the first contraction, for by dividing the first and second terms by 9, we
yds. yd. £ s. £ s. d.
have as 4 : 1 : : 29 17 : 7 9 3 the answer. As the middle term in this last stating is 1, it is evident the quotient arising from dividing the 3d. term by the 1st. must give the answer. This contraction falls directly under the first, so that there is, strictly speaking, only one contraction in this rule.

As soon as pupils are able to state the questions accurately,

and to perform the operations with tolerable expertness by the common method, they should be taught to apply the contractions, which will give them an accuracy and expertness altogether unattainable by the general rule. The importance of simple proportion is a sufficient reason why it should be understood perfectly.

Simple proportion is frequently divided into DIRECT, and INVERSE or RECIPROCAL, and separate rules and examples given for each.

SIMPLE PROPORTION DIRECT is when more requires more, or less requires less. Thus, if 1 acre of land let for 7/4.; what will 37 acres let for? Here more requires more, because 37 acres will let for more money than 1 acre. But if the question had been, if 37 acres of land let for £13 11s. 4d.; what is that per acre? In this case less would have required less, that is, 1 acre would have let for less money than 37 acres, and therefore both these cases are in simple proportion direct.

SIMPLE PROPORTION INVERSE is when more requires less, or less requires more. Thus, if 600 men perform a piece of work in 6 hours, how many men will perform the same in 24 hours? Here more requires less, that is, when more time is allowed, fewer men will be required. But if the question had been if 150 men perform a piece of work in 24 hours, how many men will perform the same in 6 hours? Here less requires more, that is, less time being allowed, more men will be required to perform the same work. Both these cases therefore are in simple proportion inverse or reciprocal.

Subdividing proportion into direct and inverse, always tends to perplex beginners, and renders complex what is naturally simple. The general rule in the text, is equally applicable to direct, and inverse proportion, and renders the method of working the question invariably the same. The examples of direct and inverse proportion are purposely written indiscriminately, that the pupil's judgment may be exercised in stating them. By attending carefully to the nature of the question, and to the directions in the general rule, little difficulty will be experienced in stating questions, either direct, or inverse, even by the youngest pupil.

In many examples in proportion, much labour may be saved, by using compound multiplication, and division instead of reducing the terms.

COMPOUND PROPORTION is so called, because the term sought is limited by the compounded ratio of four or more terms.

Or from the circumstance of its being compounded of two, or more simple proportions. This I shall illustrate by an example.

If 18 roods of ditching be wrought by three men, in 16 days, of 15 hours; how much will be done by 8 men, in 4 days, of 9 hours.

Men.	3 : 8 Roods.	
Days.	16 : 4 :: 18	$\dfrac{\overset{2}{\cancel{8}}\times\cancel{4}\times\cancel{9}\times 18}{\cancel{3}\times\cancel{16}\times\underset{5}{\cancel{15}}}=\dfrac{2\times 18}{5}=\dfrac{36}{5}=7\frac{1}{5}$ Roods.
Hours.	15 : 9	

Had the answer in this example depended entirely upon the first part of the stating, it must have been 48, because 3 : 8 :: 18 : 48, and had it depended entirely upon the second part of the stating, it must have been $4\frac{1}{2}$, because 16 : 4 :: 18 : $4\frac{1}{2}$. Again, had it depended entirely upon the last part of the stating, it must have been $10\frac{4}{5}$ because 15 : 9 :: 18 : $10\frac{4}{5}$. The ratio of 18 to the answer is therefore compounded of the ratio of 3 : 8, of 16 : 4 and of 15 : 9; consequently the continued product of 8×4×9×18 divided by the continued product of 3×16×15 must give the answer. From this explanation both the name of the rule is evident, and also the reason of the general rule in the text. The rule given in the text comprehends both what is commonly called the rule of five, and conjoined proportion. The principles upon which this rule depend are the same in every respect as for simple proportion. It would also be unnecessary to repeat here the rules for contractions as they are the same as those already given for simple proportion. In order to prevent confusion in the operation, I would recommend writing all the terms in the first column under a line, and the remaining terms above it, with the sign of multiplication between them, as in the above example. See also the example of contraction explained in the text at full length.

Questions in compound proportion may also be wrought by two or more operations of the single rule of three. The preceding example wrought by this method stands thus.

M. M. R.	D. D. R.	H. H. R.
As $\cancel{3}$: 8 : $\cancel{18}$	As $\cancel{16}$: $\cancel{4}$ :: 48	As $\cancel{15}$: $\cancel{9}$ :: 12
6	4 12	5 : 3 3
8	Roods	36
Roods 48		$\frac{36}{5}=7\frac{1}{5}$

The operation here is more tedious, and where there are fractional remainders, it renders it still more irksome to the young pupil.

There is another rule, given by Dr. Harris, in his Lexicon Technicum, frequently used for compound proportion, and is called stating by the blank. But it is applicable only when five terms are given to find a sixth, or to the rule of five as it is generall called, and is this ;—Let the principal cause of loss, or gain, interest or decrease, action or passion, be put in the first place. Let that which betokeneth time, distance of place, and the like, be put in the second place; and the remaining one in the third place. Write the other two terms under their like in the supposition. If the blank fall under the third term, multiply the first and second terms for a divisor, and the other three for a dividend. But if the blank fall under the first, or second term, multiply the third and fourth for a divisor, and the other three for a dividend. The quotient is the answer in the same name and denomination with the term which stands above the blank.

Example—If £100 in 12 months gain £5 interest; what will £70 gain in 8 months?

$$\begin{array}{ccc} \pounds & M. & \pounds \\ 100, & 12, & 5 \\ 70, & 8 & - \end{array} \qquad \frac{7\cancel{0}\times\overset{2}{\cancel{8}}\times\cancel{5}}{\underset{2}{\cancel{100}}\times\underset{3}{\cancel{12}}}=\frac{7}{3}=\pounds 2\ \ 6s.\ 8d.$$

In stating this example we put £100 in the first place, being the principal cause of gain, and 12 months which betokens time in the second place, and £5 the remaining term of the supposition in the third place. We then write £70 under £100, and 8 months under 12 months. The blank here falls under the third term, we therefore multiply the first and second, viz. £100, and 12 months for the divisor; and the third, fourth, and fifth terms, viz. £5, £70, and 8 months for the dividend; and by division we have $2\frac{1}{3}$, or £2 6s. 8d. If the blank had fallen under the £100 or under the 12 months, we would then have multiplied the £5 and the £70 for a divisor, and the other three for a dividend. When any term consists of different denominations, both it and the one with which it was compared must be reduced to the same denomination, as if they were the first and second terms of a stating in simple proportion. The other terms are not affected nor the proportion altered by this reduction.

VULGAR FRACTIONS.—In vulgar fractions the numerator is the remainder after division, and the denominator is the

divisor, and as they are supposed to have undergone no artificial preparation they are called vulgar and common fractions.

The greater the numerators of two or more fractions are which have the same denominator, the greater these fractions are, thus $\frac{4}{5}$ is greater than $\frac{3}{5}$, and $\frac{3}{5}$ is greater than $\frac{2}{5}$, &c.

A fraction is multiplied by any number, either by multiplying its numerator by that number, or dividing its denominator by it; thus $\frac{1}{12} \times 6$ is either $\frac{6}{12}$ or $\frac{1}{2}$.

A fraction is divided by any number, either by multiplying its denominator, or dividing its numerator by it; thus $\frac{8}{9}$ divided by 4, is either $\frac{2}{9}$, or $\frac{8}{36}$.

Multiplying, or dividing the terms of a fraction by the same number, does not alter its value, thus $\frac{2}{3} \times 4 = \frac{8}{12}$, and $\frac{8}{12} \div 2 = \frac{4}{6}$, but $\frac{2}{3}$, $\frac{8}{12}$, and $\frac{4}{6}$ are all equal to one another.

A fraction of any number, is equal to the sum of the like fractions, of the several parts of that number; thus $\frac{1}{3}$ of 12 is equal to $\frac{1}{3}$ of $9 + \frac{1}{3}$ of 3, for $\frac{1}{3}$ of 12 is 4, and $\frac{1}{3}$ of $9 + \frac{1}{3}$ of 3 is also 4; the same is true of any other number.

Any fraction of a number, is equal to the same number of times the like fraction of 1; thus $\frac{2}{5}$ of 4 is equal to $\frac{8}{5}$ of 1, and inversely, $\frac{2}{5}$ of 4 is equal to $\frac{1}{5}$ of 8.

The difference between like fractions of two numbers, is equal to the like fraction of the difference of these numbers; thus the difference between $\frac{1}{4}$ of 12, and $\frac{1}{4}$ of 8 is 1, but the difference between 12 and 8 is 4, the $\frac{1}{4}$ of which is also 1.

Fractions of the same denomination, which have equal numerators, but different denominators, that is the least which has the greatest denominator; thus $\frac{12}{19}$ is less than $\frac{12}{17}$.

One fraction is said to be the reciprocal of another, when the numerator of the one, is the denominator of the other, and the denominator of the one the numerator of the other; thus $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$.

Two fractions are equal when the products arising from multiplying the numerator of the one into the denominator of the other are equal; thus $\frac{1}{4}$ and $\frac{2}{8}$ are equal, because $1 \times 8 = 2 \times 4$.

When two fractions are equal, their reciprocals are also equal; thus $\frac{1}{3} = \frac{2}{6}$, and their reciprocals $\frac{3}{1}$ and $\frac{6}{2}$ are also equal.

REDUCTION OF VULGAR FRACTIONS.—The reason that reduction of fractions is placed before addition, &c. is because a knowledge of its operations is necessary to prepare fractions for addition, subtraction, &c.

CASE 1. The reason of the rule is this; if we both multiply

and divide a number by another number, the quotient is the same as the number multiplied.

The principle of the rule is this; multiplying any number by another, repeats each unit in the number multiplied as often as there are units in the multiplier, and this product being made the numerator of a fraction, and one of its factors the denominator, it is evident that the expression is equal to the number multiplied, which may again be obtained by division.

It has already been observed in the definitions, that a whole number may be expressed fractionally, by writing 1 for the denominator, which depends on the above principle.

CASE 2. The reason and principle of this rule are the same as the first. We may consider the denominator of the fractional part as the proposed denominator; and multiplying the integral part by it, the whole is reduced into parts equal to those in the numerator of the fraction, and these being added to the other, the sum is the numerator of the improper fraction, and will contain the denominator of the fractional part, as often as there were units in the original number, with the numerator of the fractional part for a remainder.

CASE 3. The reason of this rule is plain from the definition given of a fraction. Since the denominator represents the number of equal parts into which the unit is divided, it is therefore evident that so often as the denominator is contained in the numerator, so many units is the fraction equal to, and if there is a remainder, it is a fraction of the denominator.

This case is the reverse of the first and second, and the one serves as a proof of the other.

CASE 4. The reason of the rule may be shown thus; take for example $\frac{2}{3}$ of $\frac{5}{8}$, then because $\frac{2}{3}$ of any thing, is equal to $\frac{1}{3}$ of the double of that thing, consequently $\frac{2}{3}$ of $\frac{5}{8}$ is equal to $\frac{1}{3}$ of $\frac{10}{8}$, but $\frac{1}{3}$ of $\frac{10}{8}$ is equal to $\frac{10}{24}$ or $\frac{5}{12}$, therefore $\frac{5}{12}$ is the $\frac{2}{3}$ of $\frac{5}{8}$. It is evident that we have here multiplied 5 by 2, and 8 by 3 according to the rule. Had the fraction consisted of more terms, we might now have reduced $\frac{5}{12}$, and the next member of the fraction to a simple fraction in the same manner, and so on.

The principle of the rule is this; the continued product of the denominators brings them to the lowest name, and the continued product of the numerators ascertain the number of that name in the product.

It may also be remarked here, that before the rule can be properly applied, whole and mixed numbers must be reduced to improper fractions.—The work may also be shortened, and

the fraction brought out in the lowest terms by cancelling the upper and under numbers when they admit of it.

It is evident from this rule that the simple fraction will be the same in whatever order the compound fraction is placed,—nor is the result altered by putting the denominator of the one for that of the other, or the numerator of the one for that of the other. From what has been said, the method of dissolving a fraction, (that is of reducing a simple into a compound fraction) is evident. EXAMPLE. Dissolve $\frac{8}{15}$ into an equivalent compound fraction, $\frac{2}{3}$ of $\frac{4}{5}$, or $\frac{1}{3}$ of $\frac{2}{5}$ of 4, or $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{2}{5}$ of 4, &c. are all equal to $\frac{8}{15}$.

Dissolving of fractions is not of much practical utility.

A compound fraction is always known by the word OF between its parts thus, $\frac{2}{3}$ of $\frac{7}{8}$.

CASE 5. Although this case does not properly belong to reduction of fractions, nor are the examples given strictly fractional, yet I have taken the liberty of introducing it in this place, not so much from having met with it in a similar situation in some of our best writers on arithmetic, as from the circumstance that such expressions as this case embraces, may, and often do occur in actual business, which I hope will be an apology for giving it a place here. I flatter myself that the rule has also been rendered more intelligible to the young arithmetician than that commonly given for this case.

The reason of the operations will appear evident when we consider, that, by taking away the denominator of a fraction, we in effect multiply the numerator by it. Again multiplying each of the numerators by the denominator of the other, we thus multiply both the terms by the same number, and therefore do not change the value of the expression. By this operation the involved expression is brought into a simple fraction, and made subject to the same rules. EXAMPLE. Reduce $\frac{9\frac{1}{8}}{12\frac{1}{5}}$ to a simple fraction: $\frac{9\frac{1}{8}}{12\frac{1}{5}} = \frac{\frac{73}{8}}{\frac{61}{5}}$, according to the explanation given above, when we take away the divisor 8 from the upper fraction we multiply the numerator by 8, again multiplying it by 5, the denominator of the under fraction, we thus multiply $\frac{73}{8}$ by 8 and 5, which gives 365 for the numerator of the simple fraction, and when we take away 5 the denominator of the under fraction, we multiply it by 5, and then multiplying it by 8, the denominator of the upper fraction, we thus multiply $\frac{61}{5}$ by 5 and 8, which gives 488 for the new denominator. The simple fraction is therefore $\frac{365}{488}$, and it is plain we have multi-

plied both the upper and under terms by the same numbers, consequently $\frac{9\frac{1}{8}}{12\frac{1}{5}}$ is equal to the simple fraction $\frac{365}{488}$.

CASE 6.—All the operations in this case are performed by simple proportion, the reasons and principles of it are therefore the same as already explained under that rule. It is evident from the nature of proportion, that the rule will give a new denominator in all cases having the same ratio to the new numerator which the original denominator had to its numerator. But wherever the new denominator assumes the form of a mixed number, as in the example wrought in the text, the thing proposed is impossible in pure fractions, as they then become what is called complex fractions, which, as stated in the notes to last case, are not properly fractional. I have introduced some questions where the answer comes out in a complex form, with a view to accustom the student to the proper management of fractional expressions. I have preferred simple proportion in this case, as the operation is generally better understood by it, than by the rule commonly given, which is, "Multiply the given numerator by the denominator of the given fraction, and divide the product by its numerator, the quotient is the corresponding denominator sought." It is evident that the process here directed is the same as by proportion, although differently expressed.

CASE 7.—The operations in this case are also by simple proportion. In all cases where the numerator assumes the form of a mixed number or fraction, as in the 2d. and 5th. examples in the text, the thing proposed is impossible in pure fractions. My reasons for introducing such examples, and for preferring a stating by proportion, to the common method, are given under case 6th. This case and the 6th. are not generally given, yet, as a knowledge of them may sometimes be of use, and as they are easily acquired, I have introduced them. There is another rule sometimes given for this case, namely, multiply the given denominator by the numerator of the fraction, and divide the product by its denominator. The reason of the rule is this, dividing the divisor and dividend by the same number does not alter the value of the quotient. But from the definition of a fraction, the denominator is the divisor, and the numerator the dividend, therefore dividing them by any number, which divides them without remainder, does not alter the value of the fraction. When no number greater than unity divides the terms without remainder, the fraction is evidently in its least terms.

In order to discover what numbers will divide the terms of a fraction without remainder, the following observations may be of service.

When the terms of a fraction terminate in an even number, or a cipher, they are divisible by 2, or its multiples, thus $\frac{12}{34}$ is divisible by 2, and $\frac{20}{48}$ is divisible by 2 or 4.

When both terms of a fraction terminate in 5 or 0, they are divisible by 5, thus $\frac{25}{55}$, and $\frac{25}{30}$ are both divisible by 5.

When both terms have one or more ciphers on the right, an equal number of them may be cut off from each, without altering the value of the fraction, thus $\frac{600}{700} = \frac{6}{7}$.

When the sum of the digits in the terms of a fraction are divisible by 3 or 9, the terms are divisible by 3 or 9, thus $\frac{84}{135} \div 3 = \frac{28}{45}$, and $\frac{675}{783} \div 9 = \frac{75}{87} \div 3 = \frac{25}{29}$.

When any number which is expressed by the sum, or difference of several other numbers, is to be divided by any number, all its parts must be divided by that number, thus

$$\frac{6+9+12-3}{3} = 2+3+4-1 = 8.$$

A fraction may be brought to its lowest terms at once by dividing both terms by their greatest common measure, which is found thus; divide the greatest term by the least, and this divisor by the remainder, and so on till nothing remains, the last divisor is the greatest common measure, but if the last divisor be one the fraction is already in its lowest terms.—EXAMPLE. Find the greatest common measure of $\frac{846}{981}$.

```
846)981(1
    846
    ---
    135)846(6
        810
        ---
         36)135(3
            108
            ---
             27)36(1
                27
                --
                 9)27(3
                   27
                   --
```

I here divide 981 by 846, again I divide 846 by the remainder 135, and so on as in the example, and as 9 is the divisor which leaves no remainder, it is the greatest common measure of $\frac{846}{981}$, that is, the greater number which divides both without remainder.

The reason of this operation, and the truth of the rule may be deduced from the established principles, that a number which divides other two numbers, will divide their sum, and difference, and any multiple of these numbers.

In the above example, since 9 divides 9 and 27 it will also

divide their sum 36, and 108, which is a multiple of 36, and since it divides 27 and 108, it will divide their sum 135, and since it divides 135 it will also divide its multiple 810, and since it divides 810 and 36 it will divide their sum 846, and because it divides 846 and 135 it will divide their sum 981; 9 is therefore a common measure of $\frac{846}{981}$. But 9 is also their greatest common measure. For if not, some number greater than 9 will measure 846 and 981, and if so, it will also measure their difference 135, and because it measures 135 it will measure its multiple 810, and since it measures 846 and 810 it will measure their difference 36 and 108 a multiple of 36, and because it measures 135 and 108 it will measure their difference 27, and since it measures 27 and 36, it will also measure their difference 9, that is, 9 contains, or is divisible by a number greater than itself, which is absurd; therefore 9 is the greatest common measure of $\frac{846}{981}$. The truth of the rule is therefore clearly demonstrated. When a fraction is to be brought to its lowest terms, and no number can be discovered by inspecting the nature of the terms, which will divide both without remainder, we always have recourse to this rule, *as in these examples.* Reduce [illegible], [illegible], and [illegible] to their lowest terms. Ans. [illegible], [illegible], and [illegible].

CASE 9.—Before the rule for this case can be applied, it will be necessary to reduce compound fractions to simple ones, and mixed numbers to improper fractions. It will also be proper to reduce the fractions to their lowest terms, and although this is not absolutely necessary, yet it tends much to abreviate the operation.

The reason of the rule given in the text is this, the numerators and denominators of each fraction are multiplied by the same numbers, therefore their value is not altered.

This may be illustrated by the following example.

Reduce $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{5}{8}$ to a common denominator.

$$\left.\begin{array}{r} 4\times3\times8=\ 96 \\ 2\times5\times8=\ 80 \\ 5\times5\times3=\ 75 \end{array}\right\}\text{ new numerators.}$$

$$5\times3\times8=120 \quad \text{common denominator.}$$

When the numerators 4, 2, 5 are written without their denominators, they are in effect multiplied by them. Therefore 4 the first numerator is multiplied by 5, 3, 8; and 2 the second numerator is multiplied by 5, 3, 8; also 5 the third numerator is multiplied by 5, 3, 8, therefore all the numerators are multiplied by the same numbers; and the

continued product of 5, 3, 8 is the common denominator, consequently the fractions are not altered in their value.

Although the fractions are all in their least terms before reducing to a common denominator, yet they are not always so, after being reduced by this rule, in which case they may be brought to their lowest terms by dividing them by any number which divides all the numerators, and also the common denominator without remainder. Fractions may always be reduced to their least common denominator by the following rule, which is also the best general method of abbreviating the operation. Find the least common multiple of all the denominators, by dividing them by any small number, which divides two or more of them without remainder; then divide these quotients, and the numbers not divided, by any number which divides two, or more of them without remainder; and continue this process till no number greater than unity will divide any two of them; the continued product of the numbers used as divisors, and the numbers remaining undivided, is the least common denominator, the numerators to which are found by case 7.

Example. Reduce $\frac{6}{7}$, $\frac{9}{14}$, $\frac{5}{12}$, $\frac{6}{18}$ to their lowest common denominator.

$$\begin{array}{r|llll} 7 & 7, & 14, & 12, & 18 \\ 6 & 1, & 2, & 12, & 18 \\ 2 & 1, & 2, & 2, & 3 \\ & 1, & 1, & 1, & 3 \end{array}$$

$7 \times 6 \times 2 \times 3 = 252$ common denominator.

$$\left.\begin{array}{r} 7 : 252 :: 6 : 218 \\ 14 : 252 :: 9 : 162 \\ 12 : 352 :: 5 : 105 \\ 18 : 252 :: 6 : 84 \end{array}\right\} \text{new numerators.}$$

$$\frac{218}{252}, \frac{162}{252}, \frac{105}{252}, \frac{84}{252} = \text{new fractions.}$$

In order to perform this example by the rule now given, write the denominators as in the margin. It is then evident that 7 will divide 7 and 14, I therefore write 7 on the left, and divide 7 and 14 by it, writing the quotients, and the numbers not divided in another line, I then divide 12 and 18 by 6, writing the quotients as before, and lastly I divide this line by 2, the quotients are now 1, 1, 1, 3, and as no number greater than unity will divide any two of them, I then multiply the several divisors and the undivided numbers, which may be considered as divisors, together, thus $7 \times 6 \times 2 \times 3 = 252$, which is the least common denominator. I then find the numerators, by case 7th.

Reason of the rule. The least common multiple of two or more numbers, must be the product of the least factor by which these numbers can be produced, and it is evident the rule discovers these factors, for 7×2 is the least number divisible by 7 and 14, and 6×2 is the least number divisible by 12, also $7 \times 2 \times 6$ is the least number divisible by 7, 14, 12, again 6×3 is the least number divisible by 18, consequently $7 \times 2 \times 6 \times 3 = 252$ is divisible by 7, 14, 12, 18. It is also evident that 252 is the least number divisible by 7, 14, 12, 18, for the least possible factors of these numbers are also the factors of 252. Therefore the rule not only gives a common denominator, but also the least possible common denominator.

Since the new numerators to the common denominator are found by case 7, it is evident that they must have the same proportion to the common denominator as the original numerators had to their respective denominators, therefore the value of the fraction is not altered.

It may also be remarked here, that the new numerators in this case can never come out in the form of a mixed number; because the second term of the proportion is invariably a multiple of the first; therefore dividing the second term by the first, and multiplying the quotient by the third must give a whole number.

When the denominators of the given fractions are small, the least common denominators may often be discovered by inspection, without the formal application of any rule, which I shall illustrate by an example. Let it be required to reduce $\frac{1}{6}$, $\frac{3}{5}$, $\frac{5}{12}$, $\frac{7}{15}$ to a common denominator. By inspection, I see at once that no number less than 60 is a common multiple of 12 and 15, and also, that 60 is a common multiple of 6 and 5. I therefore multiply the terms of each fraction by any number which makes the denominator 60, and the fractions stand thus, $\frac{10}{60}$, $\frac{36}{60}$, $\frac{25}{60}$, $\frac{28}{60}$. When the student or accountant is acquainted with this method, he will naturally have recourse to it, whenever he is called upon to reduce fractions to a common denominator, whether the least common denominator be required or not, and it is only when it cannot be quickly discovered by inspection, that he will ever have recourse to any other method. The principle of this method is the same as last.

Another rule sometimes given is this: divide the product of any two of the denominators by the greatest common measure of each, multiply the quotient by another denominator, and divide the product by their greatest common measure, and so on, the last

quotient is the least common denominator.—EXAMPLE. Reduce $\frac{1}{4}$, $\frac{4}{12}$, $\frac{7}{9}$ to their least common denominator.
$\frac{4 \times 12}{4}$ 12, $= \frac{12 \times 9}{3} = 36$, common denominator, $\frac{9}{36}$, $\frac{12}{36}$, $\frac{28}{36}$ new fractions. The principle of this rule is the same as the two preceding. There are other rules given for reducing fractions to a common denominator, but as they are only particular cases of the general rule, it would be unnecessary to enumerate them. A thorough knowledge of this case is absolutely necessary to expertness and accuracy in fractional calculations.

CASE 10. The principle of this case is the same as case 4. The reason of the rule will appear evident from this example; reduce $\frac{1}{4}$ of a £ to the fraction of a farthing.
$\frac{1}{4}$ of $\frac{20}{1}$ of $\frac{12}{1}$ of $\frac{4}{1} = \frac{240}{1}$. It is evident that $\frac{1}{4}$ of a £ is equal to $\frac{1}{4}$ of 20/, and 20/ multiplied by 12 is equal to 240d., and multiplied by 4 is equal to 960 qrs., therefore $\frac{960}{4}$ qrs., or $\frac{240}{1}$ qrs., is evidently equal to $\frac{1}{4}$ of a £ according to the rule. If it had been required to reduce a farthing to the fraction of a £, the denominator in this case must have been multiplied by 4, 12 and 20 for a similar reason.

The rule commonly given for this case is, when the reduction is from a greater name to a less, multiply the numerator by as many of the less name, as make one of the greater; and when it is to a greater name, multiply the denominator by the same number. The principle of this rule is evidently the same, although the multipliers are not arranged in the form of a compound fraction, which I have recommended as being more consistent with fractional operations. This case comprehends the first and second rules for reduction of whole numbers, where we are taught to bring a higher denomination to a lower by multiplication, and a lower to a higher by division, which is the very operation this rule points out, for we multiply a fraction when we multiply its numerator, and we divide it when we multiply its denominator.

CASE 11.—Principle of the rule. By reducing both the given number and the integer to the same denomination, we do not alter their value, consequently the numerator bears the same proportion to the denominator which the number to be reduced does to the integer. This case is so plain that it scarcely requires any illustration. For suppose it were required to reduce 3/6 to the fraction of £, we would reduce 3/6 in-

to sixpences, and write under them the sixpences in a £. But 3/6 is equal to 7 sixpences, and a £ to 40 sixpences, therefore $\frac{7}{40}$ is the fraction required. From this the truth of the rule is evident, because when a £ is divided into 40 equal parts, 3/6 is equal to 7 of these parts. It is not necessary in all cases to reduce the given quantity to its lowest terms as directed in the general rule, because the greatest common denomination answers the same purpose and shortens the process. For this reason, in the above example, I have reduced 3/6 to sixpences, instead of pence, which would have given the fraction $\frac{42}{240}$, and this reduced to its lowest terms is $\frac{7}{40}$, as above.

CASE 12.—It has already been remarked in the introduction to fractions, that the numerator of a proper fraction is the remainder of a division, of which the denominator is the divisor. This case therefore comes directly under the general rule for compound division. The numerator is so many integers of the denomination which the fraction expresses, and the denominator is the divisor. For example, to value $\frac{5}{12}$ of a £, it is evident that $\frac{5}{12}$ of a £ is the same as £5 divided by 12, but since we cannot divide 5 by 12, we reduce £5 to shillings and divide them, which gives 8 shillings and $\frac{4}{12}$, or 4 pence, which is precisely the process in compound division, or the valuing of remainders in proportion, both of which, with their reasons, and principles are already explained. The operation may also be performed, by reducing the numerator to the lowest name, and dividing by the denominator, and then reducing the quotient to integers of a higher denomination. Thus £5 is equal to 4800 qrs. and $\frac{4800}{12} = 400$ qrs., and $\frac{400}{4} = 100$d., and $\frac{100}{12} = 8/4$ as before. But as this method nearly doubles the labour, without making the process either more accurate or more intelligible, it is scarcely ever employed.

ADDITION OF VULGAR FRACTIONS.—Reason of rule. Fractions which have not the same denominator, although they are of the same integer, are dissimilar, and cannot therefore be incorporated, neither can fractions of different denominations be incorporated, they also being dissimilar; but as soon as they are brought to the same denomination, and to a common denominator, they express so many equal parts of the same thing, and consequently the sum of the numerators placed over the common denominator, must be equal to all the given fractions.

Illustration. Add together £$\frac{5}{8}$, £$\frac{2}{3}$, $\frac{7}{8}$s. We cannot add $\frac{5}{8}$ and $\frac{2}{3}$ of a £ together, although they both belong to the same

integer, because they are not similar parts of that integer; neither can we add these fractions of a £ to $\frac{7}{8}$ of a shilling, because they are of different denominations. We must therefore reduce these fractions, first to the same denomination, by bringing them all to the fraction of a shilling, namely, $\frac{100}{8}$s., $\frac{60}{5}$s., $\frac{7}{8}$s., and then to a common denominator, we have $\frac{500}{40}$s.+$\frac{480}{40}$s.+$\frac{35}{40}$s.=$\frac{1015}{40}$s. = £1 5s. 4½d.

When whole or mixed numbers are given along with the fractions, it is best to add the whole numbers to the sum of the fractions, without involving them in the operation.

When fractions of different denominations are given, it is often better to find their value, by case 12 of reduction, and add these values, than to proceed by the general rule.

SUBTRACTION OF VULGAR FRACTIONS.—The reasons and principles of this rule are the same as already explained in addition of vulgar fractions. For the fractions being brought to a common denominator, it is evident that the difference of their numerators placed over the common denominator, must be the difference of the fractions. If the difference between several fractions, and several other fractions is required, take the sum of the one from the sum of the others, the remainder is their difference. When fractions of different denominations are given, it is often better to find their value by case 12th. of reduction, and subtract these values.

MULTIPLICATION OF VULGAR FRACTIONS.—Reason of the rule. To multiply one number by another is to repeat the multiplicand as often as there are units in the multiplier, consequently when the multiplier is less than unity, the product must be the same part of the multiplicand, which the multiplier is of unity, and this product is evidently obtained by multiplying the number by the numerator of the fraction and dividing the product by its denominator. But when the number to be multiplied is also a fraction, it is evident that the product of the numerators must be divided, both by the denominator of the multiplicand and multiplier, or by their product; therefore fractions are multiplied by writing the product of their denominators under the product of their numerators, as the rule directs, which is the same as case 4th. in reduction. I shall illustrate the reason of the rule thus explained by an example.

Suppose it were required to multiply 4 by 2, the product 8, is equal to 4 repeated twice, or as often as there are units in 2, and 4 multiplied by 1, the product is 4, for the same reason.

But $4 \times \frac{1}{2} = \frac{4 \times 1}{2} = 2$, and 2 is the same part of 4 that $\frac{1}{2}$ is of 1, again $\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$. From this explanation it appears that the product of two proper fractions must be less than either of the factors, which is directly the reverse of the effect produced in the multiplication of integers, and the reason is obvious, for the product is only such a part of the one fraction as the other is of unity. To multiply an integer or whole number by a mixed number, we may bring both factors to improper fractions, and work by the general rule; or we may multiply by the integers and fractions separately, and add the products; or we may multiply by the integral part of the mixed number, and divide the numerator of the fraction into aliquot parts of its denominator, then work by practice and add the quotients to the former product. Thus to multiply 16 by $5\frac{3}{8}$ according to the first method, $\frac{16}{1} \times \frac{43}{8} = \frac{688}{8} = 86$; by the second method $(16 \times 5) + (16 \times \frac{3}{8}) = 80 + 6 = 86$; and by the third method,

$$\begin{array}{c|c|r} \frac{2}{8} & \frac{1}{4} & 16 \\ & & 5 \\ \hline & & 80 \\ \frac{1}{8} & \frac{1}{2} & 4 \\ & & 2 \\ \hline & & 86 \end{array}$$

When both factors are mixed numbers, reduce them to improper fractions, and work by the general rule. The work may always be abreviated when we can cancel as directed in the notes to the 4th. case of reduction. When the continued product of several fractions is required, multiply all the numerators together, and all the denominators together, for the numerator and denominator of the product, thus the continued product of

$$\frac{1}{\cancel{2}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{2}}{5} \times \frac{\cancel{4}}{\underset{3}{\cancel{9}}} = \frac{1}{15}.$$

DIVISION OF VULGAR FRACTIONS.—Reason of the rule. By the process prescribed in the rule, fractions are brought to a common denominator, and the quotient is represented by the numerator of the dividend divided by the numerator of the divisor; and as they are both in similar parts of the integer they must be the quotient required.

I shall illustrate the reason as here stated by an example.

Let it be required to divide $\frac{4}{5}$ by $\frac{3}{8}$. Inverting the divisor and multiplying we have $\frac{4}{5} \times \frac{8}{3} = \frac{32}{15} = 2\frac{2}{15}$ according to the rule. But if we reduce $\frac{4}{5}$ and $\frac{3}{8}$ to a common denominator, we have $\frac{32}{40} \div \frac{15}{40}$, by taking away the common denominator, and dividing the numerator of the dividend by the numerator of the divisor, which are similar parts of the integer, we have $\frac{32}{15} = 2\frac{2}{15}$ as before. Inverting the divisor and multiplying is evidently the same as reducing the fractions to a common denominator, and dividing the numerator of the dividend by the numerator of the divisor.—The reason of the rule may also be shown thus, let it be required to divide $\frac{4}{5}$ by $\frac{3}{8}$; it is evident that $\frac{4}{5} \div 3 = \frac{4}{15}$, for we divide the fraction by multiplying its denominator by 3, but $\frac{3}{8}$ is only $\frac{1}{8}$ part of 3, therefore $\frac{3}{8}$ will be contained 8 times as often in the dividend as 3 is, consequently the quotient $\frac{4}{15} \times 8 = \frac{32}{15} = 2\frac{2}{15}$ as before. Here also it is evident that we multiply 5 the denominator of the dividend, by 3 the numerator of the divisor, and 4 the numerator of the dividend, by 8 the denominator of the divisor, which is the process prescribed in the general rule.

It must appear from these explanations that the general rule might be expressed in a way, perhaps more intelligible to the young arithmetician, namely; reduce the fractions to a common denominator, and divide the numerator of the dividend by the numerator of the divisor; but this rule is not so concise, and should only be used to explain the nature of the process.

When the numerator, and denominator of the dividend, are divisible, by the numerator, and denominator of the divisor, let them be divided thereby, thus $\frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$, which is simpler than the general rule.

Integers and mixed numbers must be reduced to improper fractions before the general rule can be applied, thus $3\frac{1}{4} \div 6\frac{1}{5} = \frac{13}{4} \div \frac{31}{5} = \frac{13}{4} \times \frac{5}{31} = \frac{65}{124}$. and $7 \div \frac{5}{6} = \frac{7}{1} \div \frac{5}{6} = \frac{7}{1} \times \frac{6}{5} = \frac{42}{5} = 8\frac{2}{5}$.

To divide a fraction by an integer, either divide the numerator or multiply the denominator by it, thus $\frac{8}{13} \div 4 = \frac{2}{13}$ or $\frac{8}{52}$. The quotient arising from dividing by a proper fraction is always greater than the dividend, thus $5 \div \frac{3}{4} = \frac{20}{3} = 6\frac{2}{3}$, and $\frac{8}{9} \div \frac{2}{3} = 1\frac{1}{3}$.

The reason of this will appear evident from the nature of division, the quotient being always equal to the number of times the dividend contains the divisor. Thus the quotient of $12 \div 2 = 6$, because 12 contains 2 six times; and $12 \div 1 = 12$ for the same reason. But $12 \div \frac{1}{2} = 24$, because 12 contains $\frac{1}{2}$ 24 times; and $12 \div \frac{1}{4} = 48$, because 12 contains $\frac{1}{4}$ 48 times.

When the divisor is less than unity, the quotient is greater than the dividend in the same proportion that unity is greater than the divisor. Division of fractions is proved by multiplication the same as whole numbers.

SIMPLE PROPORTION.—The definition, the reasons, and the principles of proportion in vulgar fractions are all the same as already explained in whole numbers, and therefore require no farther illustration here. The reason for inverting the divisor is fully explained in the notes on division of vulgar fractions. The method of operation is also fully explained in the example wrought in the text. Proportion in vulgar fractions is frequently omitted by writers on arithmetic, for what reason I know not. But I have observed without exception, during many years experience in teaching arithmetic, that young gentlemen who have studied systems in which this rule is not given, never attempt working a proportion account by fractions, however much the operation might thereby be abbreviated. This appears a satisfactory reason for giving a few examples both in simple and compound proportion, for nothing can with propriety be omitted in a system of arithmetic, the want of which tends directly to increase the labour of the practical arithmetician.

COMPOUND PROPORTION IN VULGAR FRACTIONS.—As the rule for compound proportion in vulgar fractions is the same in substance as for the whole numbers, it would be unnecessary to repeat its reasons and principles in this place. The reason for reducing the numbers in the first and second columns to the same denomination in pairs is also explained in the notes to compound proportion in whole numbers.

It has also been shown in division of fractions, that by inverting the divisor and multiplying, we actually divide by the fraction, and this effect is not altered whatever number of fractions the divisor may consist of, because the quotient is the same, whether we divide by a number all at one division, or continually by its factors.

The same reasons have induced me to give a place to compound proportion in vulgar fractions, which are already stated for introducing simple proportion; and although few examples are given under either of these rules, they will be found sufficient to make the student of arithmetic acquainted with the nature of the process, and able to apply the rules whenever occasion requires.

DECIMAL FRACTIONS.—The definition of a decimal fraction in the text is different from that which is generally given. But from a mature consideration of the nature of decimals, and of the process by which they are produced, it appears equally natural, and more intelligible than the other.

Besides, a decimal fraction is nothing more than a proper vulgar fraction expressed upon the denary scale, and as our present system of calculation is by that scale, fractions expressed upon it are much more consistent with the plan observed in whole numbers, than what vulgar fractions are, which vary their scale with every variation in the denominator.

The definition of a decimal fraction commonly given is, a fraction whose denominator is 1, with as many ciphers annexed as there are figures in the numerator, thus $\cdot 5 = \frac{5}{10}$, $\cdot 67 = \frac{67}{100}$, &c. This definition is perfectly correct, so far as the value of a decimal is concerned, yet it is unnecessary, and tends directly to perplex the young arithmetician, and to damp his spirits, by the prospect of new difficulties when entering upon this important and interesting part of the science. Instead therefore of representing them as a species of vulgar fractions, whose denominators are always suppressed, I have decidedly preferred defining them as a continuation of the numeration scale descending, and decreasing from unity *ad infinitum*, in the same ratio, and on the same principles as whole numbers decrease to unity. Farther, to show that decimals, strictly speaking, have no denominators more than whole numbers, it will only be necessary to observe the following example, viz. 222·222, here the highest place is hundreds, the second tens, and the third units, the fourth tenths, the fifth hundredths, and the sixth thousandths, thus decreasing all along in the same ratio from the left to the right, however far the whole number, or the decimal extends. From this it is obvious that the numbers are all on the same scale, and subject to the same laws, and therefore equally unnecessary to conceive the decimal having a denominator, as it is to conceive the whole number having a denominator. So long then as decimal fractions are either represented or defined by vulgar fractions, much both of their beauty and simplicity remains obscured.

Decimals have been classed under the two general heads of terminate and interminate. The terminate, or finite decimal is one which expresses exactly the value of the vulgar fraction from which it is derived, as $\frac{1}{2} = \cdot 5$, $\frac{3}{4} = \cdot 75$. The interminate, or infinite decimal, is one which, if extended ever so far, would still be less than the value of the vulgar fraction

from which it is derived. Thus the decimal of $\frac{1}{3}$ is $.\dot{3}33$ &c. but this decimal extended to any proposed length is still less in value than $\frac{1}{3}$, neither can $\frac{2}{11}$ ever be equalled in value by its decimal, which is $\cdot\dot{1}\dot{8}$ repeated for ever; the same is true of all decimals which repeat or circulate. It is worthy of observation, that repeating and circulating decimals afford an interesting specimen of a converging series. For example $\frac{2}{3}$ cannot be equalled by its decimal $\cdot\dot{6}6$, &c. for although every figure annexed to the decimal, brings it nearer to the value of $\frac{2}{3}$, yet if it were extended through boundless space, during all the revolutions of endless time, it would still be less than $\frac{2}{3}$.

DECIMAL NUMERATION.—From what is already said upon the definition of decimals, it appears almost unnecessary to add any thing to what is said in the text, respecting the method of reading and writing them. Let it suffice here to have an example or two on the common method of decimal numeration. Suppose it were required to express .6, .54, .968 in order to read them, they would be written thus, $\frac{6}{10}$, $\frac{54}{100}$, $\frac{968}{1000}$, and so of any other decimals. But this presupposes that vulgar fractions are already understood, which, however useful, is not absolutely necessary in order to acquire a knowledge of decimals. Other writers on Arithmetic have recommended a different method of reading decimals, for example, .3672 would be read thus, 3 primes, 6 seconds, 7 thirds, 2 fourths. It is evident this method might also be applied to whole numbers, but he must know little of the power and beautiful simplicity of our present numeration scale, who would discard it, in order to introduce this method. Were the original ABACUS still in use, a decimal fraction expressed upon it would appear much less mysterious to the young arithmetician, than by the application of its skeleton now used. But having surmounted the necessity of using even the semblance of the abacus in whole numbers, there appears no good reason for retaining it in the numeration of decimals.

The reason why ciphers on the right of decimals have no influence upon their value is, they do not remove the significant figures nearer to the decimal point, which is the place of their highest possible local value. For example, .56 still remains 5 tenths, 6 hundredths, although written thus .5600.

The reason why ciphers on the left of decimals diminish the value of the significant figures is, that every cipher coming between the decimal point, and a significant figure, removes it farther from the decimal point, and thus diminishes its local

value in the same ratio as whole numbers are diminished, by removing them farther to the right. For example, .5 is five-tenths, and .05 is only five hundredth parts of unity, or the tenth part of what it was in its former position, which is the same proportional diminution of value which takes place when an integer is removed a place to the right.

REDUCTION.—CASE 1. The reason of the rule is this, annexing a cipher to the numerator of a fraction, is in effect multiplying it by 10, and when divided by the denominator, each unit in the first quotient figure is a decimal, or tenth part of a unit in the numerator of the original fraction. Again, annexing a cipher to the remainder we multiply it by 10, therefore every unit in the second quotient figure is only the tenth part of a unit in the first, and so on with every succeeding figure. It has already been shown in division of whole numbers, when the divisor is not contained in the dividend, it cannot be contained in it more than 9 times after annexing a figure; consequently there cannot be more than one decimal place in the quotient, for each cipher annexed during the process. Neither can there be less than one for each cipher annexed, because if we annex two ciphers to the numerator, before it contain the denominator or divisor, we have thus multiplied by 100, and therefore the first quotient figure is hundredth parts, and must be removed to its proper place, by writing a cipher between it and the decimal point, and so on for any other number of ciphers annexed. From this the reason of the whole rule is clear; the nature of decimals manifest, and the impossibility of there being either more or fewer decimal places in the quotient, than the number of ciphers annexed in the process is evident.

Several methods have been taken to show the reason of this rule, some have shown it thus, suppose $\frac{5}{8}$ to be reduced, then 5 the numerator is equal to $\frac{5000}{1000}$ and $\frac{5000}{1000} \div 8 = \frac{625}{1000} = .625$. Although this method is quite intelligible to those who are perfectly acquainted with vulgar fractions, yet it appears doubtful if the young arithmetician can derive much benefit from it. Others have explained it in the following manner; because the denominator of a decimal fraction is 10, 100, &c. and in equal fractions the denominator of the one is to the denominator of the other, as the numerator of the one to the numerator of the other, therefore $8 : 1000 :: 5 : \frac{1000 \times 5}{8} = \frac{5000}{8} =$

$\frac{625}{1000} = .625$ as before. This method is founded on the same principles as the last, and is subject to the same objection.

When the denominator of a vulgar fraction reduced to its least terms, is 2 or 5, or any power of 2 or 5, or the product of any power of these numbers, the decimal corresponding to it is finite, but in every other case, it must either repeat or circulate. The reason is this, whenever we affix a cipher to the numerator of a vulgar fraction it becomes 10, or some multiple of 10, and no numbers but 2 or 5, or a power of 2 or 5, or the product of a power of 2, by a power of 5, can possibly measure 10 and its multiples. Bonnycastle, in his arithmetic, 11th edition, in a note on this rule, says, "if the right hand figure of the denominator of the fraction to be reduced, be an even number, an 0 or a 5, the quotient will consist of some exact number of decimals." But that learned author must not have examined this assertion with his usual care, for example $\frac{5}{14}$, $\frac{22}{36}$, $\frac{4}{15}$, terminate n all the different ways he mentions, and yet none of these produce terminate decimals, nor is it possible that any decimal can be terminate unless the denominator of the fraction be such as already mentioned. In all cases where the decimal is not terminate, it is less than the vulgar fraction; but it can always be brought nearer to the value of that fraction than any proposed difference. For when the decimal is carried out one place, it cannot want the 10th. part of a unit, when carried 2, it cannot want 100th. part of a unit, and when carried 3, it cannot want 1000th. part of a unit of the true answer. In this way we can have the answer sufficiently correct for any practical purpose, without carrying out many decimal places, but the number of decimal places must depend entirely upon the nature of the integer, and the degree of accuracy required.

When dividing any number by another, the remainder may be carried out in a decimal, by annexing ciphers, and thus carrying on the division to any degree of accuracy required.

When the decimal is to be carried out to a great number of places, the following method given in Colson's Commentary on Sir Isaac Newton's Fluctions, page 162, is perhaps the simplest yet discovered. It is this, suppose the decimal of $\frac{1}{29}$ is required, and having carried out the decimal by the common method for a few places, or till the remainder is a single figure, thus $\frac{1}{29} = .03448\frac{8}{29}$, then multiply this quotient by the numerator of the fractional part, gives $\frac{8}{29} = .27586\frac{6}{29}$, which being substituted for the fraction in the first quotient gives $\frac{1}{29} = .0344827586\frac{6}{29}$, again if we multiply this quotient by 6, the numerator of the fractional part, and substitute the product in place of the fraction, it will give us other 9 additional places of decimals, and this process may be carried to any length, and each step doubles the former number of decimals at least. This

method may be employed with great advantage whenever many decimal places are required.

CASE 2. Although this is generally given as a distinct case in decimals, yet it is easily reduced to the form of the first case. For example; let 6s. 5½d. be given, to be reduced to the decimal of a £. If we take the farthings in 6/5½ for the numerator, and the farthings in a £ for the denominator of a vulgar fraction, we have $\frac{310}{960}$, which may be reduced as in the first case. The rule in the text has been preferred to the method of first reducing to a vulgar fraction, on account of the brevity of the operation by it. It is therefore evident that the reason of this rule, and the principles upon which it is founded are the same as in Case 1st.

The reason of this rule has also been explained upon the principles of vulgar fractions, thus resuming the above example, $\frac{1}{2} = .5$d. therefore $5\frac{1}{2} = 5.5\text{d} = \frac{55}{10} = \frac{55}{120}$ s. which reduced to a decimal gives $.458\dot{3}$ s. therefore $6 ,, 5\frac{1}{2} = 6.458\dot{3}$ s. $= \frac{64583}{10000}$ s. $= \frac{64583}{200000}$ £, which being reduced, gives $.32291\dot{6}$ as before.

This case may also be performed, by multiplying the lower denominations continually by 10, as in compound multiplication, counting the integers thus produced for decimals in the order in which they are produced, and reckoning a decimal place for each multiplication. Taking the above example, 6/5½.

	s.	d.
	6	5 ½
		10
3.	4	7
		10
2.	5	10
		10
2.	18	4
		10
9.	3	4
		10
1.	13	4
		10
$\dot{6}$.	13	4

The figures here in the order in which they occur in the work give $.32291\dot{6}$ as before; and as the same number of shillings and pence recur in the two last steps, we conclude that 6 is a repeater. This method shows the nature of decimal fractions distinctly, yet it is too tedious for practice and therefore seldom used.

CASE 3. If decimal fractions were generally introduced into the counting house, this case would be of the greatest importance, and it would in general be unnecessary to carry the decimal beyond 3 places, which is exact enough for most practical purposes.

The reason for halving the shillings for the first decimal place is this; because 20 shillings make a £, 2 shillings are the 10th part of a £, therefore dividing any number of shillings by 2 reduces them to tenth parts,

or decimals of a £. . The reason for adding 1 to the farthings in the remainder, for every 24, to give the decimal correct in the second and third places is this; if there had been 1000 qrs. in a £, then 1 qr. would have been the thousandth part of a £, or 1 in the third decimal place, but 960, the qrs. in a £, wants 40 of being a 1000, and 40 is the 24th part of 960, therefore adding 1 for every 24 in the qrs. converts them into the decimal of a £. Again, the reason for prefixing a cipher when the qrs. are under 10 is this; when the qrs. are expressed in one figure, it is thousandth parts according to the rule, and must occupy the 3d. decimal place, consequently it must have a cipher before it, to remove it to the proper place in the scale.

When the qrs. amount exactly to 24, 48, or 72, adding according to the rule makes the decimal terminate, but in all other cases it may be carried out more places. The reason of this is; when the qrs. amount exactly to 24, &c. the addition according to the rule makes them exactly 1000th parts of a £. But when they are under 24, or above 24 and under 48, or above 48 and under 72, or above 72, there is always a 24th part of this defect or excess wanting to complete the decimal. For example, if they amount to 12, we add nothing; but 12 is the half of 24, and if we add 1 at 24, we must evidently add the half of this or 5 in the 4th. decimal place when they amount to 12. The student may easily make out a table for correcting every excess without regard to the directions in the general rule, which are often too complicated for a mental exercise, without long practice. For reducing the fractions $\frac{1}{24}$, $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{5}{24}$, $\frac{1}{4}$, &c. to decimals as far as $\frac{23}{24}$ a correction will be obtained for every possible case, and may be carried to any degree of accuracy. Thus, if the qrs. are 8, 32, 56, 80, the excess is 8, which is equal to $\frac{1}{3}$, or $\cdot\dot{3}$, therefore any of those cases will be corrected by writing $\dot{3}$ in the 4th decimal place.

Bonnycastle gives the following method for correcting the third decimal place; "increase the third place by 1 when the qrs. exceed 12, and by 2 when they exceed 37." This method will give the decimal too great when the qrs. are above 12 and under 24, and also when above 36 and under 48, for we thus add one when the correction amounts only to $\frac{1}{2}$, and 2 when the true correction is only $1\frac{1}{2}$. But this appears departing from the principle of the rule without either simplifying or improving the practice, which should never be done.

CASE 4.—The principle of this rule will appear evident, if we consider the process by which the decimal fraction is produced.

The reason of the rule is this. If shillings, pence, and qrs.

are reduced to the decimal of a £, by dividing them by 4, 12, and 20, it is evident that the reverse of this process or multiplying, the decimal of a £, by 20, 12, and 4, must reduce it again to shillings, pence, and qrs.

This case is so plain, and the process so similar to the reduction of whole numbers, that no explanation appears necessary after what has been said of the example in the text.

CASE 5.—From a consideration of the principles of the third case, the reason of this rule will appear evident. For the 1st. place of decimals being tenth parts of a £, the double of it must be twentieth parts or shillings. The reason of the second part of the rule is this; the second and third decimal places taken together are thousandth parts of a £, but there are 960 qrs. in a £, and because 1000 contains 25 as often as 960 contains 24, it is evident that diminishing the second and third places by 1 for every 25, the remainder is farthings. Although we might take five from the second decimal place when it amounts to or exceeds 5, and add 1 for it to the shillings, yet this is not absolutely necessary to the rule, but is sometimes given in order to simplify the mental operation of converting the remaining decimal into money. If we take 5 from the 2d. place, the decimal remaining can never exceed 48 thousandth parts of a £, or 11½d, but if 5 is not taken away, the decimal may be as high as 98 thousandth parts of a £, or 1/11½. This method gives the qrs. correct even although the decimal should extend beyond 3 places, and is therefore sufficiently true for any practical purpose. For example, 6/5¾ reduced to a decimal is ·3239583̇, but valuing the first 3 places according to this rule gives exactly 6/5¾. The correction of the decimal as explained in case 3d., causes it to run out farther, but has no influence on the correctness of the answer in the present case.—Bonnycastle in a note on this case, 11th edition, p. 112, gives the following directions, "then call the figures in the 2d. and 3d. places after 5 is deducted so many farthings, abating 1 when they are above 12, and 2 when they are above 37, and the result will be the answer. But this rule is incorrect, as it must invariably give the answer a farthing too little whenever the figures in the second and third places are above 12 and under 25, and also 1 farthing too little when above 37 and under 50.

CASE 6.—Before we can state the reasons of this case, it will be necessary to consider the nature of a repeating decimal. For example, $\frac{1}{9}$ reduced to a decimal fraction is equal to ·1 repeated for ever, therefore the sum of the infinite decreasing series .111 &c. must be equal to $\frac{1}{9}$. But if the sum of the infi-

nite decreasing series $\cdot\dot{1}$ is equal to $\frac{1}{9}$, it is evident that the sum of the similar decreasing series $\cdot\dot{2}$, must be equal to $\frac{2}{9}$, and the sum of $\cdot\dot{3}$, $\cdot\dot{4}$, $\cdot\dot{5}$, $\cdot\dot{6}$, $\cdot\dot{7}$, $\cdot\dot{8}$, and $\cdot\dot{9}$ equal respectively to $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$, $\frac{8}{9}$, and $\frac{9}{9}$, but $\frac{9}{9}$ is equal to 1. This being premised the reason of the rule will appear evident, because whatever the repeating figure may be, its denominator when reduced to an equivalent vulgar fraction must be 9, and since $\frac{9}{9}$ is equal to 1, we must carry at 9 on the right when adding or multiplying repeating decimals.

The reason for terminating the repeating figures in each product under the right hand figure of the first product is, that they are thereby made similar parts of the integer.

Some writers on arithmetic have not only asserted, but proved by algebra, that repeating figures in decimals signify ninth parts. But if it be admitted, as I think no person will deny who understands the meaning of the word, that decimals invariably signify tenth parts, it must appear strange not only to the young arithmetician, but also to many who have attained to very considerable proficiency in arithmetic, how these tenth parts are changed into ninth parts whenever they repeat; and algebra must appear a very wonderful art, by which tenth parts of any thing can be proved to be ninth parts of the same thing. These writers would obtain equal credit, and be equally well understood by nine-tenths of their readers, were they, by dint of algebra, to prove that 10 is 9. It is no doubt true, that the sum of the infinite decreasing series formed by any repeating decimal is equal to a vulgar fraction which has the repeating figure for its numerator, and 9 for its denominator; and although the pupil may yet understand nothing of the summation of an infinite series, yet he will much easier give his assent to what he does not understand, because above his present attainments, than to that which appears so directly contrary to what he already knows and believes. For this reason I have preferred representing the vulgar fraction as equal to the sum of the infinite decreasing series formed by the repeating decimal; rather than representing the repeating figures as ninth parts, which plainly involves an absurdity.

CASE 7.—The reason for adding to the right hand figure of each product, as if the circle had been repeated, is, that without this carriage the circle in the product would not have been correct. For example, let the circle be $\cdot\dot{7}\dot{6}$, and when multiplied by 4 gives $3\cdot\dot{0}\dot{4}$ of which the circle is $.\dot{0}\dot{4}$: but if the circle is repeated, namely $\cdot\dot{7}67\dot{6}$, and multiplied by 4 the pro-

duct is 3·0704, wherefore $\dot{0}\dot{7}$ is the true circle. It is therefore evident that it is necessary to carry according to the rule in order to complete the first circle. The reason for terminating each product under the right hand figure of the first product, is that the circles in the several products are thereby made similar. The reason for carrying to the right hand column of the circle when adding, as if the circle had been repeated, is the same as for carrying to the right hand products when multiplying. It may be observed here, whenever all the figures in the circle become the same in the course of reduction, that the circle is changed into a repeater; and when the repeating figure is 9, it is equal to 1 in the next higher place, as explained in Case 6. When the circle is not changed into a repeater by the several multiplications in the course of reduction, it generally retains the same number of figures as in the original circle.

CASE 8.—The reason of the rule is this, every decimal fraction whatever is a part of a unit, simply as such, or of a unit of some given denomination in money, weights, or measures, and therefore always less than that unit. But by the rule, the decimal forms the numerator of the vulgar fraction, and the denominator is unity divided into similar parts. For example, ·5, ·63, ·035 are decimals, of which the first is tenths, therefore unity reduced into tenth parts is the denominator to 5, or any decimal of one place. The second is 6 tenths, 3 hundredths, or 63 parts of which a hundred make unity, therefore unity divided into a hundred parts is the denominator to the decimal .63, or any other decimals of two places. For the same reason a unit divided into a thousand parts is the denominator to ·035, or any decimal of three places; and the same reasoning may be extended to any number of decimal places whatever. The decimal fraction, being brought into a vulgar fraction, may then be reduced into lower terms as already taught in vulgar fractions. As decimals are all tenth parts, it is evident that unity is brought into similar parts when we multiply it by 10 as often as there are places in the decimal, and therefore it must consist of 1 with as many 0s as the decimal has places, which is exactly according to the rule.

CASE 9.—The reason why a repeating decimal, when reduced to a vulgar fraction, has 9 for its denominator, has already been explained under Case 6. The reason for writing 9 under each figure in the circle for its denominator when converting it into an equivalent vulgar fraction is, that the sum of the infinite decreasing series formed by the circle, is exactly equal to a vulgar fraction which has the circulating figures for its

numerator, and as many 9s as there are figures in the circle for its denominator. For example, $\frac{1}{99}$ reduced to a decimal is $\cdot\dot{0}\dot{1}$ repeated for ever, therefore the sum of the infinite series thus formed, must be equal to $\frac{1}{99}$. Again, $\frac{5}{7}$ reduced to a decimal is $\cdot\dot{7}1428\dot{5}$ repeated for ever, but the sum of this infinite series repeated for ever is equal to $\frac{714285}{999999}$ by the rule, and this fraction reduced to its lowest terms is $\frac{5}{7}$. The same mode of reasoning may be applied to every circulating decimal, therefore the rule is correct, and the reason assigned is true.

CASE 10.—The reason of the rule is this, by the process prescribed, the two vulgar fractions, corresponding to the value of the terminate, and interminate parts of the decimal, are reduced to a common denominator and added together, and their sum is a fraction equal to the value of the whole decimal. The reason for annexing as many ciphers to the 9s in the denominator as there are figures in the finite part is this, the fraction corresponding to the circulating part is always a fraction of a fraction, which has 1 for its numerator, and 1 with as many ciphers annexed as there are places in the finite part for its denominator, and these are brought to a simple fraction by annexing the ciphers in the one denominator to the 9s in the other.

In order to render the reasons here assigned perfectly intelligible to the young arithmetician, I shall illustrate the whole process by an example. Suppose $.76\dot{5}\dot{3}$ were given to be reduced to a vulgar fraction. From what has already been explained, it must appear evident that, if the decimal had consisted only of the finite part, the vulgar fraction corresponding to it would have been $\frac{76}{100}$, and had it consisted only of the circle, the vulgar fraction corresponding to it would have been $\frac{53}{99}$. This last fraction is not the fraction of unity, but the fraction of a unit in the decimal place immediately preceding the circle, which is here the place of hundredths, therefore it is a compound fraction, or $\frac{53}{99}$ of $\frac{1}{100} = \frac{53}{9900}$. Therefore $\frac{76}{100} + \frac{53}{9900}$ is the value of the decimal $.76\dot{5}\dot{3}$. But these fractions cannot be added till they are brought to a common denominator, and it is evident we may cut off the ciphers from the denominators of each, without altering their relative values, and they then become $76 + \frac{53}{99}$ or $76\frac{53}{99}$ in the form of a mixed number. Now, instead of reducing this mixed number to an improper fraction in the common way, which would give $\dfrac{7577}{99}$, we may multiply 76 by 100, and take in 53 which gives 7653; but we have here multiplied by 1 too many, and must therefore subtract the mul-

tiplicand 76 from the product, which leaves $\frac{7577}{99}$ as before. But multiplying 76 by 100 and taking in 53, and then subtracting 76, is the same thing as annexing 53 to 76 and subtracting 76 from the number thus formed; and annexing 53 to 76 gives .7653, and subtracting 76 leaves 7577 the numerator of the vulgar fraction, which is the very process the rule prescribes. It is also evident that the fractions $\frac{76}{100}$, $\frac{53}{9900}$ are reduced to their least common denominator, by multiplying the numerator and denominator of the first by 99, which gives 9900 for the denominator, and this denominator has a 9 for each figure in the circle, and a cipher for each figure in the finite part according to the rule. From this the reason of every part of the rule must appear obvious. It may also be remarked here, that when there are whole numbers, as well as finite decimals given along with the repeating, or circulating decimal, they must be considered as belonging to the finite part, and subtracted accordingly, but we do not annex ciphers for the integral part to the denominator of the fraction.

ADDITION.—CASE 1. The reason of the rule for this case is the same as for simple addition in whole numbers; for like places stand under each other, and are therefore added together. As ten in any column make one of the next higher column in decimals as well as in whole numbers, and ten tenths make a unit, the process of addition in finite decimals is in every respect the same as in whole numbers.

CASE 2.—The reason for extending repeaters according to the rule is, that we thereby not only complete the sum in the finite part of the decimal, but also ascertain the true repeating figure, without which the sum would not be correct. The reason for carrying at 9 when adding repeaters, is already explained under case 6th of Reduction. It is quite necessary to extend repeaters as far as the longest finite part in order to have it correct, even carrying them a place farther is in most cases necessary to obtain the same end. For example, let it be required to add together $.674\dot{6} + .87\dot{5} + .32412\dot{3}$. Now, if the repeaters were not extended at all, the sum would be $1.87372\dot{3}$ which is correct only to the second place of decimals; and if the repeaters were extended only as far as the longest finite place, the sum would be $1.87433\dot{3}$, which is still deficient both in the last finite place and also in the repeating figure; but when we extend the repeaters according to the rule the sum is $1.87434\dot{8}$, and these figures would not be altered although the

repeaters were carried out ad infinitum. From this explanation two things are evident, namely, that it is absolutely necessary to extend repeaters one place farther than the longest finite part; and that it is equally unnecessary to extend them more than one place beyond it.

This case may be performed by reducing the decimals to vulgar fractions, then adding them, and reducing their sum to a decimal. This method is often preferred to that given in the text, by eminent writers, and teachers of arithmetic; but it appears to me injudicious, because it keeps the pupil ignorant of the proper management of decimals, and renders the work much more laborious. In proof of which observe the following example performed by both methods. Add together $.327\dot{3}+.69\dot{7}+.8\dot{6}$.

By vulgar fractions.

$$\left.\begin{array}{l} .327\dot{3}-327=\dfrac{2946}{9000} \\ .69\dot{7}-69=\dfrac{628}{900} \\ .8\dot{6}-8=\dfrac{78}{90} \end{array}\right\} =\frac{2946}{9000}+\frac{6280}{9000}+\frac{7800}{9000}=\frac{17026}{9000}=1.891\dot{7}.$$

Ans.

By general rule.

$$\begin{array}{r} .327\dot{3} \\ .69\dot{7}7 \\ .8\dot{6}66 \\ \hline 1.891\dot{7} \end{array}\text{ Ans.}$$

From this it is evident, that the method by vulgar fractions is much more laborious and complex than the other, and should therefore be discarded in practice, for increasing the labour, invariably increases the risk of errors. Hence, Bonnycastle's method of adding interminates is injudicious, and consequently the remarks in the 2d. edition of the arithmetic by the established schoolmasters of Scotland as to the conciseness of this method, is unfounded. The results of this rule are not approximations, as there stated, but in every respect the same as by vulgar fractions, and the rule is much more intelligible.

Case 3. The reason of the first part of the rule is, that repeaters and circulates are both considered finite so far as the longest finite part extends, and go directly to complete the finite part of the sum. The reason for extending repeaters and circulates according to the rule is, that the least common multiple of the number of figures in the several circles, is invariably the number of places to which the circle in the sum will extend. The reason for carrying according to the rule is, that the same number would have been carried had the circles been repeated, and is therefore necessary to complete the first circle.

As this case is generally but ill understood by the pupil, it

may be of some importance to explain it a little more particularly. Suppose $.675\dot{3}\dot{2}$ and $47\dot{5}3\dot{6}$ were given to be added; and having written them thus $\begin{array}{l|l} .675 & \dot{3}\dot{2} \\ .47\dot{5} & 3\dot{6} \end{array}$ we may mark the longest finite part by a perpendicular line after it as here represented, which prevents the possibility of mistaking it in the process. But there are here two places in the first circle, and three in the second, and the least common multiple of 2 and 3 is 6, therefore each of the circles must be extended 6 places to the right of the perpendicular line, to complete the circle in the sum, thus $\left.\begin{array}{l|l} .675 & \dot{3}\dot{2}3232 \\ .47\dot{5} & 3\dot{6}5365 \end{array}\right\} = 1.150\dot{6}8859\dot{8}$. If the circles had been repeated, the same figures would have returned in both numbers, in the same order as they do on the right of the longest finite part, and given the same circle in the sum as before. Although a part, or even the whole of some of the circles should stand on the left of the perpendicular line, and therefore belong to the finite part of the sum, yet this does not prevent them from being extended to complete the circle in the sum. It is absolutely necessary to extend the several circles and repetends as far to the right of the longest finite part, as the least common multiple of the number of places in each, indicates, to complete the circle in the product, but if carried farther, the same figures appear again under each other as on the right of the longest finite part, and would continue to return in the same order for ever, giving the same circle.

It was observed in the definition of decimals, that circulates are similar when they have the same number of places; and coterminous when they begin and end at the same distance from the decimal point. As this is the first case where these definitions require to be perfectly understood, it will be necessary to explain them more fully, which is best done by an example. Suppose $.\dot{6}2\dot{5}$ and $.5462\dot{9}87\dot{2}$ given to make coterminous, and similar in order to be added, they would be written thus $\begin{array}{l|l} .\dot{6}25\dot{6} & \\ .5462 & \dot{9}87\dot{2} \end{array}$ extending the circulating figures as far as the longest finite part. Then find the least common multiple of 3 and 4, the number of places in each circle, which is 12; this done, extend each circle 12 places to the right of the longest finite part, or perpendicular, thus $\begin{array}{l|l} .\dot{6}2\dot{5}6 & 256256256256 \\ .5462 & \dot{9}87\dot{2}98729872 \end{array}$. These circles are now similar, because they have the same number of

places, viz. 12; they are also coterminous, for they begin and end at the same distance from the decimal point. Again, since they are both carried out in terms of their least common multiple, the circle in the sum is complete, and its right hand place is 9, having 1 to carry to it from the left of the circle.

This case can also be performed by reducing the decimals into vulgar fractions, then adding them, and reducing their sum to a decimal. But this method is subject to the same objections here as in the last case; besides, by it the pupil is kept working in the dark all the time he is carrying out the circle, not knowing when or where it will terminate. But by the rule in the text he knows before extending the repeaters, how many places must be in the sum, and how many belong to each of the parts. For example, if $.\dot{4}372463\dot{4} + .2\dot{3}62475\dot{1} + .72\dot{4}3495\dot{8}$ were to be added, by reducing them to vulgar fractions, it is more than probable the pupil's patience would be exhausted long before he had ascertained the circle in the sum. But by the rule in the text he easily finds the circle will extend to 280 places in the sum, and as the finite part consists of three places, the whole decimal in the sum will be 283 places, and one place of integers making the answer to consist of 284 places. From a careful perusal of these remarks, it is presumed the student will have his difficulties connected with this case resolved, and his mind more than ever alive to the beauty and regularity of decimal calculations.

SUBTRACTION OF DECIMALS.—CASE I. The reasons for this rule, and for the proof are the same as in whole numbers. The reason for placing the decimal point in the remainder is, that, subtraction can neither increase nor diminish the number of decimal places, and therefore the remainder must have as many places as were in the minuend or subtrahend, which ever had most; and the operation is the same as in whole numbers.

CASE 2.—The reason for extending repeaters one place beyond the longest finite part is the same as for addition of repetends.

The reason for taking one from the repeater in the minuend before borrowing, when it is less than the repeater in the subtrahend is, that one would have been borrowed from it if the repeaters had been extended another place, therefore diminishing it by one is necessary to give the repeater correct in the remainder. For example, to subtract $.625\dot{6}$ from $.847\dot{3}$ by extending the repeaters two places we have $\left.\begin{array}{r} .847\dot{3}3 \\ .6256\dot{6} \end{array}\right\}$ the re-

mainder is $.221\dot{6}7$, in which the first repeating figure is correct, and shows clearly the reason of the rule. For it is evident in subtracting the first figure on the right, thus extended, we borrow 1 from the first repeating figure, and diminish it by 1 according to the rule. Proof by addition of repetends.

This case may also be performed by reducing the decimals to vulgar fractions, and then reducing their difference to a decimal.

CASE 3.—The reason for making circulates similar and coterminous is the same here as in addition. The reason for diminishing the right hand figure of the minuend as the rule directs, is the same as explained in last case for repeaters. To prove this case, we carry to the right of the circle, the figure which is carried to the first finite part, or the same as if the circle were repeated. The method of making circulates similar and coterminous is already explained in addition, case 3d. This case may also be performed by reducing to vulgar fractions, and the remainder to a decimal.

MULTIPLICATION OF DECIMALS.—CASE 1.—The reason of the rule is, that the product of the right hand figure of the multiplicand, by the right hand figure of the multiplier, invariably occupies a place in the product, as far removed from the decimal point as there are decimal places in both factors; and when the product does not consist of so many, it must be removed to its proper place by prefixing ciphers.

To illustrate the principles of the rule, and the reasons here assigned more fully, let it be required to multiply .2654 by .5421.

$$\begin{array}{r} .2654 \\ \underline{.5421} \\ 2654 \\ 5308 \\ 10616 \\ \underline{13270} \\ .14387334 \end{array}$$

It is here evident from the nature of decimal numeration, that the right hand figure both in the multiplicand and multiplier occupies the place of ten thousandth parts, but ten thousandth parts multiplied by ten thousandth parts or 10000×10000 gives hundred millionth parts, which occupy the 8th. place in the decimal scale, there must therefore be 8 decimal places in the product, and 8 is the number of places in both factors according to the rule. By the same method we ascertain how many ciphers stand between the decimal point and the first significant figure. For, multiplying 2 tenths by 5 tenths, the product is 10 hundredth parts, equal to 1 tenth, wherefore the highest figure in the product occupies the place of tenths, and no ciphers are required. Again, let ·036 be multiplied by .36. The highest significant digit in the multiplicand here is 3 in the place of hundredths, which multiplied by 3 in the place of tenths and 1 added from the preceding pro-

duct, gives 10 in the place of thousandths, equal to 1 in hundredths, but hundredths occupy the second place in the scale, therefore the place of tenths must be supplied by a cipher according to the rule. Decimals are all proper fractions, consequently the product of any two decimals is less than either of the factors, as already explained in Vulgar Fractions; therefore the product of two decimal factors can never produce a whole number. The reason of this rule is generally shown by converting the decimals into vulgar fractions, and their product into a decimal, which is quite consistent where the notation of decimals is given in vulgar fractions. When the multiplier is 10, 100, 1000, &c. the operation is performed by removing the decimal point as many places to the right as there are ciphers in the multiplier. The reason is, multiplying by 10, 100, 1000, &c. is performed by annexing the ciphers to the multiplicand, and then counting the decimal places from the right, removes the point as many places to the right of its former position as there were ciphers annexed.

Case 3.—The reasons for carrying at 9 on the right of each product, for extending them one place farther than the longest finite part, for carrying at 9 when adding the right hand column in repeaters; for adding to the first products on the right of circles, the carriage from the left, and for making circles in the products similar before adding, have all been explained in addition. Repeaters are extended one place farther than the longest finite part, and circulates are made similar in the several products, by terminating them under the right hand figure of the first product; for the circles in the several products, and in the sum of the products have generally the same number of places as the circle in the multiplicand. When adding the right hand column of repeaters, if 0 remain after taking out the 9s, the product is finite; and when adding circulates, if there be as many 0 or 9s on the right as there were figures in the circle, the product is finite. The reason of this has already been explained in addition.

This case may also be performed by reducing the decimals into vulgar fractions, and again reducing their product into a decimal.

Case 3.—The reason of this rule is evident from the nature of vulgar fractions, for any number multiplied by the numerator, and divided by the denominator, gives its product by the vulgar fraction.

This case may be performed without reducing the factors to a vulgar fraction, thus, If the multiplier is a pure repeater, or circulate, multiply by it as if it were finite, and repeat this

product, or the sum of the products (according as it is a repeater, or circulate) under itself, removing it as many places to the right at each step as there are figures in the circle; continue this process till the circle in the product is complete, or till it is sufficiently extended for the end in view.

$$\begin{array}{r} .\dot{4}3\dot{6} \\ 3 \\ \hline 130930930 \\ 13093093 \\ 1309309 \\ 130930 \\ 13093 \\ 1309 \\ 130 \\ 13 \\ 1 \\ \hline \dot{1}4547881\dot{2} \end{array}$$

EXAMPLE.—Multiply $.\dot{4}3\dot{6}$ by $.\dot{3}$.
The reason of this process is evident; for, if the repeating figure had been taken nine times, and beginning to multiply by the left hand figure, which is always optional, and extending the circle in the product, we would have had exactly the result here given. Had the circles been carried out one place further, there would have been 4 to carry from it to the first column given here, which completes the first circle. By continuing this process, the circle in the product may be obtained, however many places it may extend to.

Again, let $.\dot{2}9\dot{7}$ be multiplied by $.\dot{7}\dot{2}$.

$$\begin{array}{r} .\dot{2}9\dot{7} \\ .72 \\ \hline 594 \\ 20810 \\ \hline 21405405405 \\ 214051054 \\ 2140540 \\ 21405 \\ 214 \\ 2 \\ \hline \dot{2}1\dot{6}21621620 \end{array}$$

The reason of this process is also evident, for, having obtained the first product by the circle, we repeat that product, writing it two places farther to the right, and every time we repeat this process is the same as repeating the circle in the multiplier. The process is continued here, so as to repeat the circle in the product three times.

From these examples it is evident, that this is the natural process, and extremely simple when the product is required only to a few places, but when many decimals are required in the product the operation by this rule becomes very heavy, and therefore it was not given in the text. There are many other rules for multiplying, repeating, and circulating decimals, but those given embrace every possible case, and are both easily comprehended, and simple in their operation. This case may also be performed by reducing the factors to vulgar fractions, then multiplying, and reducing the product to a decimal.

CONTRACTION.—The reason of this rule will appear evident by comparing it with an example wrought in the common way. Inverting the multiplier has no effect in altering the product, for it has been shown in simple multiplication that we may begin with any figure of the multiplier. It has also been

shown in the notes to the first case of the multiplication of decimals, that tenths multiplied by tenths give hundredths, therefore units multiplied upon any decimal place, must give a decimal in the product of the same name as the place multiplied. But as decimals decrease in the same ratio as whole numbers, therefore the place of tenths multiplied into a decimal one place higher in the scale, than that under which units stands, must give a product belonging to the same place in the decimal scale as the product by unity, and so on for any other product either on the right or left of units place. Inverting the multiplier is not absolutely necessary to this contraction, and only saves the trouble of ascertaining at each step of the process what figure of the multiplicand to commence with. The reason of carrying 1 from 5 to 15, &c., from the product of the figures on the right of the one we are multiplying by is merely an approximation, to supply as near as possible what would have been carried to the right hand column had the products been brought out at full length. This operation generally gives the last figure of the decimal correct; yet it may possibly be 1 too much, or too little in the last place; but in order to have the decimal always correct in the last place, carry it out, according to the rule, two places more than required, which reject after addition. The answers to the exercises under this rule are brought out by the directions in the rule, and are all marked how much they exceed, or fall short of the true answer.

This contraction is extremely useful in raising high powers in decimals, where not many places are required in the answer, and in constructing decimal tables.

DIVISION OF DECIMALS.—CASE 1. The principles and reasons of the rule for division of decimals are the same as for whole numbers. The reason why ciphers are annexed has already been explained under case 1 of reduction. The reason why repeating or circulating figures are annexed is, that they may be extended at pleasure, and therefore can never be exhausted by division. The reason why the decimal places in the quotient must equal the excess of those in the dividend above those in the divisor is this, the product of the divisor and quotient, adding in the remainder, must always equal the dividend; but the decimals in the product are invariably equal to those in the factors; therefore the decimal places in the divisor and quotient must be equal to those in the dividend; that is, the decimals in the quotient must be equal to the excess of the decimals in the dividend above those in the divisor, which is the rule. From this, it is evident, that whenever the

decimal places in the divisor and quotient fall short of those in the dividend, the defect must be supplied by prefixing ciphers to the quotient as the rule directs. When the decimal places in the divisor and dividend are equal, the quotient is a whole number; and when the divisor is a decimal, and the dividend a whole number, the quotient is a whole number until there are as many ciphers annexed in the course of division as there are decimal places in the divisor. The decimal places in the quotient may also be determined by observing that the first figure in the quotient invariably occupies the same place, either of decimals or whole numbers which the figure in the dividend does that stands over units place in the product by the first quotient figure.

CASE 2.—The reason of this rule is evident from the notes on division of vulgar fractions. The multiplication in this case is expeditiously performed by the contraction for 9s given in notes to simple multiplication.

When the divisor and dividend are both interminate, the division may be performed thus: Make the circles in the divisor and dividend similar and coterminous; then divide, annexing the repeating or circulating figures when carrying out the division. This rule is quite consistent with the analogy of decimals, but the operation by it is often too laborious for general use. This case may also be performed by multiplying both divisor and dividend by any number which exterminates the repeater or circulate in the divisor. Both this and the last case may be performed by converting the decimals into vulgar fractions, and dividing them, and then reducing the quotient into a decimal.

CONTRACTION.—The principal difficulty experienced in this contraction is ascertaining how many figures altogether must be in the quotient. But this difficulty will be removed by observing the last method given under case 1st. for ascertaining the number of decimal places in the quotient. For example, .943265 ÷ .43625 retaining three decimals places in the quotient. It is evident that the first quotient figure must be 2, and multiplying the divisor by it, gives tenths under tenths, and if there had been a unit it would have stood under units place of the dividend, wherefore the first figure in the quotient is units; and the question requires three decimals, therefore there must be four places in the quotient; but there are five places in the divisor, and as we never retain more places than what we require in the quotient, the right hand figure must be rejected before dividing.

The reason for carrying 1 from 5 to 15, &c. in the figures

cut off from the divisor, is the same as already given for a similar carriage in contracted multiplication. This contraction often saves much labour when there are many decimals in the divisor, and few required in the quotient.

SIMPLE PROPORTION.—The reasons and principles of this rule are in every respect the same as in whole numbers, and the operation is so simple that it requires no illustration in this place. If the previous rules are understood, the young arithmetician will soon learn how and when he can apply decimals with advantage in the following part of this work. In many cases the operation may be much abbreviated by using decimals, but in others they tend to increase the labour. It is impossible to give any general rule when to apply decimals; this, therefore, must be left to the accountant's own judgment and discretion. The exercises in simple proportion in vulgar fractions may be wrought by decimals with advantage.

COMPOUND PROPORTION.—The reasons and principles of this rule are the same as given in whole numbers. The exercises in compound proportion in vulgar fractions, and some of those in whole numbers may very properly be performed by decimals. In the following rules the pupil should be accustomed to perform the exercises both by vulgar and decimal fractions, which will fix them on his memory, and teach him when to apply them profitably.

OBSERVATIONS.—I shall conclude these notes with a few observations which may be of use in the application of decimals.

Multiplying by a decimal gives a product less than the multiplicand, and dividing by a decimal gives the quotient greater than the dividend, the same as by a proper vulgar fraction, and depends on the same principles.

The reciprocal of any number is found by dividing 1 by that number, thus the reciprocal of 5 is $\frac{1}{5} = .2$, and multiplying by the reciprocal is the same as dividing by the number, thus $10 \times .2 = 2$; and dividing by the reciprocal of a number, is the same as multiplying by the number, thus $10 \div .2 = 50$.

Having already shown that a vulgar fraction whose denominator is 2, 5, or any power of 2, 5, or the product of any power of 2 by any power of 5 must give a terminate decimal, it now only remains to make a few remarks on interminate decimals. Vulgar fractions whose denominators are 3 or 9 give pure repeaters, thus $\frac{1}{3} = .\dot{3}$, $\frac{1}{9} = .\dot{1}$. Again, every vulgar

fraction whose denominator is the product of 2 or 5, or any power of 2 or 5, &c. multiplied by 3 or 9 gives a mixed repeater; and when the denominator is an even power of 2 or 5, that is the 2d. 4th. &c. power, multiplied by 3, the repeating figure in the decimal is 3, but if an odd power of 2, 5 is multiplied by 3, the repeating figure is 6. Every mixed repeater multiplied by 9 gives a terminate decimal in the product; wherefore every mixed repeater must be derived from a vulgar fraction whose denominator is 2 or 5, or some of their powers, or the product of their powers multiplied by 3 or 9.

Every power of 3, excepting 9, in the denominator of a vulgar fraction, produces a pure circulate, and the number of places in the circle is always indicated by the quotient of the denominator divided by 9, thus $\frac{1}{27} = .\dot{0}3\dot{7}$ or 3 places, but $27 \div 9 = 3$, and $\frac{1}{81} = .\dot{0}1234567\dot{9}$ or 9 places, but $81 \div 9 = 9$ and so on, for any other power of 3.

Every repeater excepting $.\dot{3}$, $.\dot{6}$, $.\dot{9}$, divided by 3, gives a pure circle of 3 places, &c., and every circle not a multiple of 3, being divided by 3, gives a circle of three times as many places as the circle divided.

Every vulgar fraction whose denominator is any power of 2, 5, or the product of any power of 2, 5, multiplied by any other number than 3 or 9 gives a mixed circulate.

Every vulgar fraction of whose denominator 2, 5, are not aliquot parts, (3 and 9 excepted) produce pure circulates.

Any vulgar fraction being given in its lowest terms, the form of the decimal corresponding to it can easily be discovered by the following rule. Divide the denominator of the fraction by 10, 5, or 2, as often as possible without remainder, the finite decimal places in the quotient are equal to the number of these divisions; if there is a remainder divide 9s by it till there is no remainder, the number of nines thus used are always equal to the number of figures in the circle of the quotient.

SIMPLE DISTRIBUTIVE PROPORTION.—By Simple Distributive Proportion, the gain or loss on company accounts is divided among the partners in proportion to their shares in the concern, or the estate of a bankrupt among his creditors in proportion to their several claims; or the effects of a testator among the legatees in the case either of a deficiency or excess of assets, or average losses in insurance among the underwriters, &c.

In simple distributive proportion, either the stock of the several partners, or the time it continues in company, are sup-

posed to be equal. It is also called Partnership, Fellowship, or Company, from its application to copartnery accounts, and although its name is optional its application is fixed.

The principles on which this rule is founded are the same as in simple proportion, and the same contractions are also applicable; but if the 1st. and 3d. terms can be cancelled, I would prefer them, as they remain the same in all the statings.

The reason of the rule given in the text is this, since the whole stock gives the whole gain or loss, it is evident that any part of the stock must give a proportional part of the gain or loss. Suppose, for example, I put in the half of a company's stock, I am entitled to the half of the gain, or if I put in the third of the stock, I am entitled to the third of the gain. Whatever proportion therefore the whole stock bears to any particular partner's stock, the same proportion must the whole gain, &c. bear to that partner's gain, &c. When the stock of each partner is the same, divide the gain by the number of partners, the quotient is the share of each. Reason of proof. The sum of all the parts is equal to the whole. When there are many partners concerned with unequal claims, as is generally the case in bankruptcy, the operations by the general rule would become extremely laborious, several methods have therefore been invented to shorten the process, a few of which I shall here subjoin.

1st. Find the gain or loss on £100, and perform the rest of the operation by compound multiplication, or by practice.

2d. Find the gain or loss on £1 of stock, and work by practice.

3d. Annex a cipher or ciphers to the whole gain or loss, divide this number by the sum of the stocks, and multiply each particular stock by the quotient, and from each product cut off as many figures from the right as you annexed ciphers to the gain or loss; the figures cut off are the remainder, the others are the integers of each particular gain or loss.

4th. When the composition in bankruptcy is agreed upon, the dividend is easily found for any sum by practice.

5th. When there are many partners or creditors, find the dividend for £1, by decimals. Multiply this dividend by 2, 3, 4 and 10, multiply the product by 10 again by 2, 3, 4 and 10, and this last product again by 2, 3, 4 and 10, place these products in the form of a table. By this method the share of any or all the creditors in a bankruptcy, or partners in a company may be calculated with ease and expedition.

These tables may be formed as directed for tabling the divisor in simple division. Examples of these methods.

1st. Method. Four merchants in company gain £498 3s. 4d., A's stock was £650, B's £875, C's £500 and D's £425, what part of the gain must each receive?

650
875
500
425
2450

Stock. Stock. Gain.

As £$\cancel{2450}$: $\cancel{100}$: : $\cancel{498}$ $\cancel{3}$ $\cancel{4}$
7 : 2 : : 71 3 4
2
7)142 6 8
gain on 100 stock = £ 20 6 8

£20 6 8
6½
10 3 4
122 0 0
A's £132 3 4

£20 6 8
8¾
10 3 4
5 1 8
162 13 4
B's £177 18 4

£20 6 8
5
C's £101 13 4

£20 6 8
4¼
5 1 8
81 6 8
D's £86 8 4

In cancelling the stating in this example I divide the first term by 350, of which the factors are 50 and 7, I then divide the 100 in the middle term by 50, and the third term by 7, which shortens the process. The reason of this method is evident; for the dividend on £100 of stock multiplied by the hundreds the partner had in stock must give his whole dividend.

2d. Method. Three merchants in company gain £350 8s. A's stock was £486, B's £685, C's £365, what must each receive.

Stock. £
as 1536 : 1 : : 350 8
20

486
685
365
1536

1536)7008(4/6¾ per £
128 6144
32 864
768 &c.

4 = ½ | 486
6 = ⅛ | 97 4
¾ = ⅛ | 12 3
1 10 4½
£ 110 17 4½
A receives.

½ | 685
⅕ | 137
⅛ | 17 2 6
2 2 9¾
£ 156 5 3¾
B receives.

½ | 365
⅕ | 73
⅛ | 9 2 6
1 2 9¾
£ 83 5 3¾
C receives.

It is evident the first stating gives us the dividend on £1 stock, and having that, the answers are readily found by practice, as in the example. In working the 1st. stating, I have divided the divisor by 12 and 4, instead of multiplying the re-

mainders by these numbers, which is the preferable method whenever it is practicable, as we thereby cancel the work.

The reason of the method is this. Whatever part the dividend upon £1 is of £1, the same part is the dividend upon each stock, of that stock.

3d Method. Three merchants in company clear £684, A's stock is £750, B's £460, C's £500, what must each receive of the gain?

750		750	460	500
460	1710)684(4	4	4	4
500	6840	£300,0	£184,0	£200,0
1710		A receives.	B receives.	C receives.

It is evident that the common multiplier is 4 tenths of a £, and cutting off the figure on the right is according to the rule for multiplying decimals. The reason of this method is the same as last.

4th Method. This method is so plain, that it requires no example for illustration, it being the same as the second method after finding the rate per £. Reason the same as the second.

5th Method. The effects of a bankrupt amount to £10016 12s. he owes in all £72848; he owes to A £256, to B £320, to C £735, to D £2560, how much must each of these creditors receive?

$$10016.6 \div 72848 = .1357 = \text{dividend on £1.}$$

Index.	Table.
1	.1375
2	.2750
3	.4125
4	.55
10	1.375
20	2.75
30	4.125
40	5.5
100	13.75
200	27.5
300	41.25
400	55.
1000	137.5

$$\begin{array}{rrr} & 200 = & 27.5 \\ 30 + 20 = & 50 = & 6.875 \\ 3 + 2 = & 6 = & .825 \\ \hline & 256. = & 35.2\cancel{0}\cancel{0} \\ & & 20 \\ \hline & & 4.0 \end{array}$$

A's dividend £35 4s.

$$\begin{array}{rr} 300 = & 41.25 \\ 20 = & 2.75 \\ \hline 320 = & 44. \end{array}$$

B's dividend £44

$$\begin{array}{rr} 700 = & 96.25 \\ 30 = & 4.125 \\ 5 = & .6875 \\ \hline 735 = & 101.0625 \\ & 20 \\ \hline & 1.25\cancel{0}\cancel{0} \\ & 12 \\ \hline & 3.00 \end{array}$$

$$\begin{array}{rr} 2000 = & 275 \\ 500 = & 68.75 \\ 60 = & 8.25 \\ \hline 2560 = & 352 \end{array}$$

C's dividend £101 1s. 3d. D's dividend £352.

In this method the first thing to be done is to divide the

effects of the bankrupt by his debts, the quotient is the decimal of the dividend upon £1.

The table is constructed thus; place the dividend upon £1 for the first number in the table, double it for the second, add the 1st. and 2d. for the 3d., &c. to whatever length you require it. These tables are generally made with all the numbers, viz. 5, 6, 7, &c. 50, 60, &c. but I consider this unnecessary, as any of these numbers are easily found, by an addition or multiplication. If there are shillings or pence in any of the stocks the table may be applied to them, by taking the parts of a £ with them by practice out of the 1st. number in the table and adding this sum for the shillings, or the shillings and pence may be reduced to the decimal of a £, and the 1st. number in the table multiplied by them, the product is the decimal to be added for the shillings and pence.

Application of the table. To find what is due to A, whose stock or money is £256, I take the number in the table opposite to 200 and write it down, then the tabular number for 50 (which is the sum of the numbers opposite to 20 and 30) and write it under the last, then the tabular number for 6, which is found by multiplying the number which stands opposite to 3 by 2, and write the product under the other numbers, the sum of these is £35.2 or £35 4s. which is A's dividend. The other dividends are found, in the same manner, as seen in the example. The operation by this method, is both simple and expeditious. The reason of this method is the same as the second method, tabling the dividend is a mere mechanical process which does not affect the reason of the rule.

The general rule given in the text, is to find the particular gain or loss of each partner, having given the particular stocks and the whole gain or loss; but it may apply equally to find the particular stocks, having given the whole stock, and the particular gains or losses, for, as the sum of the gains, &c. is to each particular gain, so is the whole stock, to each particular stock.

COMPOUND DISTRIBUTIVE PROPORTION is so called, because the share of gain or loss belonging to each partner is neither in the ratio of the stock, nor the time of its continuance, but in the compound ratio of the stock and time, that is, in the ratio of the products of the several stocks into their respective times. This relation has already been explained in the notes to compound proportion.

The general rule considers the product of each stock and time, as the particular stock or time of each partner, and works with these as in simple Distributive Proportion.

The principles upon which this rule is founded are the same as in simple proportion.

The reason of the rule is this, when the times are equal, the shares are in proportion to the several stocks; and when the stocks are equal, the shares are in proportion to their respective times of continuance, consequently when neither the stocks nor times are equal, the shares must be in proportion to their products.

The reason of the rule may perhaps be more satisfactorily explained thus, Let us take the example wrought in the text, where B had £60 for 4 months, and C £80 for 6 months. Now £60 at interest for 4 months, is equal to £240 for one month; and £80 for 6 months is equal to £480 for one month, the time being here the same, the shares of the partners must consequently be in the proportion of their stocks, that is of £240 to £489, and these sums are the products of their stocks and times.

There are a species of examples generally classed under compound distributive proportion, which cannot be solved by the general rule, such as when goods, or property of different values, are to be divided among the partners in proportion to their several stocks, &c.—Example. Three merchants, in company, have 69 pipes of wine to divide among them; A's stock was £640, B's £420, and C's £240. The wine which A received was £80, B's £60, and C's £40 per pipe; how much must each receive?

In all questions of this nature, where the property to be alloted to each partner is of a different value, the quantity cannot be in the proportion of their stocks; nor is it inversely as the value of the property, but the quantity of property which each must receive is in the proportion of the quotients arising from dividing each partner's stock, by the value of the property alloted to him. All such questions may be solved by the following rule. Divide the stock of each partner by the value of the property of which his share is to consist, and use these quotients instead of the several stocks as in simple distributive proportion. The operation of the above example stands thus.

		Pipes	hhd.	gal.	
$640 \div 80 = 8$	As 21 : 8 :: 69 :	26	1	9	= A's.
$420 \div 60 = 7$	As 21 : 7 :: 69 :	23	0	0	= B's.
$240 \div 40 = 6$	As 21 : 6 :: 69 :	19	2	54	= C's.
21					

In proving such questions as the above, it is not enough that the sum of the shares of the partners be equal to the whole

quantity to be divided, for their values also must be proportional to the stocks of the partners.

Compound Distributive Proportion is sometimes called double Fellowship, Fellowship with time, and Company with time.

COMMISSION AND BROKERAGE. The rules for this case are the same as for simple interest for a year; or they may be referred directly to simple proportion and practice.

FACTORS in the mercantile world are merchants' agents generally residing abroad, and constituted by powers of attorney to act for their employers.

Factors cannot sell the goods of their employers upon credit without particular orders, or an extra commission, with a stipulation to make good what bad debts they may meet with.

If a factor become bankrupt or die insolvent with the property of his employer in his hands, no portion of it beyond the commission can be appropriated to the payment of the factor's debts.

The indispensable qualifications in a factor, as well as the means of raising, and establishing his reputation, and securing his fortune, are frequency and punctuality in correspondence, a thorough knowledge of the value of goods, and of the rise and fall of markets both at home and abroad, diligence in executing orders, and honesty and regularity in rendering faithful accounts.

BROKERS are home agents, who transact business for a small per centage, and are generally employed by extensive merchants to effect their purchase and sales.

Brokers should always be well informed of the state of markets, the causes which affect them, and the probabilities of a rise or fall.

SIMPLE INTEREST.—It is impossible to say at what period interest first began to be exacted for money given in loan, but it is evident that its origin must be nearly coeval with the division of property, and the first dawning of commerce. For many ages exacting of interest was considered inconsistent with Christianity, and prohibited under severe penalties, but the practice still prevailed, and in proportion as the penalties were severe, the rate of interest became the more exorbitant. From the year 1255 to 1270, no less than 50 per cent. per annum was charged for the use of money; from 1270 to 1307, 45 per cent. was charged, and between 1307 and 1470, interest had fallen to 15 per cent. In England, under Henry the VIIIth., interest was permitted by law in 1545, and the rate limited to 10 per cent. In 1624 the legal interest was reduced to 8 per cent.; in 1660 it was still farther reduced to 6 per cent., and in 1714,

anno 12 Annae, cap. 16, and Sess. 2, cap. 6, it was enacted, that no person is to take for the loan of monies above £5 for the forbearance of £100, for the space of one year, and that all bonds and contracts for a higher rate of interest to be void and null, and the offender to forfeit triple the value of the sum lent. This act is still in force, regulating the legal rate of interest, and the penalty against usury. But 12 per cent. is legal interest in the East Indies, 8 per cent. in New South Wales, and 6 per cent. in Ireland.

When money is lent, and the interest, though due, remains still in the hands of the borrower, without bearing interest, or becoming a part of the principal, it is called simple interest.

When money is lent, and no rate of interest specified in the agreement, legal interest or 5 per cent. is always understood. Legal interest is also the rate due upon all debts after the term of credit is expired. Although 5 per cent. is legal interest in Britain, and any thing beyond it is termed usury, and liable to be punished; yet in other countries much higher rates of interest are tolerated. But the profits resulting from the employment of capital, are now in general so small that the market rate of interest has fallen much below the legal rate.

Case 1.—The reasons and principles of the rule for this case are the same as for compound proportion. For example, to calculate the interest of £1000 for 3 years at 5 per cent., we might state thus, $\begin{matrix}\pounds100 : 1000 \\ \text{year } 1 : \quad 3\end{matrix}$: : £5. From this stating it is evident that £100 and 1 year are the first terms in every question in this case, and the principal, rate, and number of years are the other terms of the proportion, therefore multiplying the principal by the rate, and the number of years, and dividing the product by 100, must give the answer, and is the rule in the text. It is farther evident from the stating, that we may cancel 100 in the 1st. term, and 5 in the 3d. term by 5, then multiplying the principal by the years, and dividing the product by 20 gives the answer; or which is the same thing, multiplying $\frac{1}{20}$ of the principal by the years, the product is the interest. Again for 6 per cent., the answer is obtained, by adding to itself, and at 4 per cent. by subtracting from itself $\frac{1}{5}$ of the answer at 5 per cent. By considering the nature of the question when stated by compound proportion, every contraction in this case, at any rate, and for any time, may easily be discovered. For example, the interest at 5 per cent. for 2 years is equal to $\frac{1}{10}$ of the principal, for 4 years to $\frac{1}{5}$ of the principal, for 10 years to $\frac{1}{2}$ the principal, and in 20 years the principal and interest exactly equal each other. The application of decimals in interest often saves labour.

CASE 2.—The reasons and principles of this case are also the same as compound proportion. For example, to find the interest on £1000 at 4 per cent. for 3 months, we might state thus, $\begin{array}{l}\pounds 100 : 1000 \\ \text{year } 1 : \ \frac{1}{4}\end{array} :: 4.$ In this case as in last, 1 year or twelve months and £100 are always the 1st. terms of the proportion, and the principal, rate, and time are the other terms. Many useful contractions may be discovered in particular cases by observing the stating by compound proportion. In the above example it is evident that the answer may be obtained by dividing the principal by 100, because multiplying by 4, and taking $\frac{1}{4}$ of the product gives the number which was multiplied, for the same reason, dividing the principal by 100 gives the interest at 4 per cent. for 3 months.

When the rate is 5 per cent., the interest of £1 for a year or 12 months is 1/ or a penny a month, therefore the sum considered as pence, and multiplied by the months gives the interest; thus £89 for 6 months is 7/5 × 6 = £2 4s. 6d. It may also be found in £s. by dividing the principal by the quotient of 240 divided by the months, thus £89 ÷ $\frac{240}{6}$ = 89 ÷ 40 = £2 4s. 6d. When the term of any bill or bond is expressed in months, calendar months are understood. In courts of law interest is generally reckoned for years, half years, or quarters, and this, because the months are not all the same length, and therefore taking a month as the $\frac{1}{12}$ part of a year is not quite accurate.

CASE 3.—The reasons and principles of the rule for this case are the same as in the two preceding. For example, to find the interest of £1000 for 56 days at 3 per cent. we might state it thus, as $\begin{array}{l}\pounds 100 : 1000 \\ \text{Days } 365 : \ 56\end{array} :: \pounds 3.$ Now, if we multiply the two first terms together we have 36500 for a divisor, but in the stating of every question belonging to this case, £100 and 365 days are the first terms, therefore 36500 is invariably the divisor. In this example we have 56000 × 3 for the dividend, but if we multiply 36500 the divisor, and 3 the rate by 2, we have 73000 for the divisor, and 56000 × 6, or double the rate for a dividend, which is the rule in the text; and multiplying in this way does not alter the ratio, therefore the rule is correct. The reason for multiplying by double the rate is, that 36500 multiplied by 2, gives a divisor with fewer significant figures, and therefore simpler than the other, and it is generally as easy to multiply by double the rate as by the rate itself, but any question in this case may be wrought by multiplying the principal by the rate, and days, and dividing by 36500. The reason why we do not multiply by double the rate

when it is 5 per cent., and divide by 7300 is, that taking away a cipher from the divisor is the same as annexing one to the dividend. By attending to the nature of the stating by compound proportion, a particular divisor may be obtained for every rate per cent., but it is more satisfactory, and less perplexing to have a divisor for every possible case, as it is easily remembered; and the particular divisor for 5 per cent., is the same as the other wanting a cipher, which therefore requires no great additional stretch of memory. In calculating interest for days, the day from which we reckon is not included, but the day we reckon to, is. Thus, if it were required to find the days between the 14th. of April and the 20th. of July, we would begin with the 15th. of April and count the 20th. of July, in the following manner, 16+31+30+20 = 97 days. Although this is the natural method of finding the days, yet they may be found by the following table.

Table, showing the number of days from any day of one month to the same day of any other month.

To the same day of	From any day of: Jan.	Feb.	Ma.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	301	276	245	215	184	153	123	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
Apr.	90	59	31	365	335	304	273	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	91	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

USE OF THE ABOVE TABLE.—Find the days from the 7th. July to the 7th. October? Under July, and opposite to October, there are 92, the days sought.

Find the days between the 18th. of November and 24th of

August? Under November, and opposite August is 273, the days to the 18th. August, to which add 6, the days between the 18th. and 24th. makes 279 days.

3. Find the days from 15th. April to the 6th. January. Under April, and opposite to January is 275, the days to the 15th. of January, and 275 — 9 = 266, the days sought.

4. Find the days from 20th. February to 30th. September leap year? Under February and opposite September, is 212, to which we must add the days between the 20th. and 30th. and 1 for leap year, or 11 days, making 223 days.

When the end of February leap year is in the time, a day must be added, as the table takes no notice of leap year. From the examples given, the method of using the table must appear evident.

In business it is customary to reject the lower denominations of the principal when under 10/, and when 10/, or more, to count them a £. If it were required to calculate the interest on £156 9s. 8d. they would calculate for £156, but when the sum is £156 10s. they would calculate for £157. Business men also reject fractions in the interest when under ½d. and when they are a ½d. or more, they reckon them 1d. These methods, however convenient in business are not admissible in treatises on arithmetic, therefore the best method is to reduce inferior denominations to the decimal of a £, and value the quotient mentally.

Interest for days may also be calculated by the following table given in Hutton's Arithmetic.

Num.	Interest.				Num.	Interest.				N.	Interest.			N.	Int.
	£.	s.	d.	qrs.		£	s.	d.	qrs.		s.	d.	qrs.		qrs.
100000	273	19	5	1.70	3000	8	4	4	2.41	50	2	8	3.51	.7	1.84
90000	246	11	6	0.33	2000	5	9	7	0.27	40	2	2	1.21	.6	1.58
80000	219	3	6	2.96	1000	2	14	9	2.14	30	1	7	2.90	.5	1.32
70000	191	15	7	1.59	900	2	9	3	3.12	20	1	1	0.60	.4	1·05
60000	164	7	8	0.22	800	2	3	10	0.11	10	0	6	2.30	.3	.79
50000	136	19	8	2.85	700	1	18	4	1.10	9	0	5	3.67	.2	.53
40000	109	11	9	1.48	600	1	12	10	2.08	8	0	5	1.04	.1	.26
30000	82	3	10	0.11	500	1	7	4	3.07	7	0	4	2.41	.09	.24
20000	54	15	10	2.74	400	1	1	11	0.05	6	0	3	3.78	.08	.21
10000	27	7	11	1.37	300	0	16	5	1.04	5	0	3	1.15	.07	.18
9000	24	13	1	3·23	200	0	10	11	2.03	4	0	2	2.52	06	.16
8000	21	18	4	1.10	100	0	5	5	3.01	3	0	1	3.89	.05	.13
7000	19	3	6	2.96	90	0	4	11	0.71	2	0	1	1.26	.04	.11
6000	16	8	9	0.82	80	0	4	4	2.41	1	0	0	2.63	.03	.08
5000	13	13	11	2.68	70	0	3	10	0.11	.9	0	0	2.37	.02	.05
4000	10	10	2	0.55	60	0	3	3	1.81	.8	0	0	2.10	.01	.03

To apply the table.—Multiply the days by the principal and rate, both in £s; divide the product by 100, then add the interest which stands opposite to the several parts of the quotient in the table; the sum is the answer. Thus to find the interest of £375 15s. for 72 days at 3½ per cent.?

By the Table.

	355.75
	3.5
	187875
	112725
	1315.125
	72
	2630250
	9205875
	$946.89\cancel{000}$

Opposite

900	is £2	9	3	3.12
40	—	2	2	1.21
6	—		3	3.78
.8	—			2.10
.09	—			.24
	£2	11	10½	.45

The method by the table has been recommended by Dr. Hutten as concise, to which it appears to have little claim, as may be seen by the above example, wrought by the general rule, which, besides saving the trouble of taking the numbers out of the table, has fewer figures than the other, and the division by 73, and valuing the decimal mentally, is so very simple, that there is little risk of errors. It will also be observed that this method gives a little more than by the table. Interest for days is frequently taken from interest tables without the trouble of performing the operation.

By general rule.

375.75
7
2630.25
72
526050
1841175

$73,000)189,378.\cancel{00}(2.594 =$ £2 11 10¾

146
433
365
687
657
308
292

To find the interest for any number of weeks; first find the interest for a year, then say, as 52 are to the weeks given, so is the interest for a year, to the answer.

EXAMPLE. Required the interest of £100 for 13 weeks at 5 per cent.

As $\underset{4}{\cancel{52}} : \cancel{13} :: \dfrac{£5}{4} = £1\ 5s.$ Ans.

This method is scarcely correct, as there are 52 weeks and one day in a year. It is therefore bet-

ter to reduce the weeks to days, and proceed by the general rule.

Some authors have given several cases in simple interest which are omitted here, because they fall directly under compound proportion.

BANKING-HOUSE CALCULATIONS.—Case 1. The Bank of England gives no interest on any monies lodged in it; but all the established banks in Scotland have been, and still are, in the habit of giving interest, both on deposit accounts, and on the balances of cash and current accounts. But the difficulty of investing their capital profitably has for some time obliged them to reduce considerably the rate of interest allowed; and unless a change take place in their favour in the money market, they must soon, for their own safety, withhold interest altogether on deposit accounts.

The calculations in this case are such as generally occur on deposit accounts, that is, when money is lodged in the bank and drawn as occasion may require, or lodged merely for interest.

It is evident if the interest is calculated upon the principal till the date of the first payment, and then on the several balances for the time between each payment, the sum of these must be the whole interest due. The answer would come out nearly the same if we multiplied and divided each product as it arises, but this would require a division for each payment, and thus more than double the work; besides, from the loss of remainders, it is not quite so accurate.

Questions in this case may also be calculated by finding the product of the principal by the days, and double the rate, till the time of final settlement, and subtracting from it the sum of the products arising from multiplying the several payments by double the rate, and by the days between the time they were made and the final settlement, and dividing the remainder by 73000. This gives the same result as the other, but would allow the business of accountants in banking houses to accumulate to such an extent as to become perfectly unmanageable. The settlement of these accounts might also come upon them when least at leisure, and besides the risk of errors in the hurry and bustle of business, the party requiring the settlement might be subjected to very inconvenient delay. But as accuracy and dispatch are the two great requisites in an accountant, he should employ every leisure moment in carrying forward his calculations.

Case 2.—The principle of the calculations in this case is the same as interest for days.

A Cash Account with a bank, is one, where the party is authorised to draw to a certain specified extent without being obliged to lodge any money, and this in consequence of a third party becoming responsible to the full amount of the sum specified.

An Account Current, is when a person keeps an open account with a bank, lodging and drawing his own money as occasion may require.

These accounts make up a considerable portion of the business of every banking house, and should be well understood in a commercial country.

The form given in the exercises in the text is that of a bank pass book, the left hand column of which is written by the person in whose name the account stands when he draws money, and the right hand column by the clerk or teller in the bank when money is lodged. The second form given in the example wrought in the text, is that kept by the accountant who has the charge of calculating the interest, &c.

The balances here are multiplied by the days as in last case, and extended in different columns when due to different parties, as represented and explained in the example wrought in the text.

To ascertain whether the several additions and subtractions are correct, find the difference between the Dr. and Cr. sides of the pass book, and if this is the same as the balance in the accountant's form, this part is right. To ascertain whether the days are correctly found, add up the days in the accountant's form, and if they are the same as the days between the commencement and termination of the account, they are correctly taken.

I shall here subjoin an example to illustrate a form for bank accountants, which I am not aware of having ever been introduced, and which possesses the advantage of enabling the accountant to tell at a single glance to whom the balance and interest are due, and to settle the longest account with accuracy and expedition when called for.

Required a statement of the following account on the 30th. December, allowing the bank 4 and AB 3 per cent. on balances.

Dr.—Mr. A. B's Cash Account with the Royal Bank.—*Cr.*

April 18.	To Cash	£260	April 30.	By Cash	£290
May 16.	To ——	200	June 15.	By ——	420
July 10.	To ——	500	Aug. 12.	By ——	200
Aug. 20.	To ——	110	Sept. 30.	By ——	400
Nov. 30.	To ——	620	Oct. 10.	By ——	300
Dec. 20.	To ——	300	Dec. 6.	By ——	100

		£	Days.	Products.	
April 16.	To Cash.	260	12 =	24960	Dr.
—— 30.	By ——	290			
	Cr.	30	16 =	2880	
May 16.	To ——	200			
				22080	Dr.
	Dr.	170	30 =	40800	
June 15.	By ——	420			
				62880	Dr.
	Cr.	250	25 =	37500	
July 10.	To ——	500			
				25380	Dr.
	Dr.	250	33 =	66000	
Aug. 12.	By ——	200			
				91380	Dr.
	Dr.	50	8 =	3200	
—— 20.	To ——	110			
				94580	Dr.
	Dr.	160	41 =	52480	
Sept. 30.	By ——	400			
				147060	Dr.
	Cr.	240	10 =	14400	
Octr. 10.	By ——	300			
				132660	Dr.
	Cr.	540	34 =	110160	
Nov. 13.	To ——	620			
				22500	Dr.
	Dr.	80	23 =	14720	
Dec. 6.	By ——	100			
				37220	Dr.
	Cr.	20	14 =	1680	
—— 20.	To ——	300			
				35540	Dr.
	Dr.	280	10 =	22400	
				57940	Dr.

	£	s.	d.
Balance due to Bank	280		
Interest do. do.	0	15	10½
Due Bank 30th Dec.	£280	15	10½

```
73)57.940(.793
   511          15/10½
   ---          Interest.
    684
    657
    ---
     270
     219
     ---
```

Now if A. B. had enquired how his account stood during any period of its currency, suppose on the 30th. December, the accountant has only to multiply 280 by 10, and the product by 8, double the per cent., add this product to the one immediately preceding, both being Dr., and divide the sum by 73000 as in the example; the information required might thus be furnished in 5 minutes. Farther, were bankers to keep a separate book in which the accountant could carry forward the statement of the accounts to be annually rendered, or produced at settlement, it is evident that any account, however extended, might be settled in a few minutes; for nothing farther would be necessary than to divide the last number in the accountant's form by 73000 as in the example, for the interest, and adding this to, or subtracting it from the balance in the statement book, the account is made up, and may be rendered by simply cutting it out of the statement book. The same thing might be done by keeping a double form on the same page. As this method is practicable without additional expense, and its dispatch a desideratum in business, there appears no good reason why it should not be adopted in every banking-house. The several balances by this method must be multiplied by the days, and double the rate, before placing them in the column of products, and therefore require a few more figures than the method in common use; but the work is done when the accountant would otherwise be disengaged, and is consequently a saving, not a loss of time. Farther, to the recommendation of having the accounts always made up, it is also exempted from heavy additions, which are ever attended with risk of errors.

It is customary with bankers to exact a per cent., and sometimes more than what they allow on balances, but this is perfectly justifiable on the ground of the expense of conducting the business, although there were no other reason.

BONDS.—In calculating interest on bonds, or progressive accounts, where the payments are made at intervals greater than a year, it is customary to add the interest to the principal before subtracting the payment; this practice has for many years been sanctioned by the Court of Session. The method of calculating the interest on the bonds in this treatise is by decimals, and valuing the quotient mentally, which I consider the simplest. I have preferred multiplying the principal by the whole of the days between the payments, instead of calculating for the years and days separately. When the end of February in leap year is within the currency of the complete year, I only count the year 365 days, for it is evident if the interest had

been calculated for the years separately, the additional day would not have been noticed. But when February leap year terminates in the time counted for odd days, we reckon it 29 days.

CASE 3.—The calculations in this case are made by interest for days or months; the principle is therefore the same as in these cases.

What bankers call discount, is the interest of the sum specified in the bill, from the day it is paid, till three days beyond the term of the bill. When commission is charged, it is always calculated upon the sum specified in the bill, unless a provision is made to the contrary. After deducting the discount, and commission when it is allowed, the remainder is called the proceeds, and is what the holder receives. The reason why the interest is calculated for three days beyond the term of the bill is, that diligence cannot be used on bills in Britain or Ireland, nor any expenses incurred till three days after the term of the bill is expired. The days thus allowed are called days of grace, and vary in different countries from 3 to 30 days. When the term of a bill is expressed in months, calendar months are understood, and bankers seldom discount bills which have more than 3 months to run. Thus, if a bill were dated on the 20th December at 4 months, it would not be discounted till the 20th January, and interest would be charged till the 23d of April.

Although what is here explained is the common and established custom of discounting bills, yet there are other methods of a ruinous nature, which men in straitened circumstances often have recourse to in order to raise money to meet some urgent demand, and so conducted as to elude the laws against usury. In such cases it is not uncommon for a man to discount his own bill with some petty banker, or discounter of bills, paying at the rate of 5 per cent. and ½ per cent commission, and when the term of the bill is expired, and the accepter finds it inconvenient to pay the money, the bill is renewed, and the same interest and commission charged; and as long as the lender is not alarmed for the principal, he will continue to renew the bill, thus securing more than 8 per cent. for his money. But there is a still more lucrative manner of discounting bills sometimes practised. A man is in the immediate want of cash, and offers his acceptance at 3 months to a money broker, who purchases the acceptance of a bill for £100, for £80 or £90. I shall subjoin an example of these methods, which may serve as a caution against engaging in such transactions.

1st. Discounted my own bill for £100 dated 1st. March at 2 months, and renewed the same on the 1st. May and 1st. July,

paying discount at 5 per cent. per annum, and commission ½ per cent. each time, and retired the bill on the 1st. September; at what rate did I pay interest?

	£0	16	8
Interest for 2 months . . .	0	10	0
Commission ½ per cent.	1	6	8
			6
Interest and commission for a year .	8	0	0
Interest of discount and commission .	0	2	7½
	£8	2	7½

It is evident that interest is here paid at more that 8⅛ per cent., yet such methods of procuring money are acted upon every day. 2d. Being straitened for cash I sold my own acceptance of a bill for £100, payable in three months, for £80 ready money; what rate of interest did I pay?

$\begin{matrix}\cancel{8}\cancel{0} : 100 \\ \cancel{3} : \cancel{1}\cancel{2}\end{matrix} :: \cancel{2}\cancel{0} = £100$ per cent. per annum. Thus for the use of £100 for a year, I pay £100. Whoever borrows money at this rate must soon find himself a bankrupt.

STOCK JOBBING CALCULATIONS.—The stocks are the debts contracted by government, in the name of loans, at different periods since the revolution, for defraying the expense of the public service. Public records are kept of these loans which amount to an acknowledgment that the nation owes to the lenders, or holders, the sum specified in these records. The interest of these sums is paid from a fund appropriated by government out of the revenues of the country for that purpose, and therefore they are called funded debts. The interest on these debts is paid half yearly, and the stock or principal is transferable at the pleasure of the holder. The calculations in stock-jobbing are all connected with the selling and purchasing of these funds. The real value of property in the funds is fluctuating, for although the rate of interest is fixed, yet many causes operate in changing the market value of the principal, or stock frequently in the course of a single day, and a knowledge of these causes is necessary to speculate with advantage in the public funds.

A large proportion of the national debt is vested in two funds bearing interest at 3 per cent. on the nominal capital. The largest of these is the 3 per cent. consols, or consolidated annuities, so called from having been formed by uniting several funds which were formerly kept separate, and the 3 per cent. reduced annuities, which are so called from having had the original rate of interest reduced from 4 to 3 per cent. Besides

these there are several other stocks which generally receive their names from the rate of interest granted. These are the 4 per cent. consols, the new 4 per cents., the 3½ per cent. consols, the 3 per cent. imperial annuities, so called because the loans of which they are composed were for the Emperor of Germany, and sanctioned by the British government. All these are permanent annuities; there are also long annuities which terminate in January 1860, and are sold at so many years purchase.

The interest, dividends, or annuities on the 3 per cent. consols, 3 per cent. imperial, new 4 per cents. South Sea stock, and new South Sea annuities, are all payable at the bank of England on the 5th. January, and 5th. July; and on all the other stocks on the 5th. April and 10th. October.

The transfer books of every description of stock are shut nearly a month before the day of paying the dividends, and remain shut about a fortnight after that date. The person in whose name the stock stands when the transfer books are shut, receives the interest for the preceding half year.

When stock is sold the purchaser becomes entitled to the interest due upon it, and this is the reason why the 3 per cent. consols, and the 3 per cents. reduced, which are payable at different terms, never sell at the same price. The same reason applies equally to the 4 per cents., and new 4 per cents. The difference of price between these stocks arises entirely from the difference of interest due upon them at the time, and it often requires considerable nicety of calculation to ascertain which of them is the preferable investment.

When a new loan is authorised by parliament it is made payable by instalments, and a certain portion of nominal stock, bearing interest at 3, 3½ and 4 per cent., and a small portion of the long annuities is generally offered by the chancellor of the exchequer for each £100 to be advanced, and he who gives £100 for the least portion of these different stocks is always preferred. During the progress of the loan, or till all the instalments are paid up, the several stocks thus united are transferable together, and are therefore called OMNIUM; but when sold separately they are called Subscription Receipts, or SCRIPT. When any person pays up the whole of the instalments at once, his receipt is in the language of exchange alley called "heavy horse;" and the receipt for one or two instalments is called "light horse," and is better adapted for speculation than heavy horse.

The capital of the bank of England amounting to £14553000, the nominal capital of the East India Company, amounting to £6000000 steg., and the capital of the South Sea Company are

called stock, and transferable like Government stock, but they do not belong to what is termed the funds, or funded debt of the country.

There are other sums raised by government, for the payment of which no special provision has been made, and are therefore termed unfunded debts; under this head may be classed Exchequer, Navy and Ordinance bills.

Exchequer bills are drawn for £100, £200, £500, or £1000 each, and bear interest from date, till they are paid off at the rate of 2d. per cent. each day.

Navy bills are payable 90 days after date, and if not then paid, they bear interest till paid. Ordinance bills are subject to similar regulations. The whole of these bills are generally paid off once a year by government.

India bonds are issued by the East India Company for £100, £300, £500, or £1000 each, they bear interest from date at 4 per cent., and are payable on the 31st. March and 30th. September.

The business of stock-jobbing is almost entirely transacted by stock-brokers, who find it their interest to keep the art of dealing in the stocks as mysterious as possible; they are allowed $\frac{1}{8}$ per cent., or 2/6 for every £100 of stock which they buy or sell.

Every person is at liberty to transact his own business on stock-exchange without the assistance of a broker, if he is on the spot and finds so disposed; but unless he has considerable experience in the alley he will not find this much to his profit.

To enter into a minute detail of all the particulars respecting the transfer of stock, the groundless rumours circulated to produce a temporary rise or fall in the market, the artifices resorted to in order to serve some interested purpose, and the endless variety of BULL and BEAR transactions entered into by brokers, and others, where no real stock is concerned, are sufficient to fill volumes, and cannot therefore have a place here.

CASE 1.—The reason why the rule directs to deduct $\frac{1}{8}$ from the selling price is this; the broker charges $\frac{1}{8}$ per cent. on every £100 of stock which he sells, and retains the sum in his hands as a matter of right, thus diminishing the selling price by $\frac{1}{8}$ per cent., or 2/6. The rule gives the sum which the broker must remit to his employer. The rule for this case is evidently simple proportion, and requires no explanation.

CASE 2.—The reason why $\frac{1}{8}$ is added to the selling priceis, the broker who makes the purchase, charges $\frac{1}{8}$ or 2/6 per cent. for his trouble, and this $\frac{1}{8}$ must be added to the selling price per cent. The rule gives the whole sum for which the broker is

entitled to draw upon his employer; and is simpler than calculating the purchase, and brokerage separately. The operations in this case are sometimes performed most expeditiously by decimals, sometimes by vulgar fractions, and are so simple as to require no further explanation.

CASE 3.—The reason of the rule for this case is obvious, for as the selling price increased by the brokerage is to the whole sum to be invested, so is £100 stock to the whole quantity of stock the proposed sum will purchase. The $\frac{1}{8}$ per cent. is added for the same reason as in last case. In this case there is often a fraction in the divisor, and generally the operation is simplified, by converting it into a decimal, and carrying out the decimal in the quotient three places, and valuing mentally. This rule gives the whole sum which the stock will cost the purchaser, including brokerage.

CASE 4.—The only difficulties in stock-jobbing calculations are found in questions connected with this case. The rule in the text is simple proportion, and might have been expressed thus. As the selling price of £100 stock increased by the brokerage, is to £100, so is the dividend on £100 stock to the rate per cent.

When the interest due on the stock at the time of purchase is to be taken into account, which should always be done when we wish to ascertain with accuracy in what fund we can vest money to the greatest advantage, we must find the interest due on £100 stock, from the time when the preceding dividends on that particular stock were due, subtract this interest from the selling price, and proceed with the remainder by the rule. This is the customary method of calculating in these cases, and the examples of this nature given in the text are calculated accordingly. Thus, to find what interest would arise from money vested in the 3 per cent. consols on the 25th. of May, when selling at $87\frac{3}{8}$ per cent. We would first find the interest of £100, for 140 days, which are the days between the 5th. January, when the last dividend was due, and the 25th. of March, by the following stating; as $\overset{\text{days}}{181} : \overset{\text{days}}{140} :: \overset{\pounds}{1{\cdot}5}$ to the interest due. There are 181 days between the 5th. January and the 5th. July, and in this time £1 10s. of dividend becomes due on £100 stock. The above stating gives £1 2s. $1\frac{1}{4}$d. due on the 25th. May, and subtracted from the selling price, or £87 7s. 6d., leaves £86 5s. $4\frac{3}{4}$d., with which we proceed by the rule. In this way we can ascertain which stock affords the best investment on any given day, and also how much it is preferable to any other stock, when the selling price of both is

given. Although this is the method of calculation generally used, yet it is not perfectly correct, for instead of subtracting the interest due upon the stock, we should only subtract the present worth of this interest payable at the time the first dividends on that particular stock become due. But this method is laborious and therefore seldom practised. Tables showing the present value of the interest due on the several descriptions of stock for every day between the different terms of paying the dividends might easily be calculated, and would form a useful manual both for the broker and the private gentleman.

The long annuities are calculated differently from other stocks, and the following rules may therefore be of use to the young accountant.

To find the value of an annuity. Multiply the annuity by the number of years purchase, subtract $\frac{1}{8}$ per cent. from the product, the remainder is the value.

To find what sum will purchase any proposed annuity. Multiply the annuity by the number of years purchase, add $\frac{1}{8}$ per cent. of the product to itself, the sum is the money required.

To find what annuity any proposed sum will purchase. Subtract $\frac{1}{8}$ per cent. of the proposed sum from itself, divide the remainder by the number of years purchase, the quotient is the annuity.

The $\frac{1}{8}$ per cent. taken notice of in the rules for the different cases of stock-jobbing calculations is on account of brokerage, but when a man transacts his own business in the stocks, the $\frac{1}{8}$ per cent. should be left out of the calculation altogether.

I shall here subjoin a few remarks which may enable the pupil to understand the statements given in the newspapers of the public stocks.

Price of stocks, April 22, 1829.

3 per cent. cons. $87\frac{7}{8}$, 88, $\frac{1}{8}$, $\frac{1}{4}$.	Bank Stock 210, 11.
3 — — red. $87\frac{1}{8}$, $\frac{1}{4}$.	India Bonds 48, 50.
$3\frac{1}{2}$— — cons. $96\frac{1}{4}$, $\frac{3}{8}$	Exch. Bills 59, 60.
4 — — cons. $102\frac{7}{8}$, 3	Long Annuities——.
4 — — 1826. $104\frac{1}{8}$.	3 per cent. cons. for 20th May $88\frac{1}{2}$.
India Stock $231\frac{1}{2}$, $2\frac{1}{2}$.	

Remarks.—3 per cent. cons. £100 of this stock at the opening of the market was worth £87 17s. 6d., it advanced to £88, £88 2s. 6d., and at the close of the market was selling for £88 5s.; thus gradually advancing during the day.

3 per cent. red. In the morning £100 of this stock sold for £87 2s. 6d., and at the close of the market it brought £87 5s. having thus advanced $\frac{1}{8}$ or 2/6 per £100.

3½ per cent. con. £100 of this stock sold for £96 5s. and at the close of the market it brought £96 7s. 6d.

4 per cent. cons. £100 of this stock sold for £102 17s. 6d. in the morning. and closed at 103.

4 per cent. 1826. £100 of this stock continued during the day to sell for £104 2s. 6d. without variation.

India stock. £100 of this stock sold in the morning for £231 10s., and at the close of the market it brought £232 10s., which was an advance of £1.

Bank stock. £100 of this stock sold in the morning for £210; and at the close of the market for 211.

India bonds. A bond for £100 sold in the morning for £102 8s., and at the close of the market for £102 10s. The 48, 50 here, signify the shillings per cent. of advance upon the sum specified in the bill. Thus a bill for £1000 would cost £1024 in the morning, and 1025 in the evening.

Exch. bills 59, 60. P. M. That is an exchequer bill for £100 sold in the morning for £102 19s., and at the close of the market for 103.

Long annuities, or Bank annuities.—Shows that there was no business done in this stock that day. But if it had been written 17⅝; it would have signified they were selling for 17⅝ years' purchases.

3 per cent. cons. for 20th. May, 88½. This shows that some bargains had been made in anticipation of a rise in the market, to pay at the rate of £88 10s. on the 20th. May, for every £100 specified in the agreement.

It sometimes happens that India bonds, Exchequer bills, &c. are at so much discount, that is, sell for a few shillings per cent. less than the sum specified in the bill. It also frequently happens that the price of the several stocks fluctuate, that is, both rise and fall in the course of a single day, and is expressed thus, 3 per cent. con. 87⅜, ½, ¼, which shows that £100 stock, in the early part of the day sold for £87 7s. 6d., that it advanced to £87 10s., and again fell to £87 5s.

When, "shut," is written after the name of any stock, it signifies that the transfer books are shut for the payment of the dividends, and during that time no transfers can be made.

INSURANCE OFFICE CALCULATIONS.—The insurer is the party who takes upon him the risk. The insured is the party exempted from risk by the payment of a stipulated sum. The sum paid is called premium, and the paper on which the agreement is written is called the policy. The policy duty is a tax paid to government, of 3/ per cent. on policies for land insurance, 2/6 per cent. on sea insurance, from one part of the

United Kingdom to another, and 5/ or ¼ per cent. for any other voyage, but brokers often charge a little more to compensate for the advance of money and the trouble of filling up the policy.

CASE 1.—The rule in the text is simple proportion, and might have been expressed thus. A's £100 is to the sum insured, so is the rate agreed upon to the premium; the rule for finding the policy duty may also be considered as belonging to proportion, and therefore requires no explanation here.

Land insurances are generally effected by chartered companies called insurance companies. All the calculations in this case are extremely simple and require no farther illustration. Policy duty both on land and sea insurances is charged on the fraction of £100, thus £250 pays the same policy duty as £300.

When a part only of the value of the property is insured and a total loss is sustained, the insurers must pay up the whole sum specified in the policy. When insurance is effected to the whole value of the property, and either a total or partial loss is sustained, the insurers must make good the loss, either by paying the value in money, or replacing the goods by others of equal value, and it is customary with insurers to reserve this privilege to themselves by a special clause in the policy.

When only part of a property is insured, and the rest risked, and a partial loss is sustained, it is evident that in equity, the insurers and the owner of the property should sustain the loss in proportion to the sums risked. Thus, suppose insurance was effected to the amount of £500 on property worth £1000, and a loss of £400 was sustained, the insurer should not be liable for more than £200.

CASE 2.—The rule for this case is the same as for last. Although there are several sea insurance companies, yet the great proportion of business in this department is transacted by private individuals who subscribe insurance policies, for whatever sum they may feel disposed, and from this circumstance are called under-writers. The business of sea insurance is generally effected by the agency of insurance-brokers, who are allowed 1/ for every £ or guinea of premium, according as it is expressed in £s or guineas.

When a broker receives orders to effect an insurance he draws out a paper mentioning for whom the insurance is to be effected, on his own or some other person's account, the amount to be insured, the particular kind of goods, the name of the ship and master, where and when the risk is to begin and end; the different risks for which the insurers become bound; at what places the ship is to call; the rate of premium to be given;

how much must be paid in case of loss, and at what time; the year and day in which the policy is executed; with every other necessary particular relative to the adventure.

The broker having obtained under-writers to the amount required, he fills up the policy, accompanied with an acknowledgment of the receipt of premium, which, however seldom obtained at that time, is considered necessary in the eye of the law, in order to obligate the under-writers to make good a loss, and this policy is signed anew by the under-writers.

In time of war it is customary to charge a very high premium, with a stipulation to return so much if the ship sail with convoy and arrive safe, and this return is always a part of the premium paid to the under-writers. But if a ship deviate from the course prescribed in the policy without any legal reason, the under-writers are freed from any responsibility in case of a loss.

When merchants act as agents, they generally charge ½ per cent. for effecting an insurance, and 2 per cent. for settling a loss; but brokers charge only ½ per cent. for settling a loss, and this charge is made upon the sum recovered.

It frequently happens that the value of the goods shipped falls short of what is specified in the policy, in which case the deficiency is called short interest, and a return is made of the under-writer's premium upon the difference between the value of the goods shipped and the sum insured, but no part of the brokerage, policy duty, or agent's commission is ever returned; and under-writers deduct ½ per cent. from the sum returned for short interest as a compensation for their trouble, unless they are prohibited by the terms of the policy.

Insurance brokers in England are prohibited by law from acting as under-writers.

CASE 3.—It is evident the rule for this case is also proportion, and might have been expressed thus; as the difference between £100 and the sum of the premium policy and commission per cent. is to £100, so is the value of any property to the sum which will cover it.

Many of the public ordinances regarding insurance enjoin particularly, that the insured run part of the risk themselves as the best method of preventing frauds, but these injunctions are seldom complied with, and the common practice is almost every where in this respect opposed to the law. When therefore a case in dispute is brought before a judge, and there is no appearance of fraud having been committed, although the insured has run no part of the risk, the under-writers are generally obliged to fulfil their contract, thus deciding according

to custom rather than law. The calculations, in this case, are therefore such as custom has introduced in opposition to the spirit of the laws.

Merchants seldom insure to cover their property, yet it is sometimes done, and custom has rendered it practicable in all cases, therefore the rule should be understood.

The rule given in the text is scarcely correct, as it takes no notice of the expense of recovering a loss from the under-writers, and only applies when they make good a loss without discount, or the necessity of employing an agent to recover the loss. To make the calculation quite correct, it will be necessary to deduct the expense of recovery along with the premium, &c. from £100 for the first term of the stating. I shall illustrate this by an example. How much must be insured to cover £100, premium 5 guineas per cent. policy 5/, and commission $\frac{1}{2}$ per cent. supposing an agent charges 2 per cent. for recovering a loss?

$$£100 - (£5\ 5 + 5s. + 10s.) = \frac{100 \times 100}{100-6} = \frac{10000}{94} = £106\ 7s.$$

8d. By Rule.

$$£100 - (£5\ 5 + 5s. + 10s.\ £2) = \frac{100 \times 100}{100-8} = \frac{10000}{92}\ £\ 108$$

13s. 11d. By Note.

From this it is evident that £108 13s. 11d., and not £106 7s. 8d., must be insured to cover £100 according to the conditions of the question. In all such cases the whole expense of recovery must be subtracted along with the premium, &c. for the first term, whenever we would calculate with perfect accuracy.

When insurance is to be effected to cover property on a voyage out and home, or any similar insurance; first find how much must be insured to cover it out, then find how much must be insured to cover this sum home.

CASE 4.—This case is wrought by simple proportion.

The calculation of averages is often attended with considerable difficulty, arising principally from the manner in which the policy is expressed, and which has often terminated in law suits. Too much care cannot be taken to express the policy so clearly, and unequivocally, that the insurers may see clearly the extent of their risk, and that the insured may understand what they have to expect in the event of loss or damage.

Averages are either GENERAL or PARTICULAR.

General or Gross Averages are borne equally by all concerned in the ship and cargo, in proportion to the value of their property.

Particular Averages either of the ship or cargo are borne by those at whose risk the property is, which has sustained the damage, without affecting those whose property is not damaged.

As this department of insurance is the most intricate, and an acquaintance with it of the greatest importance both to the insurers and the insured, I shall subjoin a few observations on each of these heads, which may serve to show the general principle in a manner quite intelligible to all concerned.

Whatever the master of a ship in distress deliberately resolves on doing with the advice and consent of the officers and men, such as cutting away masts and cables, throwing goods overboard, &c. for the preservation of the ship and cargo, come under the head of general averages. But to render this jettison legal, it must be shown that the ship was in distress, that the action was in consequence of a deliberate consultation between the master and men, that the sacrifice was absolutely necessary to save the remainder, and that the saving of the ship, and remainder of the cargo, was actually owing to the means thus used with this view alone. Besides the loss thus sustained by jettison, the following expenses come under the head of general averages, viz.

All the damage which a ship may sustain in her rigging, hull, or cargo in defending her against an enemy.

Any reasonable gratuity which a master may promise to the sailors, to animate them to a desperate resistance when attacked by an enemy.

The expense of extraordinary attendance on officers and sailors wounded in the ship's defence, and the rewards promised by articles before the engagement takes place, to the widows and children of such as lose their lives in the action.

All extraordinary pilotage, &c. which may be incurred, when obliged to take shelter in a harbour from springing a leak, or some other damage, or in bringing her to a place of safety when chased by an enemy.

The necessary expenses of getting the ship clear when run aground, such as the charges for unloading and demurrage, carterage, porterage, &c. when goods are to be carried over land; perquisites to proprietors of lands for liberty to digg off the ship when driven above high water mark, with the expense of performing the work; the expense of repairing the damaged works of the ship under water occasioned by such accident;

rewards given to the sailors for extra labour in preserving the ship and cargo; charges of victualling, and guarding the ship while thus detained, these, with similar charges, together with interest, insurance premium, and commission on monies laid out for the general good, go to the amount of gross averages.

The value of all goods and stores taken away by privateers, or other commissioned ships, promising to pay for them.

The expense of extraordinary quarantine, and other unavoidable accidents; such as holes cut in the ship to run out the water, or to convey it to the pumps when she is filled by shipping a sea; when goods are lost from having been put into any other vessel, or lighter, and there perished when otherwise they must have been thrown over board; together with the hire of anchors, and cables to replace those thrown away by jettison, and all similar expenses belong to gross average.

Charges of salvage also belong to general average, such as retaking a vessel captured by the enemy, which is $\frac{1}{8}$ of the real value when she is retaken by one of his Majesty's ships, and $\frac{1}{8}$ when by any other ship, and also the expense of preventing the loss of ship and cargo by shipwreck, pirates, fire, &c. but ransoms paid to an enemy are not borne by under-writers, and therefore do not belong to general averages?

General averages are always paid, however small they may be, and the best method of calculating them, is to ascertain what the ship, freight and cargo, if no jettison had taken place, would have produced if sold for ready money, at the time the ship arrived; then say, as this sum is to each particular person's share thereof, so is the whole loss to each person's share of that loss.

In computing general averages for masts, sails, cables, &c. only two-thirds of the expense is admitted into the account of averages, the new being considered one-third better than the old.

It would carry us beyond our limits to enter more particularly upon this part of the subject of insurance, which is so diversified, and so complicated as still to leave room for law suits, notwithstanding the united wisdom of commercial Europe has been engaged for generations in framing ordinances for its regulation.

PARTICULAR AVERAGES arise from damage sustained either by the ship, or the goods belonging to any particular person, from the common accidents of the sea, without any regard to the safety and preservation of the remainder of the ship and cargo. These losses are borne by the party at whose risk the goods are. Thus, supposing 10 hhds. of sugar and 20 hhds. of coffee had been shipped in Jamaica for A B of Leith, and that A B had insur-

ed the sugar at £20 a hhd. and risked the coffee, and that in the course of the voyage the vessel had lost one of her masts, and sustained other damage in her rigging during a storm, and by shipping a sea, 3 hhds. of the sugar and 4 hhds. of the coffee were destroyed. It is evident the damage sustained was the effect of accident, and therefore the proprietors of the ship have no claim either upon A B, or the insurers of the sugar, neither have the under-writers any claim, either upon A B, or the proprietors of the ship; that is, the shipowners must bear the whole damage sustained by the ship, A B bears the loss of the 4 hhds. coffee, and the under-writers must pay A B £60, the price of the 3 hhds. of sugar which was lost. Again, had the sugar been damaged, so as to sell for £12 per hhd. whereas if sound it would have brought £24 per hhd. it is evident the damage here amounts to half the value, therefore the under-writers must pay half the value specified in the policy, or £10 for every hhd. which was damaged.

There are regulations by which under-writers are exempted generally, from particular averages under 3 per cent., and on tobacco, sugar, hemp, flax, hides, and skins, under 5 per cent., unless the ship is stranded, and on corn, seed, flour, fruit, fish, and salt, no average is paid unless the ship is stranded, if a special provision is not made to the contrary in the policy.

The return of premium is not allowed on the sum claimed for particular averages.

The reason why particular averages are not paid when under 3 per cent. is to prevent the under-writers from being harassed with trifling charges; and in this way the intention of the insured may sometimes be disappointed, and a very considerable loss sustained, without any redress. For example, if insurance was effected on 101 bales of goods valued in the policy at £10100, and 3 bales were destroyed, it is evident by this custom, the under-writers would not be answerable, although the insured has sustained a loss of no less than £300.

This is certainly a hardship which ought to be guarded against, and which may be done in several ways, a few of which I shall point out. 1st. It may be guarded against, by a special clause in the policy, stating a certain fixed sum upon the whole amount of goods, beyond which if damage is sustained particular average shall be settled. 2d. When the goods are valuable let them be insured by parcels, in different policies. Thus, if the above 101 bales had been divided into 5 parcels, and insured in 5 distinct policies, the whole 3 bales destroyed must have been accounted for by the under-writers. 3d. Let every bale, box, or barrel, be numbered, and valued in the po-

licy, and so much insured upon each individually, and thus the damage upon every bale, box, or barrel, must be accounted for if it amount to 3 per cent. This last method is perhaps the most satisfactory for all parties, and secures against any considerable loss. For if a single bale is valued at £100, and if it has sustained damage to the amount of £3, the under-writers are bound to make it good.

The Barratry of the master and crew, that is running away with ship or cargo, or fraudulently disposing of them, should also be guarded against by special provision in every policy of insurance. For although it seldom happens, yet there are cases of it, and it may again occur when least expected.

Before a loss can be settled it is necessary to show how much is insured upon the ship and goods in other offices; to produce the protest made by the master upon oath; the bills of sale and custom-house registers, together with the bills of lading signed by the master, with every other information which the insurers may judge necessary and are able to procure, but if fraud can be proved against the insured, the insurers are generally freed from their obligation.

It is customary in London for insurance companies to pay £98 per cent. in a week after the adjustment of the loss, and with under-writers to pay the same per cent. in a month. In Edinburgh and Glasgow it is customary for the insurance offices and under-writers, to grant bills at 3 months immediately after settling the loss.

COMPOUND INTEREST.—The rule for compound interest is the same as for simple interest, only considering the amount at the end of each term of paying the interest as a new principal.

The laws of Britain do not allow compound interest, but every person is at liberty to uplift the interest of money the day it becomes due, and to lend it out again at interest, either to the person by whom it was due, or to any other, thus realizing all the advantages of compound interest.

There may be substantial political reasons for prohibiting compound interest by law, but this law does not appear to have its foundation in justice, which may be illustrated satisfactorily by a single example. Suppose A has lent to B £100 on the 1st. of January, at 5 per cent. interest, payable annually, it is evident that on the 1st. of January following A is entitled to £5 of interest, and that this sum is as legally his property as the £100 of principal first lent, therefore if B retains this £5 in his hands, he has then £105 of A's money, which at the same rate of interest must yield £5 5s. at the end of the second year. But the law says it shall only yield

£5, thus diminishing considerably the stipulated rate of interest. The law which so ordains would be improved by inserting some such clause as the following, "when interest is called for when due, and not paid, it shall thenceforth become part of the principal, and bear interest accordingly." The method of calculating compound interest by the rule in the text is extremely laborious, and therefore seldom resorted to. See more on this subject, with the method of calculation by tables, under the head of compound interest and annuities.

DISCOUNT.—This rule may with propriety be called true discount, when compared with the method in general use among bankers and merchants. To illustrate this, and the principle on which the rule is founded, I shall suppose that A is indebted to B £100, but not payable till the end of a year. It is evident that the money is A's for a year, and that B has no right to it, nor interest in it for that time. But if A can improve his £100 at the rate of 5 per cent., it will at the end of a year amount to £105. The question therefore is, if B wishes the money paid immediately, how much discount should he allow to A? The answer given to this question by the general practice of merchants and bankers is, that he should be allowed £5. Now this is evidently too much, for if A improve his £5 at the same rate which we supposed he could do the £100, he will be a gainer of 5/ at the year's end by this transaction, that is, by the whole interest of the discount allowed, therefore the common practice is incorrect. But the discount found by the rule for this case, which A should receive, is £4 15s. 2$\frac{6}{7}$d.; and this sum put out at the same rate of interest for a year amounts exactly to £5, which is the sum A would have gained by retaining the £100 in his own hands; this rule is therefore correct. When the time is short, the difference between the discount obtained by this rule, and the common practice, is so small as scarcely to deserve notice, but when the time is long, the difference is almost incredibly great, as it increases in the same ratio as the square of the times, in all such cases the common rule should never be applied. It may be remarked here, that Smart's tables of discount are calculated on the true principles of discount. The operations in this rule are all performed by simple interest, and simple proportion, the principles of which have already been explained. The calculations of discount are often very much simplified by the use of decimals, for the application of which observe the following rule.—As the amount of £1, for the given time, and at the given rate, is to £1, so is the interest of the debt for the same time, and at the

same rate, to the discount required. EXAMPLE.—What is the discount of £86 16s. 3d. due 3½ years hence, at 4½ per cent. simple interest?
4.5÷100=.045 interest of £1 for a year at 4½ per cent.
(.045×3.5)+1=1.1575 amount of ditto for 3½ years.
(86.8125×.1575)÷1.1575=11.813=£11 16s. 3¼d. discount.
and £86 16s. 3d.—£11 16s. 3¼d.=£74 19s. 11¾d. present value.

I here divide the rate, or interest of £100 by 100, which gives the rate of £1, and this multiplied by the time, gives the interest of £1 for the given time, to which if £1 be added, we have the amount of £1. Again, if we multiply the proposed sum by the interest of £1 for the given time, we have its interest for the whole time; and as the second term of the proportion is always £1, it is evident that dividing the whole interest by the amount of £1, we have the true discount of the proposed sum for the time, and at the rate proposed.

EQUATION OF PAYMENTS. This rule has been the source of many warm and lengthened disputes among writers on arithmetic. To enter upon these, would be as unprofitable to the pupil, as it is foreign to my purpose. The rule given in the text is the one in general use, and when the times are short, its results are sufficiently accurate for practice. But it is liable to the same objections as the common rule for discount, because it is founded on the same principle, that the interest of the money which remains unpaid after it becomes due, should be equal to the interest of the money which is paid before it is due, which is not correct in principle, for it should only equal the true discount of what is paid before it is due. The true rule was invented, and its truth established, by the ingenious Alexander Malcolm of Aberdeen, in 1730, the substance of whose rule is as follows. To the sum of the two first payments, add the continued product of the first payment, the time in years between the payments, and the interest of £1 for a year; and call their sum the FIRST NUMBER. From the square of this number, take four times the continued product of the two payments, the interest of £1 for a year, and the time; and call the square root of the remainder the SECOND NUMBER. Subtract the second number from the first, divide their difference by twice the product of the first payment by the interest of £1 for a year, the quotient is the equated time for the two first payments. When there are more than two payments; consider the sum of the two first as one payment to be made at the equated time, then find as before the equated

time for this sum and the third payment, and so on for any number of payments.

EXAMPLE. A debt is due, £200 in a year, and £500 in four years hence; what is the equated time to pay the whole at once, allowing interest at 5 per cent. per annum?

$(200 + 500) + (200 \times 3 \times .05) = 730$ FIRST NUMBER.

$\sqrt{[730^2 - (200 \times 500 \times 3 \times .05 \times 4)]} = 687.6772$ SECOND NUMBER.

$(730 - 687.6772) \div (200 \times .05 \times 2) = 2$ years, 42 d. 9 h. + after the first payment was due, and therefore 3 y. 42 d. 9 h. the whole equated time. Although this is the true method, yet it is extremely tedious even with two payments, and with three or four payments it would be intolerable in business. Instead, therefore, of submitting to the drudgery of this rule, it would answer the purpose nearly as well to find the equated time by the common rule, and then let the difference between the interest of the sums retained after due, and the true discount of those paid before due, be settled by cash.

BARTER.—The calculations in bartering are all performed by simple proportion, and require no further explanation here. When the second rule in the text can be applied, it gives the answer much more expeditiously than the other, and ought therefore to be preferred. It may be observed here, that in all barter transactions neither party is understood to be directly gainers, the whole being merely a matter of mutual accommodation.

PROFIT AND LOSS.—The calculations in profit and loss are all performed by simple proportion. It was customary among the old writers on arithmetic, neither to give particular rules, nor different cases in this important part of the work, and their example is still followed by some modern authors. But many of the operations in profit and loss are of such a nature, that the majority of pupils cannot be expected to comprehend them clearly without particular rules. In short, there are few rules in arithmetic where false results have more frequently been obtained, even by writers on the subject, from a want of clearly comprehending the nature of the transaction. I shall here subjoin the reason of the rules given for the several cases.

CASE 1.—The reason of this rule is evident, for whatever proportion the prime cost of any goods bears to the gain or loss upon them, £100 must bear the same proportion to the gain or loss per cent.

CASE 2.—The reason of the rule is this, whatever proportion

£100 bears to £100 with the gain per cent. added, or the loss per cent. subtracted, the prime cost must bear the same proportion to the selling price.

CASE 3.—The reason of the rule is this, £100 increased by the gain or diminished by the loss per cent. must invariably bear the same proportion to £100, as the selling price does to the prime cost.

CASE 4.—Reason of the rule. It is evident when two selling prices are given, and the rate of gain or loss by one of them, that the price whose rate is given must always be to the other, as £100 with the gain added, or loss subtracted, is to £100, plus the gain, or minus the loss per cent. of the second selling price.

CASE 5.—Reason. It is evident from the nature of proportion, if £100 minus the proposed discount give £100, that the selling price must give the advanced price, which will admit of the proposed discount, and leave the same profits.

CASE 6.—Reason. It is evident, that, whatever proportion the proposed rate bears to £100, by which it is produced, the whole gain or loss must bear the same proportion to the prime cost by which it was produced. But if the gain on any quantity of goods be added to the prime cost, or the loss on them subtracted from it, the sum in the one case, and the remainder in the other must be the selling price.

From a consideration of the reasons of the rules for the several cases, the principles on which they are founded will readily be discovered.

MERCANTILE COMPOSITIONS.—Many of our late writers on arithmetic have considered the rules of ALLIGATION, OR MERCANTILE COMPOSITIONS unworthy of notice. Others have condescended to give them a place, while they represent them as worthless in business, and unwieldy in practice, by any engine less powerful than algebra. But to me these rules certainly appear in a very different point of view. Perhaps at no former period was the practice of forming compositions of goods of different qualities more universally prevalent than what it is at present. What substantial reasons then can be assigned for withholding from the pupil, during the whole course of his education, the knowledge of a rule which in all probability he may be called upon to put in practice immediately on entering upon the business of life. It is vain to urge the advantages of algebra in this particular case, so long as it is not made an indispensable requisite, for being admitted into the shop or counting house. Besides, the operations in alligation are so simple, and their results so satisfac-

tory by arithmetic, that every individual who may be called upon in the course of business to make such compositions should be thoroughly acquainted with its several rules. By the assistance of these rules, the compounder will be enabled to mix his goods in any proportion, without the least risk of injuring himself, or defrauding the public.

CASE 1.—The reason of the rule is this; by the process prescribed we find the value, or quality of all the ingredients of which the mixture is composed, which, divided by the number of ingredients, gives the price or quality of one of the composition.

CASES 2 and 3.—The reasons and principles of these rules are the same as those for single distributive proportion already explained.

CASE 4.—From the circumstance of linking or binding together the several simples which are less than the mean rate, with those which are greater than it, this rule received the name of Alligation. But as this name appears inapplicable to the other cases, and conveys no idea of its practical application, I have preferred calling it Mercantile Compositions, which is equally applicable to the several cases, and points out its practical use. Besides linking even in this case is not absolutely necessary; for it is evident we may take the difference between the mean rate and the simples, and place them properly without linking, as its use extends no farther than to assist the eye in tracing the proper position of the several differences.

The reason of the rule is this; by taking the difference between the mean rate and two simple, one of which is greater and the other less than it, and placing them alternately, there is exactly as much gained by selling the one above its real value as there is lost by selling the other under its value; therefore a mixture of the quantities obtained by the rule, at the mean rate, must be equal in value to the simples of which it is composed at their respective rates. Again, whatever be the number of simples, proceeding with them according to the rule, the loss and gain on every pair will balance one another, and consequently the loss and gain on the whole will also balance one another. I shall illustrate this by an example; suppose tea at 7/, 9/, 12/ per lb. is to be mixed so, that the compound may be worth 10/ per lb. Having placed the numbers and taken the differences as in the margin, we find that 2 lb. at 7/ and 3 lb. at 12/ will make a composition worth 10/. But by selling 2 lb. at 10/, which is worth only 7/, there is a

$$10\left\{\begin{array}{ll} 7 & = 2 \\ 9 & = 2 \\ 12 & = 3+1 = 4 \end{array}\right.$$

gain of 6/, and by selling 3 lb. at 10/, which is worth 12/, there is a loss of 6/, the gain and loss balancing each other. Again, 2 lb. at 9/, and 1 lb. at 12/ make a composition worth 10/. But by selling 2 lb. at 10/, worth only 9/, there is a gain of 2/, and by selling a lb. at 10/, worth 12/, there is a loss of 2/, which again balance one another. Again, 2 lb. at 7/, 2 lb. at 9/, and 4 lb. at 12/ make a composition worth 10/, but upon 2 lb. at 7/ there is a gain of 6/, and upon 2 lb. at 9/ there is a gain of 2/, in all a gain of 8/, and upon 4 lb. at 12/ there is a loss of 8/ which balance one another. This relation between the gain and loss is not affected by the number of simples, for the gain and loss on each pair balance one another, and must therefore do so on the whole.

The answers in this case are not limited to what is obtained by linking, for it is evident if we either multiply or divide all the results in last example by any number, the products and quotients still have the same ratio to each other, and therefore answer the conditions of the question, consequently every question here admits of an infinite series of answers.

It is not necessary that all the simples less in value than the mean rate should be linked with all that are greater according to the rule, for by linking a less with a greater in any order, gives a correct answer, and yet different by every different method of linking. To illustrate this I shall take the example in the text, and link it in every possible way.

1st. 28. $\left\{\begin{array}{l}18\\23\\30\\36\end{array}\right.$ 2nd. or 28. $\left\{\begin{array}{l}18\\23\\30\\36\end{array}\right.$ 3d. or 28. $\left\{\begin{array}{l}18\\23\\30\\36\end{array}\right.$ 4th. or 28. $\left\{\begin{array}{l}18\\23\\30\\36\end{array}\right.$

5th. or 28. $\left\{\begin{array}{l}18\\23\\30\\36\end{array}\right.$ 6th. or 28. $\left\{\begin{array}{l}18\\23\\30\\36\end{array}\right.$ 7th or 28. $\left\{\begin{array}{l}18\\23\\30\\36\end{array}\right.$

All these different methods of linking give different results equally correct; and by applying multiplication and division to them, we obtain seven infinite series of answers to the question.

When multiplying or dividing the results, it is not necessary that we should multiply or divide them all, for when only one less than the mean rate is linked with one greater, we may multiply or divide one pair of results without altering the others, or we may multiply one pair, and divide another, and still have the answer correct. The reason of this is evident from what has already been explained. The reason why the general rule directs to link all the simple rates which are

greater than the mixture rate, with all those that are less than it, is, that the pupil may know by what method of linking to obtain the answer given, without the trouble of working by all the methods before obtaining the answer; but having found the answer given, the teacher may with propriety cause the answers to be brought out to the several questions by all the different methods of linking, and the accuracy of them to be proved by the balance of gain and loss.

The endless variety of answers of which this case admits, instead of being a disadvantage, only tends to render it more generally useful.

CASE 5.—The method of finding the quantities of the several simples by linking, is the same here as in last case, and admits of the same varieties. The second part of the rule is the same as simple distributive proportion, and requires no further illustration. But if there are more than one of the simples limited, it will be necessary to find their mean rate by case 1st. and to proceed with it as the rate of a simple whose quantity is equal to all the limited simples. But if all the other rates be less than the mean rate, and the rate of the limited simples also be less, then the question is impossible.

CASE 6.—The principles of this case are in every respect the same as last, and admits of the same varieties in linking and bringing out the simples as Case 4.

In this case, as well as the two preceding, when any of the simples is of little or no value compared with the others, such as water with wine, alloy with gold and silver, &c. it is always written 0.

EXCHANGE.—The calculations in exchange being all performed by rules already explained it would be superfluous to notice them here.

The doctrine of exchanges is too extensive and too intricate to admit of being fully discussed in a treatise of this nature. All that can be attempted here is a few cursory observations, to illustrate some of its leading principles. Those who wish to study the subject thoroughly, will find it ably and fully discussed in the "UNIVERSAL CAMBIST," published by Mr. Kelly, which is acknowledged to be the most correct and comprehensive system of exchanges in any language.

Exchange, as already remarked, is transacted by bankers and merchants by means of bills of exchange, settling the mutual claims of distant countries without remitting money. For, suppose A of Edinburgh owed to B of Amsterdam £100, and C of Amsterdam owed to D of Edinburgh £100, these debts

might be settled, if D would draw a bill upon C for £100 and sell it A, he would remit it to B, who would receive the amount from C, and thus the several claims would be answered without remitting money.

PAR OF EXCHANGE.—The par between different countries is estimated by the weight and fineness of the coins compared, and generally remains the same so long as these are not altered. The principal exceptions to this are, when the coins compared are not of the same metal, that is, the one silver, and the other gold, which is the case between Britain and most places on the continent of Europe, our standard being gold, and theirs generally silver. In these cases the par fluctuates with the relative value of gold and silver bullion. Gold and silver are also in a small proportion more valuable in countries remote from the mines, occasioned by the expense and risk of transporting them.

COURSE OF EXCHANGE.—The course of exchange between two countries is seldom at par, for this can never happen unless the exports and imports between them are precisely equal. Suppose that France exports to England the same value of goods that England exports to France, then the exchange is at par, that is, the mutual debts of the two countries are equal. But if the exports from England to France, exceed the imports from France, then the balance of trade is against France; and the demand in Paris for bills on London to remit this balance will exceed the supply, and accordingly they will rise in value; whereas the supply in London of bills on Paris will exceed the demand, and consequently they will fall in value, and the exchange will be favourable to London, and must continue so till the imports from France equal or exceed the exports to it. The course of exchange can scarcely ever exceed the expense of remitting money, or bullion, for then merchants would remit money to pay their foreign debts in preference to purchasing bills. Again, when the course of exchange is against any country, it acts as a premium for exporting goods to that country to which it is in debt, as the bills obtained in payment are more valuable on that account. Besides, through the management of bill merchants in effecting circular exchanges, the fluctuations of the course of exchange are generally confined within very narrow limits. From the combined operation of these causes, the course of exchange can never for any length of time be very much either above or below par.

USANCE.—Double usance, and half usance signify double the time, or half the time generally allowed for paying bills of exchange, thus, usance with France is 30 days, double usance 60

days, and half usance 15 days after date. When a bill is presented for acceptance it is generally allowed to remain with the person for a night, and any writing put upon the bill, which does not imply a refusal, is esteemed a legal acceptance.

In every exchange, a fixed sum of the money of one country is always given for a variable sum of the other. Thus in exchanges between London and Amsterdam, London gives £1 sterling for a variable number of shillings and pence Flemish. But in exchanges with Russia, London gives a variable number of pence for a rouble. In the first case, the higher the course of exchange the better for London, for it is more profitable to receive 36/ for £1 sterling, than 34/. In the second case, the lower the exchange the better for London, for it is more profitable to receive a rouble by paying 3/4 for it, than to pay 3/8. Therefore, in all cases, the higher the exchange the more profitable for the country which gives the fixed sum or rate, and the worse for the other.

Inland Exchanges.—The exchange between London, and the rest of Great Britain is always in favour of London. This arises partly from the great commerce of London, but chiefly from the demand for bills on London to remit revenue. Bankers in Edinburgh and Glasgow give drafts on London at 20 days date for money lodged with them, and at 30 days date for remitting revenue. In other inland remittances, bankers charge stamp duty, postages, and a small per centage.

Exchange with Ireland.—The same causes operate in producing an unfavourable exchange between Ireland and London, as between the other parts of the United Kingdom and London. Britain gives to Ireland the fixed sum, and receives the variable, therefore the higher the exchange the better for Britain and the worse for Ireland. The course of exchange with Ireland varies from 5 to 20 per cent.

Exchange with West Indies.—In Jamaica the ratio of currency to sterling is fixed at $\frac{7}{5}$, that is, £100 sterling is equal to £140 currency; but bills of exchange on London generally sell at a premium from 5 to 20 per cent. The ratio between sterling and currency is not fixed in any other of the West India islands.

Exchange with America.—In North America accounts are kept as in Britain, and bills on London are generally at a considerable premium. In the United States accounts are kept in £s. sh. and d. as in England, and also in dollars, dimes and cents; thus, 10 cents = 1 dime, 10 dimes = 1 dollar, 10 dollars = 1 eagle. The current value of the dollar varies considerably in different states; thus, a dollar in Connecticut,

Massachusetts, New Hampshire, Vermont, and Virginia is 6/ currency, it is therefore $\frac{3}{4}$ of stg. In North Carolina and New York it is 8/ currency, or $\frac{9}{16}$ of stg. In Delaware, Maryland, New Jersey, and Pennsylvania, it is 7/6 currency, or $\frac{3}{5}$ of stg. In South Carolina and Georgia it is 4/8 currency, or $\frac{27}{28}$ of stg. The exchange with Britain is either at par, or so much per cent. above or below par according to the balance of trade.

Exchange with France.—The franc and livre at one time were synonymous, the difference now existing was not intentional, but originated from an inaccuracy in the mint in 1795, by which the five franc piece was made too weighty and was worth 101¼ sous instead of 100, and the francs were afterwards made in the same proportion. As France gives to Britain the variable for the fixed sum, the higher the course of exchange the better for Britain.

Accounts in France are now generally kept in francs, centimes, and decimes. The Napoleon or Louis is equal to 24 francs. The Louis and double Louis are of gold, the francs of silver, and the centimes of copper.

Exchange with Holland.—There are two kinds of money used in Holland, the current and the bank money, of which bank money is the most valuable, the difference between them is called agio, and is estimated at so much per cent. Exchanges between Amsterdam and Britain are always transacted in £s sh. and d. Flemish banco. But by the laws of the Netherlands accounts should be kept in florins and cents. which is not the case invariably, as some still use guilders and stivers in their accounts. Florins and guilders are synonymous. In exchanges with Amsterdam, London gives the fixed sum, and receives the variable, therefore the higher the course of exchange the better for Britain.

Exchange with Germany.—Bank and current money are both used in Germany as well as in Holland, the agio varying from about 18 to 25 per cent. Accounts here are kept in marks, schillings, and phenings Hambro, both banco and current, also in £s sh. and groats Flemish, which with them is an imaginary money, like British sterling. Exchanges with Britain are effected by giving a variable number of shillings and groats Fl. for £1 sterling. Hamburg money was formerly called lubs, a contraction for Lubec, the place where it was coined; but lubs is now seldom used, the word Hambro being generally employed in its room. The higher the course of exchange between Hamburg and London, the better for London. From the table in page 173 it will appear that accounts are kept differently in different places of Germany.

Exchange with Russia.—The par mentioned in page 174 is upon the silver rouble—the rouble bank notes were originally of the same value as the silver rouble, but are now worth little more than a fourth part of its value, and are so fluctuating that no par can be fixed with them. Exchanges with Britain are now given in rouble bank notes. The lower the course of exchange with Russia the better for Britain.

Exchange with Sweden.—Accounts here are kept in rixdollars, shillings, and runstyckens. The higher the course of exchange with Sweden the better for Britain.

Exchange with Denmark and Norway.—The currency here is Rigsbank paper money, which for a long time has been at a very heavy discount. Exchanges with Britain are generally specified in paper rixdollars, which are worth little more than a third of the silver rixdollar, but from its fluctuations, no permanent agio can be specified between them. The par given in the text is upon the silver rixdollar. Accounts are kept here in rixdollars, marks, and shillings. The higher the course of exchange with Denmark the better for Britain.

Exchange with Prussia.—From the table in page 175 it will be observed, that accounts are kept differently in different places of Russia. Besides what are there mentioned, it may be of importance to state, that in Brandenburg and Pomarania accounts are kept a little different, for example, at Berlin, Stettin, &c. 12 pfenings = 1 good groshen, 24 good groshen = 1 rixdollar. The higher the course of exchange with Prussia the better for Britain.

Exchange with Poland.—The florin and groshen of Little Poland and Prussia, are nearly double the value of those of Great Poland given in the text. The exchanges between Great Poland and Britain are transacted through Holland.

Exchange with Spain.—In drawing bills of exchange on Spain, it is necessary to insert "payable in effective," otherwise they may be paid in Exchequer bills, called "vales reales," which are less valuable, being always at a considerable discount. Accounts are kept differently in different provinces of Spain. Some in reales, and Maravedies Vellon, as in Madrid, Malaga, &c. some in reales, &c., old plate, as in Cadiz, Seville, &c.; some in libras, sueldos, and dineros, as in Alicant, Barcelona, &c. divided like the £ sterling. But in whatever manner accounts are kept, bills of exchange are expressed in plate money. The Vellon money is copper, and

the plate money is silver, of which there are three kinds in Spain, namely, old plate, new plate, and Mexican plate, but old plate only is used in foreign exchanges. The lower the course of exchange with Spain the better for Britain.

EXCHANGE WITH PORTUGAL.—Accounts are kept here in rees, and milrees. The exchange with Britain is by the milree, for a variable number of pence sterling. The lower the course of exchange with Portugal the better for Britain.

EXCHANGE WITH ITALY.—It will be observed from the statement in page 188, that different places in Italy exchange with Britain on different pieces, and that some places give the fixed sum, some the variable sum. The lower the course of exchange with Rome, Naples, Genoa and Leghorn, and the higher with Venice and Milan, the better for Britain. The exchanges between Britain and Italy are often transacted more profitably through France.

EXCHANGE WITH TURKEY.—The coins of the Turkish empire are at present very far below standard, which has occasioned their exchange value to fall much below their current value. At Constantinople, and throughout most of the empire, accounts are kept in piastres, paras, and aspers. A purse is 500 aspers, a jux is 100000 aspers. The lower the course of exchange with Turkey the better for Britain.

I shall here subjoin a few remarks on the monies used in the East Indies. In the BOMBAY PRESIDENCY, accounts are kept in rees, quarters, and rupees, thus 100 rees = 1 quarter, 4 quarters = 1 rupee, 15 rupees = 1 gold mohur. The rupee is worth about 2/ sterling, but in the financial accounts, submitted to Parliament, it is estimated at 2/3, which then bears a batta of 16 per cent. against current rupees. In the MADRAS PRESIDENCY, accounts are kept in star pagodas, fanams, and cash, thus 80 cash = 1 fanam, 42 fanams = 1 star pagoda = 4½ current rupees in the company's accounts. The star pagoda is generally estimated at 8/ sterling, but is not worth quite so much. In the BENGAL PRESIDENCY, accounts are kept in current rupees, annas, and pice, thus 12 pice = 1 anna, 16 annas = 1 rupee. But the company's accounts are kept in Sicca rupees, annas, and pice, which bear a batta of 16 per cent. against current rupees, that is, 100 Sicca rupees are equal to 116 current rupees, and 16 Sicca rupees = 1 gold Mohur. The current rupee is generally estimated at 2 shilling sterling.—1000 rupees = 1 lack, 100 lacks = 1 crore of rupees. In the MYSORE COUNTRY, accounts are kept in dudus, fanams, canterraia pagodas, thus 18·2 to 26 dudus = 1 fanam, 10 fanams = 1 pagoda, 4 pagodas = 1 gold mohur. On the MALABAR COAST 3 copper pice = 1 anna, 16 annas

= 1 rupee, this rupee is equal to that of Bombay. At CANTON and throughout the whole of China, accounts are kept in tales, mace, candaries, and cash, thus 10 cash = 1 candarie, 10 candaries = 1 mace, 10 mace = 1 tale. Cash is the only money in China, and is composed of 6 parts of copper and 4 of lead, and 1000 cash are worth about 6/8 sterling.

The weight and measures of the several countries mentioned in the text, are given in such a form as to enable the pupil to reduce them into the British standards, or the reverse. These tables are not generally met with in treatises on arithmetic, but it is hoped they will not be the less acceptable on that account.

ARBITRATION OF EXCHANGE.—This case is generally divided into simple and compound arbitration, according as the course of exchange between three or more places is given. But as the one falls directly under simple, and the other under compound proportion, which should now be well understood, I have classed them under one head, without annexing any particular rule. What is called the chain rule on the Continent, is generally used for calculating arbitrations when more than three places are concerned, and as it is simpler than the other method, I shall subjoin it here.—RULE. Draw a line (——) and on its right place the term whose value is required, for the first consequent, then under the line on the left, place that term which is of the same kind as last consequent, for the first antecedent, and on its right place as consequent, the term to which it is equal. Proceed in the same manner with all the remaining terms, observing to have each antecedent in the same denomination as the preceding consequent. Then the continued product of the consequents, divided by the continued product of the antecedents, gives the answer. The work is cancelled as in compound proportion.—EXAMPLE. A of Edinburgh has credit in Leghorn for 1360 pezze, for which he can draw at 4/2 per pezze; but not liking this offer, he orders them to be sent to Venice at 94 pezze per 100 ducats, thence to Cadiz at 320 maravedies per ducat, thence to Lisbon at 630 rees per peso, thence to Amsterdam at 50 grotes per crusado of 400 rees, thence to Paris at 56 grotes per crown, thence to Edinburgh at 31½ pence per crown; how much was this better than the direct exchange, allowing £9 3s. 4d. for the expense of circular remittance?

Antecedents.		Consequents.	Antecedents.		Consequents.
————		1360 pezze.	400 rees	=	50 grotes.
94 pezze	=	100 ducats.	56 grotes	=	1 crown.
1 ducat	=	320 marave.	1 crown	=	31½ pence.
272 marave.	=	630 rees.			

$$\frac{\overset{5}{\cancel{1360}} \times \cancel{100} \times \overset{10}{\cancel{320}} \times \overset{30}{\cancel{630}} \times 50 \times \cancel{94}}{\cancel{94} \times \cancel{2112} \times \underset{4}{\cancel{400}} \times \underset{7}{\cancel{80}} \times \cancel{3}} = £312\ 10\ 0$$

	9	3	4
Circular =	303	6	8
1360 × 50d. direct =	283	6	8
gain by the circular =	£20	0	0

Having drawn a line under antecedents, opposite to it under consequents, I write 1360 pezze, the sum whose value is sought, then under antecedents and below the line, I write 94 pezze, which is the same denomination as last consequent, and under consequents their value in ducats; then under antecedents I write 1 ducat the same denomination as last consequent, and its value 320 maravedies under consequents, and so on with all the other terms. By this rule the weights and measures of different countries are generally compared. Every merchant engaged in foreign trade must have occasion frequently to draw and remit bills of exchange, he will therefore find his interest in being thoroughly acquainted with the arbitration of exchange, by which he may often gain more than those who are ignorant of its advantages do by the ordinary profits of their business. But in finding a profitable channel for drawing and remitting, it should always be kept in mind, that the circular exchange must exceed the direct by more than the additional expenses attending the circular, with interest for the additional time required, otherwise the direct is the preferable.

INVOLUTION.—It is of importance to observe, when a high power of any number is required, that any power of a number multiplied by itself gives a power double of that which was multiplied, thus the fourth power multiplied by itself gives the eighth power. Again, any power of a number multiplied by any other power of it, gives a power whose index is the sum of the indices of the powers multiplied, thus, the fifth power multiplied by the fourth power gives the ninth power, &c. Also any power of a number divided by the number itself gives a power whose index is one less than the power divided, thus the sixth power of a number divided by the number itself gives its fifth power. The expressions square and cube, &c. applied to abstract numbers is improper, for it is evident that multiplying a number by itself cannot produce a superficies, neither can a number multiplied twice into itself produce a solid body; therefore the second, third, &c. power would be more appropriate.

SQUARE ROOT.—The reason for dividing into periods as the rule directs is, no number multiplied into itself can do more than double the number of figures, thus $99 \times 99 = 9801$, and 99 is the greatest number that two figures can express. Neither can any number multiplied by itself want more than one figure to double the number of places, thus $10 \times 10 = 100$, for the same reason the periods in decimals must always be complete. The root must therefore invariably consist of as many places of integers and decimals as there are periods in each. The principal upon which the rule is founded is this; "the square of the sum of any two numbers, is equal to the sum of the squares of these numbers, together with twice their product." Thus 43 is the sum of $40 + 3$ and $43^2 = 40^2 + 3^2 + (2 \times 40 \times 3)$, and taking 40^2 from each there remains $43^2 - 20^2 = 3^2 + (2 \times 40 \times 3)$, but $3^2 + (2 \times 40 \times 3) = (80 + 3) \times 3$, and $(80 + 3) \times 3 \div 80 + 3 = 3$, the least number. Therefore, to find the second part of the root, we must divide the remainder after subtracting the square of the greater part, by double the root of the greater part, adding the quotient to the divisor before multiplying. But by the rule we disregard the local value of the figure placed in the root, which remains to be determined by the subsequent figures, as in simple division, and this defect is supplied in the divisor, by annexing the second figure in the root to double the first, instead of adding it, which is the rule. When there are more than two figures in the root we always consider those already determined as the greater part, and proceed with them as before, till the root is complete, or sufficiently extended. The reason for rejecting units place in the increased remainders when finding a quotient figure is, that it corresponds to the figure which is to be annexed to the divisor. The square root is proved thus, multiply the root by itself, to the product add the remainder, and the sum is the number whose root was to be extracted. A knowledge of the square root is indispensable in practical mathematics.

CUBE ROOT.—The reason for dividing the proposed number into periods of three places is, the cube of no number can never have more than three times its number of places, nor ever want more than two figures of having three times as many places. Thus $9 \times 9 \times 9 = 729$, again $10 \times 10 \times 10 = 1000$; in the first case the number is the greatest which can be expressed with one figure, and its cube has only three places; in the second, the number is the least which can be expressed by two figures, and its cube wants only two figures of having three times as many places. Again, the cube of any number com-

posed of two parts, is equal to the cube of the two parts, with three times the square of the greater part by the less, and three times the product of the greater part by the square of the less. Thus 45 is composed of the two parts 40 and 5, and $45^3 = 40^3 + 5^3 + (40^2 \times 5 \times 3) + (40 \times 5^2 \times 3)$, that is $45^3 = 40^3 + (40^2 \times 3 + 40 \times 5 \times 3 + 5^2) \times 5$, therefore if we divide the difference between the given number, and the cube of the greater part, by three times the square of the greater part, plus three times the product of the greater part by the less, plus the square of the less part, the quotient will be the less part; which is the rule given in the text. It is evident the divisor here consists of three parts, only one of which is known; but as it is always much greater than the other two, we use it as a trial divisor, and having found a quotient figure by it, we complete the divisor, then multiply and subtract. The reason why we annex two ciphers to the first part of the divisor, and one to the second part is, that we thereby retain the local value of the parts. Again, when the root consists of more than two figures, we may always consider the figures determined, as the greater part, and from what is shown above, it is evident that their square multiplied by three, and two ciphers annexed must give the new trial divisor, because $45^2 \times 3 = 40^2 \times 3 + 6 \times 40 \times 5 + 3 \times 5^2$, and this expression is equal to $(40^2 \times 3 + 40 \times 5 \times 3 + 5^2) + (40 \times 5 \times 3) + (5^2 \times 3)$, that is, to the last complete divisor, plus its second part, and double of its third part, which is the rule, and much simpler than the other when there are many figures in the root.

By the application of the square, and cube roots, we can often extract the root of higher powers, thus to extract the fourth root, we may extract the square root, and then the square root of that root; and for the sixth root, we may extract the square root, and then the cube root of that root. Generally, when the index of the power whose root is to be extracted is divisible by 2 or 3, we may take the square or cube root accordingly, which gives a power whose index is that of the given power divided by 2 or 3. But as there are some cases which may occur in Progressions, and Annuities that cannot be performed by the application of the square and cube root, I shall here annex a rule for extracting the roots of all powers. General Rule. Divide the given number into periods, each having as many places as the index of the required root has units, beginning on the right of integers, and left of decimals. Find the first figure of the root by trial, and subtract its corresponding power from the first period. Involve the part of the root found, to a power whose index is one

less than the index of the required root, and multiply it by the index of the required root, by this product divide the last remainder increased by the first figure in the next period, and write the quotient for the next place of the root. Involve the whole root thus found to a power, whose index is equal to the index of the required root, and subtract it from as many periods of the given number as are taken in. Increase the remainder, and find a new divisor as before, and proceed in the same manner till the work is finished.

EXAMPLE.—Required the fifth root of 1258284197543.

$$
\begin{array}{lrl}
 & & 125,82841,97543(263 \\
2^5 = & & 32 \\
2^4 \times 5 = & 80 &)938 \text{ increased remainder.} \\
 & & 12582841 \\
26^5 = & & 11881376 \\
26^4 \times 5 = & 2284880) & 7014659 \text{ increased remainder.} \\
 & & 1258284197543 \\
263^5 = & & 1258284197543
\end{array}
$$

Many other rules have been given for extracting the roots of high powers, and the process by some of them is shorter than by this; but I consider that the simplicity of the rule more than compensates for the additional labour, besides when a very high root is required there are generally but few figures in it. I shall here subjoin a table of the squares, and cubes of the nine digits.

Roots =	1,	2,	3,	4,	5,	6,	7,	8,	9.
Square =	1,	4,	9,	16,	25,	36,	49,	64,	81.
Cube =	1,	8,	27,	64,	125,	216,	343,	512,	729.

This table may be extended to any proposed power. It may be observed, that, whenever we annex a period of ciphers in the extraction of any root, the root required is interminate, running out in a decimal fraction, which neither repeats, nor circulates, and with the exception of the square root, no method has yet been discovered which shows the law by which they are continued.

SINGLE POSITION.—Both Single and Double Position have been called the RULE OF FALSE, the RULE OF TRIAL AND ERROR, and the RULE OF SUPPOSITION, but this last name is perhaps the most appropriate. The rule is founded on the principle that the results are proportional to the suppositions. In solving questions by this rule it is advisable to use a

number upon which the several operations can be performed without fractions, as the work is thereby less complicated. The questions belonging to this case may be solved by Double Position, and also by other rules.

Double Position.—This case is called double, because it requires two numbers supposed, to discover the number sought. The rule is founded on the principle, that, the differences between the true number and the assumed numbers are proportional to the differences between the result given, and the results obtained by the suppositions. The rule given by Bonnycastle for double position is simpler than that in the text, but as it does not apply to all the questions belonging to this case, it appears to be injudicious as a general rule. The following is Bonnycastle's rule; "Take any two convenient numbers and proceed with them separately according to the conditions of the question, noting the result obtained from each. Then, as the difference of these results, is to the difference of the supposed numbers, so is the difference between the true result and either of the former, to the correction of the number belonging to the result used, which correction being added to that number, when it is too little, or subtracted from it when it is too great, will give the answer required."—Now, this rule will not apply when the result of the operations to be performed on the required number, gives that number, or some multiple or part of it, and not a known number. Had Bonnycastle been aware of this circumstance, he could not have given it as a general rule, or supposed it "better adapted to practice than any other previously devised." It may be observed that, when any root, or power of the required number is involved in the result given in the question, the rule gives only an approximation to the required number, which may always be brought nearer to the truth than any proposed difference. When therefore the rule is used in approximations, the numbers supposed should be taken as near as possible to the truth, and having found the result by the first supposition, use it, and either of the other numbers which is nearest to it, or any other number which may appear still nearer to the truth, with which perform the same operation, and use the result as before, and so on till the approximation is sufficiently accurate. This approximation, besides its use in arithmetic, is often the most convenient method of approximating the roots of equations, and of ascertaining the value of unknown quantities in complicated expressions in algebra.

ARITHMETICAL PROGRESSION.—Although this name

has for ages been given to the Rule to which it is here annexed, yet it cannot be vindicated from the charge of being improperly applied, because the series belongs to geometry in common with arithmetic. Both arithmetical and geometrical progressions have been more properly named, the one progression by differences, the other progression by quotients.

CASE 1.—In every arithmetical series, the sum of the extremes is equal to the sum of any two terms equally distant from them, and to double the middle term when the number of terms is odd. Therefore the sum of the extremes multiplied by the number of terms must give a number equal to twice the sum of the series, or the half of it is the sum, which is the rule. The reason of the rules for the other cases may all be derived from the principle here stated, and may be given as a useful exercise to the advanced pupil. An arithmetical mean between two numbers is half their sum.

GEOMETRICAL PROGRESSION.—In every continued geometrical series, the product of the two extremes is invariably equal to the product of any two terms equally distant from them, or to the square of the mean when the terms are odd. This is evident, because the greatest extreme differs from the one next less, in the same ratio as the second term varies from the first, whether the series be increasing or decreasing. Every geometrical series is produced by multiplying the first extreme by the ratio for the second, the second again by the ratio for the third, and so on. In every infinite decreasing series, the last extreme may alway be considered as nothing. Case 4 was inadvertently limited to an increasing series, but it is equally applicable to a decreasing series, and is the rule by which we find the value of a repeating or circulating decimal. Thus $\dot{3}$ repeated for ever, is a decreasing geometrical series, whose ratio is 10, and multiplying 3 by 10 and dividing the product by 10—1, is equal to $\frac{3}{10} \times 10 = \frac{30}{10} \div 9 = \frac{30}{90} = \frac{1}{3}$; again $\dot{1}\dot{5}$ repeated for ever, is a decreasing series whose ratio is 100, and $\frac{15}{100} \times 100 = \frac{1500}{100}$, & $\frac{1500}{100} \div 99 = \frac{1500}{9900} = \frac{15}{99} = \frac{5}{11}$. From the principles here stated the reason of the rules for the several cases may easily be deduced. Progressions have been but partially treated of; for a minute, and clear elucidation of this subject, the advanced pupil may consult Alexander Malcom's Arithmetic, Aberdeen, 1730.

PERMUTATIONS AND COMBINATIONS.—These cases being of little importance in real business, I shall not enter upon an explanation of them here, they are fully explained in the work already referred to for progressions.

COMPOUND INTEREST AND ANNUITIES. Under this head I have only introduced such cases as most frequently occur in business, for this reason no notice is taken of annuities at simple interest which are unjust in principle, and never acted on in practice. CONSTRUCTION OF THE TABLES. TABLE 1. This table is constructed by writing the amount of £1 at the several rates, for the first year, then the square of these amounts for the second year, their cube for the third year, and so on for any number of years. These involutions are performed by contracted multiplication of decimals.—APPLICATION. It is evident that the tabular numbers, are the amount of £1 for their corresponding rates and times, therefore any of these numbers multiplied by a proposed sum, must give its amount for the time and rate corresponding with the tabular number. If the time required is not in the table, it may easily be found, for supposing we required the tabular number for 15 years at 4 per cent., it is the product of the numbers opposite to 10 and 5 years under 4 per cent., and for 60 years at 5 per cent. it is the product of the tabular number opposite 30 years, under 5 per cent. by itself, and so on for any number.

TABLE 2. This table is formed by dividing £1 by the corresponding numbers in the first table, and these numbers represent the present value of £1 for their corresponding time, and rate.—APPLICATION. It is evident that the present value of £1, multiplied by any number of £s, will give their present value; therefore the tabular number corresponding to any time, and rate, being multiplied by a proposed sum, must give its present value for the same time and rate.

TABLE 3. This table is formed by writing a unit for the first number in it, the second number is the first number in Table 1st. increased by a unit, the third number is the sum of the first and second numbers in Table 1st. increased by a unite, the fourth is the sum of the first, second, and third increased by a unit, and so on.—APPLICATION. It is evident that the amount of £1 annuity, multiplied by any proposed annuity, will give its amount, therefore if the tabular number corresponding to any proposed rate, and time, be multiplied by any annuity, the product is the amount for the time and rate.

TABLE 4.—This table is formed by writing the first numbers in table 2d. for its first, then the sum of the two first for its second, and the sum of the three first, for its third, and so on; these numbers are the present value of £1 annuity for their corresponding time and rate. APPLICATION.—Since the tabular numbers are the present value of £1 annuity for their respective rates and times, it is evident that any tabular

number, multiplied by a proposed annuity, must give its present value for the time, and rate, corresponding to the tabular number.

TABLE 5.—The numbers in this table are found by dividing £1 by the corresponding numbers in table 4th. APPLICATION.—Since the tabular numbers represent the annuity which £1 will purchase, at their respective rates, and times, it is evident that any tabular number multiplied by a given sum, must give the annuity which that sum will purchase for the time and rate corresponding to the tabular number.

It is presumed that these remarks will enable the pupil to extend the tables at pleasure, and to apply them properly to their different purposes.

TABLES 6 and 7.—The construction of these tables, and their application to life annuities is a subject both too complicated, and too extensive to admit of being treated satisfactorily in such a work as the present; and although their application is simple, the principles on which they are founded cannot be perfectly understood without some knowledge of algebra. Those who wish to understand this subject perfectly, may consult Price on annuities, where they will find it treated at full length, both in theory and practice; the writings of Morgan, Moivre, and Simson on annuities may also be consulted with advantage. Tables 6 and 7 are founded on the principles which these authors have laid down, by the joint application of the doctrine of chances, and compound interest to the Northampton tables of the actual duration of human life.

DUODECIMALS.—Since 12 duodecimal inches make a square foot, each duodecimal inch is equal to 12 square inches; and since 12 duodecimal solid inches make a cubic foot, one duodecimal solid inch is therefore equal to 144 cubic inches. When the number of feet in the factors is great, instead of using the general rule, it is often better to multiply the feet by the feet, then take aliquot parts for the lower denominations of the multiplier out of the multiplicand, and aliquot parts for the lower denominations of the multiplicand out of the feet only of the multiplier, the sum of these is the answer. When yards are given in the factors, they should be reduced to feet before applying the rule. In measuring logs, &c. when they are not the same breadth and depth at both ends, which is generally the case, add the breadth at one end, to the breadth at the other, and the depth at one end to the depth at the other, then the half of these sums, is the mean breadth and depth. When a round tree tapers irregularly, the girt should be taken in several places, and the sum of these divided by the

number of places, is the mean girt. Although this rule is sanctioned by custom, yet the content of round timber found by it, is less than the truth by nearly $\frac{1}{4}$ part of the whole. When trees are girt with the bark on, an allowance is made for it, by deducting about $\frac{1}{10}$ of the whole girt for oak, and less or more for other wood, in proportion to the thickness of the bark. I shall here subjoin a few rules for the measurement of round timber, by which the results are more accurate —Multiply $\frac{1}{5}$ of the girt by double the length of the tree, and the product is the truth within about a foot in 190.—Multiply the square of the mean girt by the length of the tree, and from $\frac{1}{11}$ of the product subtract $\frac{1}{8}$ of itself, the remainder is the answer, and differs from the truth by only 1 foot in 2300.—If a tree be cut exactly in the middle the two parts will measure by the common rule more than the whole tree, or any other two parts into which it can be cut.—If a tree be cut where the girt is $\frac{1}{3}$ of the greatest girt, the greater end will measure the most possible; and this part often exceeds the measure of the whole tree before it was cut.—To cut a tree so that the greater end may measure as much as the whole tree uncut. To four times the sum of the girts at the two ends, add their difference; multiply this sum by the difference of the girts, and from the square root of the product, subtract the difference between the difference of the girts and twice their sum, multiply this remainder by the length of the tree; and the product divided by twice the difference of the girts gives the length to be cut off from the smaller end; the greater end will then measure as much as the whole tree before it was cut.

TONNAGE OF SHIPS. Besides the rule given in the text, several other methods are employed to find the tonnage. If the measure be taken from the light mark, to the full draught of water when laden for the depth of the ship, and this be multiplied by her length and breadth, the product divided by 100 for war ships, and by 94 for merchant ships, gives the tonnage. Again shipwrights in London multiply the length of the keel, by the breadth, and by half the breadth of the ship, taken from outside to outside; and this product they divide by 100 for war ships, and by 94 for merchant ships to give the tonnage. In the Royal Navy the following method is used. From the height of the hawse holes at the foreside of the stern, and also from the back of the main port at the height of the wing transom, let fall perpendiculars, from the distance between these perpendiculars, subtract $\frac{3}{5}$ of the extreme breadth of the ship, and as many times $2\frac{1}{2}$ inches as there are feet in the height of the wing transom above the upper edge of the keel, the remainder is

the length of the keel for tonnage, which multiplied by the extreme breadth, and the product again by half the breadth, this last product divided by 94 gives the tonnage.

ARITHMETICAL TABLES OF WEIGHTS AND MEASURES.—The several tables of weights according to the old and new system, are best compared by the troy grains in each. Although the several standard measures may be readily and accurately converted from the old, into the new, and the reverse, by means of the cubic inches in each; yet I shall here subjoin a few remarks, to render that operation as simple and expeditious as possible.

WINE MEASURE.—To convert any denomination of old wine measure into imperial, Multiply .83311093 by the old, the product is the new in the same denomination as you multiplied by. To convert imperial measure into old, Multiply 1.200320346 by the imperial, the product is the old in the same denomination as you multiplied by.

ALE MEASURE.—To convert the old measure into imperial, Multiply 1.01704454438 by the old, the product is the new in the same denomination you multiplied by. To convert the imperial into the old, Multiply .98324113439 by the imperial, the product is the old in the same denomination.

SCOTCH LIQUID MEASURE.—To convert standard Scotch gallons into imperial, multiply 2.9834466989 by the Scotch, the product is the imperial. To convert imperial gallons into Scotch, multiply .3351828749 by the imperial, the product is the Scotch gallons.

DRY STRICKEN MEASURE.—To convert Winchester measure into imperial, Multiply .969447267 by the Winchester, the product is the imperial in the same denomination. To convert imperial measure into Winchester, multiply 1.031515621 by the imperial, the product is the Winchester in the same denomination.

STANDARD DRY HEAPED MEASURE.—To convert the old standard heaped measure into imperial, multiply 1.09462485932 by the old, the product is the new in the same denomination as you multiplied by. To convert imperial measure into the old standard, multiply .91355498779 by the imperial, the product is the old in the same denomination as you multiplied by.

SCOTCH STANDARD WHEAT, &c. MEASURE.—To convert Linlithgow wheat firlots into imperial bushels, multiply .9905968464 by the firlots, the product is the imperial bushels; and to convert imperial bushels into wheat firlots, multiply 1.0094919527 by the bushels the product is the firlots. To convert Linlithgow wheat bolls into imperial quarters, multiply .4952986486 by the bolls, the product is the quarters; and to

convert imperial quarters into Linlithgow wheat bolls, multiply 2.0189839055 by the quarters, the product is the bolls. To convert Linlithgow wheat chalders into imperial quarters, multiply 7.9247783786 by the chalders, the product is the quarters; and to convert imperial quarters into Linlithgow chalders, multiply .126816494 by the quarters, the product is the chalders.

SCOTCH STANDARD MEASURE FOR BARLEY, OATS, &c.— To convert the Linlithgow barley firlot into imperial bushels, multiply 1.4451066454 by the firlots, the product is the bushels; and to convert imperial bushels into barley firlots, multiply .6919904514 by the bushels, the product is the firlots. To convert Linlithgow barley bolls into imperial quarters, multiply .7453250393 by the bolls, the product is the imperial quarters; and to convert imperial quarters into Linlithgow bolls, multiply 1.3384209087 by the quarters, the product is the bolls. To convert barley chalders into imperial quarters, multiply 11.9252006302 by the chalders, the product is the quarters; and to convert imperial quarters into Linlithgow chalders, multiply .0834173356 by the quarters, the product is the chalders.

SQUARE MEASURE.—To convert Scotch acres into imperial acres, multiply 1·261183449 by the Scotch acres, the product is the imperial acres, and to convert imperial acres into Scotch, multiply .79290606 by the imperial, the product is the Scotch acres.

N. B.—When the quantities to be converted are small it will be unnecessary to use the whole of the decimal given above; but the more places that are used, the more accurate will be the result. Any rules which I have seen for converting the old wine, ale, Winchester, and dry measures, into imperial, are improperly limited to a gallon or bushel; but it is evident, since the multiples, and sub-multiples of these tables are all the same, that whatever ratio a gallon, or bushel, of the one bears to the other, the same ratio must exist between any denomination of the one and the same denomination of the other. It may still farther be remarked, that by the late acts of Parliament 1824 and 1825, for establishing uniformity in weights and measures, the imperial yard is declared to be the unit, or only standard measure of extension, from which all other measures of extension, whether lineal, superficial, or solid, shall be derived; and that the distance between the centres of the two points in the gold studs, in the straight brass rod in the custody of the clerk of the House of Commons, whereon the words and figure "STANDARD YARD, 1760," are engraved, is the standard of this yard. The standard of the imperial troy pound, is the brass weight of one pound troy, made in 1758, now in the custody of the clerk

of the House of Commons. This pound is divided into 5760 grains, and 7000 of these grains is declared to be the standard avoirdupois pound. The standard imperial gallon contains ten pounds avoirdupois of distilled water, weighed in air, at the temperature of 62 degrees of Fahrenheit's thermometer, the barometer being at 30 inches, and is the unit of the measures of capacity for all liquids, and dry goods not measured by heaped measure. The bushel, containing eight imperial gallons is the standard unit of dry heaped measure, its form being round, with a plain and even bottom, and its dimensions 19½ inches from outside to outside. In using the bushel it is to be heaped in the form of a cone having the outside of the bushel for the outside of its base, and its height to be at least six inches above the measure. The measures for the subdivisions of the bushel are to be made cylindrical, having the diameter at least double the depth; and in using them, the height of the heap is to be three-fourths of the depth of the measure, the outside of the measure being the extremity of the base.

QUESTIONS FOR EXAMINATION.

NUMERATION AND NOTATION.—1. What does arithmetic as a science treat of?—2. What does arithmetic as an art treat of?—3. Is arithmetic of much use in a commercial country?—4. What are the characters used in arithmetical calculations?—5. What are these characters called?—6. Can every finite number be expressed by these characters?—7. By whom was arithmetic first brought into Europe?—8. What is the sign of equality, and what is it called?—9. What is the sign of addition, and what is it called?—10. What is the sign of subtraction, and what is it called?—11. What is the sign of multiplication, and what is it called?—12. What is the sign of division?—13. What is the sign of proportion?—14. What is the sign of the square root, and what is it called?—15. What does numeration teach?—16. What is meant by the single value of a figure?—17. What is meant by the local value of a figure?—18. How much is the single value of a figure increased by every place it is removed towards the left?—19. What is the use of the cipher?—20. How is the numeration scale divided in Britain, and how read?—21. How is it divided on the continent of Europe?—22. What are the 1st, 2d, and 3d places on the right of a number called?—23. What are the 4th, 5th, and 6th places on the right called?—24. What is the second period called, and how many figures are in it?—25. What are the 3d, 4th, 5th, 6th, 7th, 8th, 9th, and 10th periods called?—26. How many places would you remove 7 from the right to make it seventy

million, and how many to make it seven hundred billion?—27. What place in the scale does tens of trillions occupy?—28. What place does hundreds of thousands of quadrillions occupy?—29. What place does hundreds of quintillions occupy?—30. What place does tens of thousands of sextillions occupy?—31. What place does hundreds of nonillions occupy?—32. How would you read a large number expressed in figures?—33. What does notation teach?—34. When a number is given in words, how do you express it in figures?—35. What is the best method of learning notation?—36. Is the Roman notation still in use?—37. By what letters did they express their numbers?—38. What was the single value expressed by these letters?—39. When a letter was repeated, what did it signify?—40. When a letter representing a less number was placed before one representing a greater number, what did it signify?—41. What did a letter representing a less number placed after one representing a greater number signify?—42. How much did an inverted Ↄ placed after IↃ increase its value?—43. What effect had a C not inverted placed after IↃ?—44. What was the effect of annexing and prefixing C to CIↃ?—45. How much did a line placed over a letter increase its value?

SIMPLE ADDITION.—1. What are the fundamental rules of arithmetic?—2. What is a simple, and what is an abstract number?—3. What does simple addition teach?—4. Is it equally applicable to simple and abstract numbers?—5. How do you write numbers for addition?—6. What do you understand by writing like places under each other?—7. On what principle is addition founded?—8. Why do you set the right hand figure under the column added?—9. Why do you add the other figures to the next higher column?—10. Why do you write the whole sum of the left hand columns?—11. How do you prove addition?—12. Why do you cut off the under number in preference to the upper?—13. What is the reason that the sum of the second addition added to the number cut off gives the same as the first sum?—14. What are the methods of proof given in the notes?—15. On what principles is the third method founded?—16. Under what circumstances does this method prove the work right when it is actually wrong?

SIMPLE SUBTRACTION.—1. What is simple subtraction?—2. How do you place the numbers for subtraction?—3. Is it necessary that the least number be placed below?—4. How do you perform simple subtraction?—5. Must you always take the numbers which occupy the same place in the scale from each

other ?—6. When you cannot take the under figure from the upper what do you do ?—7. Why do you place the one you borrow, on the left of the other ?—8. Has this the same effect as adding 10 ?—9. Why is the one taken from the next figure equal to 10 ?—10. When there are ciphers between the figure which is too little and the next significant figure, what becomes of them ?—11. What is the reason they become nines ?—12. Do any of the figures in the subtrahend change their value ?—13. Does any figure but 0 in the minuend change its value ?—14. What is the upper number called ?—15. What is the under number called ?—16. What is their difference called ?—17. How do you prove subtraction ?—18. What is the reason of this proof ?—19. What is the second method, and its reason ?—20. When two, or more numbers are to be subtracted from two or more numbers, how would you proceed ?

SIMPLE MULTIPLICATION.—1. What is multiplication ?—2. What is the number to be multiplied called ?—3. What is the number by which you multiply called ?—4. What is the number obtained by multiplying called ?—5. Have these numbers any common name ?—6. Is the truth of the operation affected by making the multiplicand the multiplier, and the multiplier the multiplicand ?—7. How do you place numbers for multiplication ?—8. How do you multiply when the multiplier is a single figure ?—9. How do you proceed when the multiplier has more figures ?—10. Why do you place the right hand figure of each product under the figure you are multiplying by? —11. Having found the several products, how do you find the total product ?—12. How do you prove multiplication ?—13. On what principle does this proof depend ?—14. What other methods do you know for proving it ?—15. Which of the methods is the most natural ?—16. How do you prove multiplication by division ?—17. What is the reason of this proof ?—18. What is the other method of proof ?—19. How do you prove multiplication by casting out the nines ?—20. On what principle does this method depend ?—21. What are the errors which this method does not detect ?—22. How do you multiply by 10, 100, 1000, &c. ?—23. What is the reason this method gives the true product ?—24. How do you work when there are 0's on the right of factors ?—25. On what principle is this method founded ?—26. When the multiplier is the product of numbers under 12, how do you proceed ?—27. What is the reason of this method ?—28. When a figure in the multiplier repeated any number of times under 13, gives another figure or figures in it, how is the work performed most concisely ?—29. May the

figures in the multiplier be taken in any order?—30. What is chiefly to be attended to in such cases?—31. On what principles does this fourth contraction depend?—32. How do you multiply by numbers between 12 and 20 in one line?—33. What is the principle on which this method depends?—34. When there are one, or more ciphers in the multiplier, and not on the right of it, how do you proceed?—35. When writing the multiplier may ciphers be left out?—36. Since ciphers are passed over when multiplying, and yet cannot be left out of the multiplier, what effect have they?—37. If the multiplier has any figure in units place, and two or more 1's on its left; how would you bring out the product at once?—38. When the multiplier consists of any two figures; how would you bring out the product at once?—39. How is the product obtained at once when the multiplier has any digit on the right, 1 on the left, and ciphers between?—40. If there are several 1's on the left, how do you proceed?—41. How do you bring out the product at once when the multiplier has one in all its places?—42. When the multiplier has any other digit than 1, or 9 in all its places, how is the product produced with the least labour?—43. On what principle does the six preceding cases depend?—44. How do you multiply when the multiplier is all nines?—45. After annexing the ciphers to the multiplicand, why do you then subtract the multiplicand from itself thus increased?—46. How do you multiply any number consisting of 9's by itself, or by any other number consisting of 9's?—47. What is the shortest method of multiplying by 5?—48. What is the reason this method gives the true product?—49. Can this principle be extended to many cases?—50. Can the principles of the contractions given in the text, and notes, be extended to other cases not given?—51. How do you table the multiplicand?—52. Is the work shortened by this method?—53. What then is the advantage resulting from it?—54. How is the work performed when the table is formed?—55. Is there any other method of tabling the multiplicand?—56. How do you table the multiplicand by Napier's rods?—57. When the multiplicand is tabled by the rods, how do you take out the product by the several figures?—58. Can the factors change places without altering the product?—59. If several numbers are to be multiplied together, is the last product the same in whatever order they are taken?—60. What is this product called in reference to these factors?—61. If one, or both factors be divided into any number of parts, and each part of the one be multiplied by the whole or each part of the other, is the sum of these products equal to the product of the two factors?—62. What is the reason that the sum of the products of

all the parts of one number, by the whole, or all the parts of another, is equal to the product of those numbers?—63. When the multiplier contains a fraction of which the upper figure is 1, how do you multiply?—64. What is the reason you do not place the first figure of the product by the figure in units place of the multiplier, under the tens place of the product by the fractional part?—65. When the upper figure of the fraction in the multiplier is greater than 1; how do you proceed with it?—66. What is the reason of this process?

SIMPLE DIVISION.—1. What is division?—2. What is simple division?—3. What is the number to be divided called?—4. What is the number you divide by called?—5. What is the number of times the divisor is contained in the dividend called?—6. If any thing remain after division what is it called?—7. How do you perform division by the general rule?—8. What is the principle on which this rule is founded?—9. When the divisor is large, how do you find how often it is contained in the several parts of the dividend?—10. When you require two places of the dividend to contain the first figure of the divisor, under what place of the dividend do you write the first figure of the product by the quotient figure?—11. How do you know when the quotient figure is too great?—12. How do you know when the quotient figure is too small?—13. How do you know when the quotient figure is correct?—14. When the divisor and dividend have the same number of places, can it contain the divisor more than 9 times?—15. What is the reason it cannot contain the divisor above 9 times?—16. If the divisor and dividend have the same number of places, but the divisor is not contained in the dividend, by annexing another figure to the dividend, can it contain the divisor above 9 times?—17. What is the reason it is not then contained above 9 times?—18. How do you prove division by the method given in the text?—19. What is the reason of this method of proof?—20. How many methods to prove division are given in the notes?—21. What is the first of these, and the reason of it?—22. What is the second method, and the reason of it?—23. What is the third method of proof, and the reason of it?—24. What is the first contraction, and what is its principle?—25. What is division called when performed by this contraction?—26. What is the second contraction, and the reason of it?—27. Are the places cut off, what would remain by the general rule?—28. What is the third contraction, and its principle?—29. How do you find the true remainder in this contraction?—30. What is the fourth contraction, and its principle?—31. What is the

rule in the text to find the true remainder?—32. What is the principle on which this rule is founded?—33. What is the first method in the notes to find the remainder?—34. What is the reason this method gives the true remainder?—35. What is the second method in the notes to find the true remainder, and on what principle is it founded?—36. How do you perform division by the Italian method?—37. What is the principle on which this method is founded?—38. In what does this method differ from the general rule?—39. Can you table the divisor as you did the multiplicand?—40. What are the advantages of this method?—41. How do you find the quotient figure by this method?—42. On what principle is this method founded?—43. How do you divide when the divisor contains a fraction?—44. When you multiply the divisor and dividend by the same number, do you alter the value of the quotient?—45. Why then do you multiply them by the under figure of the fraction?—46. How would you divide when the divisor ends with 5?—47. Why do you multiply by the digit which gives most ciphers?—48. Why do you multiply the dividend by the same figure?—49. On what principle is this method of division founded?—50. How do you divide when the divisor consists of the same figure repeated in all its places, and how is the remainder found?

REDUCTION.—1. What is reduction?—2. Into how many cases is reduction generally divided?—3. What is the first case of reduction?—4. How is reduction descending performed? 5. Why do the denominations change at every step of the process?—6. When there are lower denominations what is done with them?—7. Is the last product in reduction descending always the same value as the number which was given to be reduced?—8. What then is the effect produced by the multiplications?—9. What is the second case of reduction, and how is it wrought?—10. How do you know to what denomination remainders belong?—11. Why are the denominations changed by every division?—12. Is the last quotient and remainders equal to first dividend?—13. What then is the effect produced by division in this case?—14. What is the third case of reduction, and how performed?—15. What is the principle on which this rule is founded?—16. How do you prove the several cases of reduction?—17. How do you convert £s into guineas, and guineas into £s?—18. What is the shortest method of converting ells English into yards, and yards into ells English?—19. Can you give any general rule by which many questions belonging to this case may be contracted?

COMPOUND ADDITION.—1. What is compound add;

tion ?—2. How do you write down numbers for compound addition ?—3. How do you perform compound addition ?—4. How does compound addition differ from simple addition?—5. On what principle is the rule for this case founded ?—6. In compound addition, can there be a sum under any denomination in which the answer is not required, equal to or greater than a unit of next higher denomination ?—7. What two rules are employed in compound addition ?—8. Are the several denominations added as if they were separate accounts in simple addition ?—9. What is the reason you divide the sum of the farthings by 4 ?—10. Why do you divide the pence by 12, and the shillings by 20 ?—11. When there is a remainder, what is done with it ? 12. What do you do with the quotient of those divisions ?—13. Can compound addition be performed, and the several denominations brought out properly, without division ?—14. What is the methods by which you do this ?—15. What objection is commonly made to this method ?—16. For whom is this method principally useful ?

COMPOUND SUBTRACTION.—1. What is compound subtraction ?—2. How do you write the numbers for compound subtraction ?—3. How do you perform compound subtraction ?—4. If any denomination of the subtrahend is greater than the corresponding one of the minuend, how do you proceed ?—5. What do you understand by adding a unit of next higher denomination to any place of the minuend ?—6. According to this rule do you ever add to the subtrahend ?—7. How do you prove compound subtraction ?—8. On what principle does this rule depend ?—9. Wherein does this rule for compound subtraction differ from that commonly given in books on arithmetic ?—10. By which of these rules is the process most distinct ?—11. If the minuend and subtrahend both consist of two or more numbers, how do you perform the operation ?—12. What two rules do these questions involve ?—13. Can the remainder of any denomination be equal to or greater than a unit of next higher denomination ?

COMPOUND MULTIPLICATION.—1. What is compound multiplication ?—2. What two rules enter into the operation of compound multiplication ?—3. What is the general rule for performing compound multiplication ?—4. Can the sum of all the remainders ever amount to a unit of the highest denomination ?—5. What is the principle on which the general rule is founded ?—6. When the multiplier is composite, and the product of numbers under 13, how do you perform the operation ?—7. What

is the principle on which this rule is founded ?—8. Under what circumstances does this rule become of little practical use ?—9. When the multiplier is not a composite number, how do you proceed with it ?—10. On what principle is this rule founded ?—11. Is the result the same when proceeding by the rule, whether the composite number taken be greater or less than the given multiplier ?—12. When the multiplier is large, how would you perform the operation ?—13. On what principle is this rule founded ?—14. Why do you multiply the multiplicand by ten as often wanting one as there are figures in the multiplier ?—15. When we have multiplied the multiplicand by ten, as directed in the rule, if we consider the multiplicand as units place, what name is appropriate to the several products ?—16. What is the rule given in the notes for this case ?—17. When there is a cipher in any place of the multiplier, do we repeat the corresponding place of the multiplicand when bringing out the products for the answer ?—18. Does this rule admit of general application ?—19. What is the fifth rule for compound multiplication ?—20. Does this rule belong strictly to multiplication ?—21. When there is a fraction in the multiplier, what is done with it ?—22. When the value, weight, &c. of one article is given, to find the value weight, &c. of any number of articles, how would you do it ? 23. When the wages earned, provisions consumed, interests of money gained, &c. in a year, month, or day, &c. is given, to find how much it would amount to in any given number of years, months, weeks, or days, &c. how would you do it ?—24. What is the best method of ascertaining in all mixed questions, when multiplication is required ?

COMPOUND DIVISION.—1. What is compound division ?—2. Are there not several ways of putting questions in compound division ?—3. Can you repeat the several ways of expressing the question ?—4. What is the reason that all great quantities in money, weight, and measure, have been subdivided into lesser quantities, or denominations ?—5. When any inferior denomination stands unconnected with a superior denomination, may we with propriety express in it a value, or quantity, greater than the next higher denomination of the species to which it belongs ?—6. What properly constitutes a compound, or mixed number ?—7. What is the general rule for performing compound division ?—8. When both the divisor and dividend are whole numbers, can the quotient, or answer, be a compound number ?—9. How is the true quotient found, when the several denominations of the dividend are not an exact multiple of the divisor ?—10. By proceeding by the rule, are all the

parts of the dividend divided by the divisor?—11. How do you prove compound division?—12. What is the reason of this proof?—13. What is the second rule given for compound division?—14. What is the principle on which this rule is founded?—15. What is the third rule given for compound division? 16. What is the principle on which it is founded?—17. What is the fourth rule?—18. To what particular class of questions does this rule apply?—19. What other rule may these questions be performed by?—20. What is the principle on which the fourth rule is founded?—21. What is the first contraction, and how is it performed?—22. On what principle does this contraction depend?—23. What is the second contraction, and how is it performed?—24. On what principle does this second contraction depend?—25. What is the third contraction, and how is it performed?—26. What is the reason this contraction gives the true result?—27. When the value, weight, or measure, &c. of any number of articles is given to find the value, &c. of one, how is it found?—28. When the value, &c. of an unknown number of articles is given, and also the value, &c. of one, how do you find the number of articles?—29. Having given the quantity of work performed, of wages earned, &c. in a given number of years or months, &c. to find how much it amounts to in one year or month, &c., how would you do it?—30. When it is required to find the time in which a given sum of wages may be earned, a given quantity of work performed, &c. at a certain rate per year, or month, &c. how would you proceed?—31. What is the best method of discovering, in all mixed questions, when division is required?

PRACTICE.—1. What is practice?—2. Of what does practice consist?—3. From what does practice derive its name?—4. How do you find the value of any number of articles, when the value of one is given?—5. Is this rule general?—6. May not the examples under this case be performed more concisely by some of the subsequent rules?—7. How do you find the value of one, when the value of any given number is known?—8. When the price of one is any aliquot part of a penny, shilling, or pound, how do you calculate?—9. What is an aliquot part of a number?—10. What is the reason of the rule for case 3d? 11. What is the general rule, in the note to case 3d?—12. When the price is less than a shilling, and not the aliquot part of a shilling, how do you proceed?—13. What is the reason of this process?—14. When the price is above one shilling, and less than two, and not the aliquot part of a £, how do you calculate? 15. What is the reason that this process gives the answer?—

16. When the price is in shillings, how do you calculate?—17. What is the reason this process gives the answer?—18. How would you proceed by this rule, if there were pounds and shillings in the price?—19. When the price is in shillings and lower denominations, how would you calculate?—20. What is the reason of this process?—21. When the price is in pounds, and lower denominations, how would you calculate?—22. What is the reason of this process?—23. When the price is greater, or less than £1, by any aliquot part of a £, how would you calculate?—24. Does the same rule apply when the price is greater or less than a shilling, by any aliquot part of a shilling?—25. What is the reason of this rule?—26. When there is a fraction in the price for which parts cannot easily be taken, how would you calculate?—27. What is the reason of this process? 28. When there is a fraction in the quantity, how would you calculate?—29. What is the reason of these methods?—30. How would you calculate these cases by the rule in the note to case 3d.?—31. When the quantity is in several denominations, and the price of the highest given, how would you calculate?—32. What is the reason of this process?—33. How would you calculate this case by the rule in the note to case 3d?—34. If the price of the highest denomination in the question is not given, but must be found from knowing the price of some lower denomination, how would you proceed?—35. When the price of a dozen is any number of shillings, what is the price of one? 36. When the price of one is any number of pence, what is the price of a dozen?—37. When the price of a gross is any number of shillings, what is the price of a dozen?—38. When the price of a dozen is any number of pence, what is the price of a gross?—39. When the price of a ton is any number of £s, what is the price of a cwt.?—40. When the price of a cwt. is so many shillings, what is the price of a ton?—41. When the price of a cwt. is given, how would you find the price of a lb.?—42. When the price of a lb. is given, how would you find the price of a cwt.?—43. When the rate per day is given, how would you find the rate per annum?—44. When the rate per day is given, how would you find the rate for all the work days in a year?

COMMERCIAL ALLOWANCES.—1. What are commercial allowances?—2. What is gross weight?—3. What is draft?—4. What is tare?—5. What is real tare?—6. What is customary tare?—7. What is proportionate tare?—8. What is average tare?—9. What is tret?—10. What is cloff?—11. Are the allowances of tret and cloff generally given?—12.

What is neat weight?—13. When draft is the only allowance, how do you find the neat weight?—14. What is the reason of this rule?—15. When the whole tare is given, how do you find the neat weight?—16. What is the reason of this rule?—17. When the tare is at so much per box, &c. how do you find the neat weight?—18. What is the reason of this process?—19. When the tare is at so much per cent., or per cwt., how do you find the neat weight?—20. What is the reason of this process?—21. When both tare and tret are allowed, how do you find the neat weight?—22. What is the reason of this process?—23. When tare, tret, and cloff are allowed, how do you find the neat weight?—24. What is the reason of this process?—25. Are there any other cases where allowances must be made distinct from those already mentioned?—26. Can you give an example connected with any such case?—27. What is the general rule for calculating all such cases?—28. What is the reason of this rule?

SIMPLE PROPORTION.—1. What is proportion?—2. Why is this rule called SIMPLE PROPORTION?—3. Why is it called the RULE OF THREE?—4. Why is it called the GOLDEN RULE?—5. What is the rule for stating simple proportion?—6. What number do you write in the third term?—7. How do you know whether the answer must be greater or less than the third term?—8. If the answer must be greater than the third term, how do you place the remaining terms?—9. If the answer must be less than the third term, how do you place the remaining terms?—10. If the first and second terms are not in the same denomination, what is the first thing to be done?—11. If there are different denominations in the third term, what is done with it?—12. Having reduced the terms, how do you proceed?—13. When you divide the product of the second and third terms by the first, what is the quotient?—14. How do you know what denomination the quotient is in?—15. If there is a remainder after division, what denomination is it in?—16. How do you find the value of the remainder?—17. What is the first method of proving simple proportion?—18. What is the reason of this method of proof?—19. What is the second method of proof, and the reason of it?—20. What is the ratio of one number to another?—21. How do you express the ratio of one number to another?—22. What is the ratio of 12 to 4, and of 4 to 12?—23. What are the terms of a ratio, and what are they called?—24. When are two ratios equal to one another?—25. When two ratios are equal, are their terms proportionals?—26. What

are the first and last terms of a proportion called ?—27. What are the middle terms of a proportion called ?—28. What is the product of the means equal to ?—29. When the product of two numbers is equal to the product of other two numbers, are these numbers proportionals ?—30. How would you place the numbers to make them proportionals ?—31. What is the reason that the product of the second and third terms, divided by the first term, gives the fourth term of the proportion, or answer ?—32. Is ratio altered by multiplying or dividing its terms by the same number ?—33. If two ratios are equal, are they altered by multiplying or dividing the antecedents or consequents in each by the same number ?—34. What is the general contraction in the rule of three ?—35. What is the first particular contraction ?—36. What is the second particular contraction ?—37. May not both these be comprehended under the general contraction ?—38. Is not the operation shorter, and the risk of error less by contraction, than by the general rule ?—39. How is simple proportion generally divided ?—40. What is direct proportion ?—41. Give an example of it ?—42. What is inverse, or reciprocal proportion ?—43. Give an example of it ?—44. Is the general rule equally applicable to direct and inverse proportion ?—45. What kind of questions in proportion are easier solved by compound multiplication and compound division ?

COMPOUND PROPORTION.—1. What is compound proportion ?—2. Why is it called compound proportion ?—3. What is the general rule for compound proportion ?—4. How is it performed by simple proportion ?—5. Do you know any other method of performing it ?—6. Repeat the rule for stating by the blank.—7. Does this rule apply to all cases of compound proportion ?—8. Does the general rule in the text apply to all cases ?—9. Upon what principle does this rule depend ?—10. Do the contractions for simple proportion apply here ?—11. How do you arrange the terms before cancelling ?—12. What is done when any pair of terms are in different denominations ?—13. When you reduce any pair of terms, is it necessary to reduce all the terms, although in the same denomination when taken in pairs ?

VULGAR FRACTIONS.—1. What is a fraction ?—2. What is a vulgar fraction ?—3. Why is it called a vulgar fraction ?—4. How is a vulgar fraction expressed ?—5. What is the upper number called ?—6. What is the under number called ?—7. What common name applies to both these num-

bers?—8. What does the under number represent?—9. What does the upper number represent?—10. Suppose the upper number of a fraction is 8 and the under 17, how would you read it?—11. What is a proper fraction?—12. What is an improper fraction?—13. What is a simple fraction?—14. What is a compound fraction?—15. What is a complex fraction?—16. What is a mixed number?—17. How is a whole number reduced to a fractional form?—18. If two fractions have the same denominator, but different numerators, which of them is the greatest?—19. How is a fraction multiplied by a whole number?—20. How is a fraction divided by a whole number?—21. If we multiply or divide the terms of a fraction by the same number, do we alter its value?—22. Is the fraction of any number, equal to the sum of the like fractions, of the several parts of that number?—23. Is the fraction of any number equal to the same number of times the like fraction of unity?—24. Is the difference between like fractions of two numbers equal to the like fraction of the difference of these numbers?—25. When fractions of the same denomination have the same numerators, but different denominators, which is the greatest?—26. How do we invert a fraction, or find its reciprocal?—27. How do we know that two fractions are equal, if their numerators and denominators are not the same?—28. Are the reciprocals of two equal fractions equal?

REDUCTION.—Case 1.—1. What is reduction of vulgar fractions?—2. What is the reason that reduction of fractions is placed before addition, subtraction, &c.?—3. How do you reduce a whole number to a fraction having a given denominator?—4. Is the fraction thus produced equal to the whole number?—5. What is the reason of this?—6. What principle is this rule founded on?

Case 2.—1. How do you reduce mixed numbers to improper fractions?—2. What is the reason of this?—3. Is the improper fraction thus obtained equal to the mixed number?

Case 3.—1. How do you reduce an improper fraction to a mixed number?—2. What is the reason of this method?—3. Is the whole or mixed number thus obtained equal in value to the improper fraction?—4. What cases are proved by this?

Case 4.—1. How do you reduce compound to simple fractions?—2. Explain the reason of this by an example.—3. What is the principle on which this rule is founded?—4. When whole or mixed numbers are given, how are they made subject to the rule?—5. Can we cancel in this case?—6. When we cancel all the terms of the given fractions which admit of it, is the simple fraction in its least terms?—7. Does the order in

which the compound fraction is placed alter the value of the simple fraction ?—8. Would its value be altered if we write the numerators above the wrong denominators, or the denominators under the wrong numerators ?—9. Is the simple fraction equal in value to the compound fraction ?—10. What is understood by dissolving a fraction ?—11. How would you dissolve $\frac{24}{35}$ into an equal compound fraction ?—12. Is dissolving fractions of much practical use ?—13. How is a compound fraction known ?

CASE 5.—1. How do you reduce a complex fraction to a simple one ?—2. What is the reason of this operation ?—3. Does this case properly belong to fractions ?—4. Are the examples given here strictly fractional ?—5. Since this case neither belongs properly to fractions, nor the examples given strictly fractional, for what reason is it given here ?—6. Is the simple fraction obtained by the rule equal to the complex fraction ?—7. Can you explain the reason of this by an example ?

CASE 6.—1. How do you reduce one fraction to another of equal value, having a given numerator ?—2. On what principles is this operation founded ?—3. Has the new fraction the same value as the original fraction ?—4. How do you know that it is equal ?—5. Is this case always possible in simple fractions ?—6. Do you know any other rule for performing this case ?—7. Is not the process prescribed by this rule the same as by the other ?

CASE 7.—1. How do you reduce one fraction to another of equal value having a given denominator ?—2. On what principles is this operation founded ?—3. Is this case always possible in simple fractions ?—4. Has the new fraction the same value as the original fraction ?—5. How do you know it is the same value ?—6. Do you know any other rule for this case ?—7. Is the operation by this rule the same as by the rule in the text ?

CASE 8.—1. How do you reduce a fraction to its least terms ?—2. What is the reason that the operations prescribed in this rule does not alter the value of the fraction ?—3. How do you know when a fraction is in its least terms ?—4. When the terms of a fraction terminate in 0, 2, 4, 6 or 8, what are they divisible by ?—5. When they terminate in 0 or 5, what are they divisible by ?—6. When they terminate in one or more ciphers on the left, how would you reduce them to lower terms ?—7. When the sum of the digits in the terms of a fraction are divisible by 3 or 9, how would you reduce them ?—8. When any number expressed by the sum, or difference of two or more numbers, is to be divided by any number, how would

you do it ?—9. How is a fraction brought to its lowest terms at one division ?—10. How is the greatest common measure of two numbers found ?—11. What is the reason that this method gives the greatest common measure ?—12. Demonstrate the truth of this operation by an example.

CASE 9.—1. How do you reduce fractions to a common denominator ?—2. How are fractions prepared for the operations of this rule ?—3. What is the reason that this rule gives the common denominator without altering the value of the fraction ?—4. Demonstrate the truth of this process by an example.—5. If the fractions are all in their least terms before reducing to a common denominator, do they always remain so after being reduced ?—6. How do you reduce fractions to their least common denominator ?—7. What is the reason this rule gives the least common denominator ?—8. Demonstrate the truth of this reason by an example.—9. How do you prove that the value of the fractions is not altered by this process ?—10. Do the numerators in this case ever come out in the form of a mixed number ?—11. What is the reason they cannot come out in the form of a mixed number ?—12. When the denominators are small, can the common denominator be found by inspection ?—13. How do you find the common denominator by inspection ?—14. When the least common denominator is found by inspection, should not all the other operations be performed mentally ?—15. When this method is perfectly understood, do we not naturally attempt the reduction by it, in preference to any other method ?—16. On what principles is this rule founded ?—17. Can you illustrate this method by an example ?—18. What is the other method given in the notes, to find the greatest common measure ?—19. On what principles is this rule founded ?—20. Can you illustrate this method by an example ?—21. Do expertness and accuracy in fractional calculations depend much upon a perfect knowledge of this case ?

CASE 10.—1. How do you reduce fractions from one denomination to another without altering their value ?—2. On what principle is this rule founded ?—3. Demonstrate the reason of the rule by an example.—4. Are the multipliers not altered by writing 1 for their denominators ?—5. What is the rule commonly given for this case ?—6. Is not the principle of this rule the same as the one in the text ?—7. What are the cases in reduction of whole numbers which this case comprehends ?—8. How do you know that it includes both these cases ?

CASE 11.—1. How do you reduce a quantity to a fraction

of a given denomination ?—2. On what principle is this rule founded ?—3. Demonstrate the truth of this rule by an example.—4. Is it necessary in all cases to reduce the given quantity to its lowest terms ?

CASE 12.—1. How do you value fractions ?—2. On what principle is this rule founded ?—3. What is the numerator considered to be in this case ?—4. By what other method may this operation be performed ?—5. Is not this method more tedious than by the general rule ?

ADDITION.—1. What is addition of vulgar fractions ?—2. How do you add fractions ?—3. Wherefore do you bring fractions to a common denominator before adding them ?—4. What is the reason that they must be in the same denomination before you can add them ?—5. Having brought fractions to the same denomination, and to a common denominator, how do you know that the sum of the numerators placed over the common denominator is the sum of the fractions.—6. When whole or mixed numbers are given along with the fractions, what is the best method of performing the operation ?—7. When fractions of different denominations are given, what is the best method of adding them ?

SUBTRACTION.—1. What is subtraction of vulgar fractions ?—2. How do you prepare fractions for subtraction.—3. What is the reason you prepare them in this manner ?—4. How do you subtract them when they are prepared ?—5. How do you know that the fraction thus obtained is the difference between the given fractions ?—6. If the difference between several fractions and several other fractions were required, how would you find it ?—7. When fractions of different denominations are given, what is the best method of finding their difference ?

MULTIPLICATION.—1. What is multiplication of vulgar fractions ?—2. How do you multiply vulgar fractions ?—3. What is the reason of this rule ?—4. Demonstrate this reason by an example, both where the multiplicand is a whole number, and where it is a fraction.—5. What case in reduction is the same as multiplication ?—6. What is the reason that the product of two proper fractions is less than either of them ?—7. What are the two different ways in which we multiply a fraction by a whole number ?—8. How many different ways can we multiply a whole number by a mixed number ?—9. Can you illustrate these methods by an example ?—10. When both

fractions are mixed numbers, how do you find the product ?—11. Can you cancel in multiplication of fractions ?—12. How do you find the continued product of several fractions ?

DIVISION.—1. What is division of vulgar fractions ?—2. How do you divide fractions ?—3. What is the reason that the operation prescribed by the rule gives the true quotient ?—4. Can you demonstrate this reason by an example ?—5. Can you demonstrate the reason of the rule by an example in any other way ?—6. Is the process prescribed in the rule the same as reducing the divisor and dividend to a common denominator, and then dividing the numerator of the dividend by the numerator of the divisor ?—7. Can you illustrate this by an example ?—8. How would you express the general rule, to show the nature of the process more clearly ?—9. What is the reason this method is not used in preference to the method prescribed by the rule in the text ?—10. When the terms of the dividend are divisible by the terms of the divisor, how would you perform the operation ?—11. What are the two methods of dividing a fraction by a whole number ?—12. Whether is the quotient greater or less than the dividend when the divisor is a whole number ?—13. What is the reason that the quotient is greater than the dividend when the divisor is a proper fraction ?—14. In what proportion is the quotient greater than the dividend when the divisor is a proper fraction ?—15. How is division of fractions proved ?

PROPORTION.—1. How do you state questions in simple proportion in vulgar fractions ?—2. How do you work them ?—3. Are the definitions, the reasons, and the principles of simple proportion in vulgar fractions the same as already explained in whole numbers ?—4. Why do you invert the divisor ?—5. Is proportion in vulgar fractions always given in systems of arithmetic ?—6. Is there any good reason why it should ever have a place in books on arithmetic ?

COMPOUND PROPORTION—1. How do you state questions in compound proportion in vulgar fractions ?—2. How do you work them ?—3. Wherefore do you invert the terms in the first column ?—4. Is the effect produced by inverting the divisor not changed when the divisor consists of two or more fractions ?—5. What is the reason that it is not altered ?

DECIMALS.—1. What is a decimal fraction ?—2. How

is a decimal fraction distinguished ?—3. What is the common definition of a decimal ?—4. What is the objection to this definition ?—5. What does this definition presuppose ?—6. Is the notation and numeration of decimals the same as whole numbers ?—7. What is the highest place of the decimal scale called ?—8. What is the second place of the scale called ?—9. What is the 3d, 4th, 5th and 6th places of the scale called ?—10. What is the easiest method of reading decimals ?—11. Do you know any other method of reading decimals ?—12. Do ciphers on the right of decimals alter their value ?—13. What is the reason that they do not alter their value ?—14. Do ciphers on the left of decimals alter their value ?—15. What is the reason they alter the value of decimals ?—16. Under what general heads are decimals classed ?—17. What is a terminate decimal ?—18. Does a terminate decimal express the full value of the vulgar fraction from which it is derived ?—19. What is an interminate decimal ?—20. Can an interminate decimal be extended to equal in value the vulgar fraction from which it is derived ?—21. Do interminate decimals approach nearer to the value of the vulgar fraction at every step ?—22. Into how many classes are interminate decimals divided ?—23. What is a pure repeater ?—24. What is a mixed repeater ?—25. What is a pure circulate ?—26. What is a mixed circulate ?—27. When are circulates similar ?—28. When are circulates coterminous ?—29. Are the operations of addition, subtraction, &c. performed the same way in decimals as in whole numbers ?

REDUCTION.—Case 1.—1. What is reduction of decimals ?—2. How do you reduce a vulgar fraction to a decimal ?—3. What is the reason of this rule ?—4. What is the reason that annexing a cipher to a remainder gives a decimal in next inferior place ?—5. What is the reason that there must be a decimal in the quotient for every cipher annexed ?—6. What is the reason that there can neither be more nor fewer decimals in the quotient than the number of ciphers annexed ?—7. Can you illustrate the reason you have given by an example ?—8. Do you know any other methods of shewing the reason of this rule ?—9. Can you illustrate this method by an example ?—10. Do you know any other method of shewing the reason of this rule ?—11. Can you illustrate this method by an example ?—12. When the denominator of the vulgar fraction is 2 or 5, or any of their powers, or the product of their powers, is the corresponding decimal terminate, or interminate ?—13. What is the reason that the decimal is always terminate in these cases ?—14. What is Bonnycastle's opinion as to vulgar fractions

which produce terminate decimals?—15. If the denominator of a vulgar fraction is neither 2 nor 5, nor any of their powers, nor a multiple of any power of 2 by any power of 5, can the decimal be terminate?—16. What is the reason it cannot be terminate in other cases?—17. When the decimal is not terminate, can it equal the value of the vulgar fraction from which it is derived?—18. Can you bring an interminate decimal nearer to the value of the vulgar fraction than any proposed difference?—19. Suppose it were required to bring a decimal nearer to the value of a vulgar fraction than the millionth part of a unit, how far would you extend it?—20. When dividing any number by another, how do you carry out the remainder in a decimal?—21. What is Colson's method of finding the decimal, when many places are required?—22. Can you illustrate this method by an example?

CASE 2.—1. How do you reduce lower denominations to the decimal of a higher denomination?—2. What is the reason of this rule?—3. Can you prove that the method prescribed in the rule, is the same as reducing the lower denominations to the fraction of the higher, and then reducing that fraction to a decimal?—4. Can this rule be explained upon the principles of vulgar fractions?—5. Can you illustrate the reason of the rule by an example upon these principles?—6. Can this case be performed entirely by multiplication?—7. Can you illustrate this method by an example?—8. Do you know any reason why this method is not introduced in practice since it shows so clearly the nature of decimals?

CASE 3.—1. How do you reduce £s, shs, and pence to the decimal of a £, mentally?—2. What is the reason you halve the shillings for the first decimal place?—3. For what reason do you add 1 for every 24 farthings in the remainder to give the 2d and 3d decimal places?—4. When the qrs. in the remainder are expressed by one figure, for what reason do you write a cipher in the second decimal place, and not the figure expressing the farthings?—5. How do you know in this case when the decimal is terminate?—6. What is the reason the decimal is terminate in these cases?—7. When the farthings are not exactly 24, 48, or 72, how would you complete the decimal by the method in the notes?—8. What is the reason that the 24th part of the excess or defect completes the decimal?—9. Do you know any method by which a table could be made out for correcting the decimal in every possible case of this rule?—10. If the farthings amount to 42, what would the correction be?—11. Whether would you add or annex this correction?—

12. What is Bonnycastle's rule for correcting the third decimal place ?—13. What are the objections to this method ?

CASE 4.—1. How do you value a finite decimal ?—2. What is the reason of this rule ?—3. When ciphers occur on the right of the decimal in the course of reduction, what is done with them ?

CASE 5.—1. How do you value the decimal of a £, mentally ?—2. What is the reason you double the figure next the decimal point for shillings ?—3. What is the reason that the 2d and 3d places diminished by 1 for every 25 give farthings ?—4. Might we not take 5 from the second decimal place when it amounts to or exceeds 5, and add 1 to the shillings ?—5. What advantage attends this method ?—6. Does valuing mentally the 3 first places of the decimal give the farthings correct, when the decimal extends to more places ?—7. What is the reason of this ?—8. Can you illustrate the reason you have given by an example ?—9. What is Bonnycastle's method of valuing the second and third places ?—10. Into what error does this method lead us ?—11. How would you value the decimal of a cwt. mentally ?

CASE 6.—1. How do you value a single repetend ?—2. What is the reason you carry at 9, and not at 10 in the several products of the repetend ?—3. What is your reason for terminating the repeating figure in each product, under the right hand figure of the first product ?—4. Why do you carry at 9 when adding the first column ?—5. Whether are repeating decimals 9th or 10th parts ?—6. Has it not been proved that repeaters are 9th not 10th parts ?—7. Is the sum of the infinite decreasing series, formed by a repeating decimal, equal to a vulgar fraction whose numerator is the repeating figure and its denominator 9 ?—8. Can you illustrate this by an example ?

CASE 7.—1. How do you value a circulating decimal ?—2. What is the reason you carry in this manner to the right hand figure of each product ?—3. Can you illustrate this reason by an example ?—4. What is your reason for terminating each product under the right of the first product ?—5. What is your reason for carrying thus to the right hand column when adding ?—6. When all the figures in the circle become the same in the course of reduction, what becomes of the circle ?—7. When all the figures in the circle become nines, what are they equal to ?—8. When the circle is not changed into a repeater in the course of reduction, does it retain the same number of places as in the original circle ?

CASE 8.—1. How do you reduce a finite decimal to a vulgar fraction ?—2. What is the reason of this ?—3. Having brought

the decimal into a vulgar fraction, may it not often be reduced to lower terms ?—4. How do you reduce a unit into parts, similar to the decimal ?—5. Must it not therefore consist of 1, with as many ciphers annexed as there are places in the decimal ?

CASE 9.—1. How do you reduce a pure repeater or circulate to a vulgar fraction ?—2. What is the reason you write 9 for the denominator of a pure repeater ?—3. What is the reason you write 9 under each figure of a circle, for its denominator ?—4. Can you illustrate the reasons you have given by an example ?

CASE 10.—1. How do you reduce a mixed repeater, or circulate to a vulgar fraction ?—2. What is the reason of this rule ?—3. What is the reason you annex as many ciphers to the nines in the denominator as there are finite places in the decimal ?—4. Can you illustrate the rule, and the reasons you have given, by an example ?—5. When there are whole numbers given along with the finite part of the decimal, must they also be subtracted along with the finite part ?—6. Do you annex a cipher to the denominator for each place in the whole number thus subtracted ?

ADDITION.—CASE 1.—1. How do you add terminate decimals ?—2. What is the reason of this rule ?—3. Is the addition of finite decimals, the same as whole numbers ?

CASE 2.—1. How do you add repeating decimals ?—2. What is the reason you extend the repeating figure one place farther than the longest finite part ?—3. What is the reason you carry at 9 when adding repeaters ?—4. Is it necessary to carry repeaters as far as the longest finite part, to have the finite part correct in the sum ?—5. Is it not necessary in most cases to carry them a place farther to have the finite part correct ?—6. Can you illustrate this by an example ?—7. Is it not therefore absolutely necessary to carry repeaters one place farther than the longest finite part ?—8. Does it not also appear from the example you have given that it is unnecessary to extend them more than one place beyond the longest finite part ?—9. Can you perform this case by reducing the decimals to vulgar fractions ?—10. How would you perform an example by this method ?—11. Is not this method often preferred ?—12. What are the objections to it ?—13. Is not Bonnycastle's method of adding interminate decimals therefore injudicious ?

CASE 3.—1. How do you add circulating decimals ?—2. What is the reason that both repeaters and circulates are counted finite, as far as the longest finite part ?—3. What is

the reason you extend repeaters, and circulates as far to the right of the longest finite part as the least common multiple of the number of places in the several circles indicates ?—4. What is the reason you carry at 10 from the right of the longest finite part, to the right-hand column of the circle ?—5. If a part, or the whole of a circle, is taken up in completing the finite part, does this prevent its being extended to complete the circle ?—6. Is it quite necessary to extend all repeaters and circulates as far to the right of the longest finite part, as the least common multiple indicates ?—7. Is it unnecessary to extend them farther ?—8. What would be the consequence if they were extended farther ?—9. How do you make circulates similar, and coterminous ?—10. Take two or more circulating decimals, not having the same number of places in the circle, and one of them at least, having a finite part, and make them similar and coterminous.—11. How would you perform this case by vulgar fractions ?—12. Can you perform an example by this method ?—13. What objections have you to this process ?—14. What are the advantages of the rule in the text over this method ?—15. Supposing 4 circulating decimals were given to be added, the 1st having 5, the 2d 7, the 3d 9, and the 4th 11 places in the circle, how many places would the circle in the sum consist of ?

SUBTRACTION.—Case 1.—1. How do you subtract terminate decimals ?—2. How do you prove subtraction of terminate decimals ?—3. What is the reason of this rule, and its proof ?—4. What is the reason of placing the point according to the rule ?

Case 2.—1. How do you subtract repeating decimals ?—2. What is your reason for extending repeating decimals one place beyond the longest finite part in subtraction ?—3. What is the reason you diminish the repeating figure by one before you borrow to it ?—4. Can you illustrate this reason by an example ?—5. How do you prove this case ?—6. Can this case be performed by reducing to vulgar fractions ?—7. Are there any objections to the use of that method in this case ?

Case 3.—1. How do you subtract circulating decimals ?—2. What is your reason for making circulates similar, and coterminous in subtraction ?—3. What is your reason for diminishing the right hand figure of the circle in the minuend by 1, when its left hand figure is less than the corresponding figure of the subtrahend ?—4. How do you prove this case ?—5. Can this case be performed by reducing to vulgar fractions ?—6. Can you perform an example in that way ?

MULTIPLICATION.—CASE 1.—1. How do you multiply when both factors are terminate?—2. What is the reason that you must have as many decimals in the product as there are in both factors?—3. When there are not so many figures in the product as there are decimal places in the factors, what is the reason you prefix ciphers?—4. Does the product of the right hand figure of the multiplicand, by the right hand figure of the multiplier, always occupy a place as far removed from the decimal point, as the number of decimals in both factors indicate?—5. Can you illustrate this by an example?—6. How do you ascertain without performing the operation, whether there should be any ciphers, or how many, between the decimal point and the first significant figure?—7. Are all decimals proper fractions?—8. Can the product of any two decimal factors give a whole number?—9. Is the product of any two decimals less than either of the factors?—10. What is the reason of this?—11. How is the reason of this rule generally shown?—12. When the multiplier is 10, 100, &c. how do you multiply?—13. What is the reason of this process?

CASE 2.—1. How do you multiply when the multiplicand is a repeater or circulate?—2. What is the reason you carry at 9 on the right of each product in repeaters?—3. What is your reason for extending repeaters in the products, one place farther than the longest finite part?—4. What is the reason you carry at 9 when adding the right hand column of the products?—5. In multiplying circles, what is the reason you add to each product of the first figure on the right, what was to carry from the left of the circle?—6. What is your reason for making the circles in the several products similar?—7. In adding the products, why do you add to the right hand of the circles what was to carry from their left?—8. What is the reason that circles in the products are made similar by terminating them under the right hand figure of the first product?—9. If nothing remains after taking the 9s out of the right hand column of the products of a repeater, is the sum finite?—10. When adding the products of a circulate, if there be as many ciphers or 9s on the right as there were figures in the circle, is the product finite?—11. What is the reason that the product is finite in these cases?—12. Can this case be performed by reducing first to vulgar fractions?—13. Can you perform an example by that method?

CASE 3.—1. How do you multiply when both factors repeat or circulate?—2. What is the reason of this rule?—3. Cannot this case be performed without reducing either factor to a vulgar fraction?—4. How is this done?—5. Can you illustrate this

method by an example?—6. What is the reason of this process? —7. Is not this the natural method of multiplying circulating decimals?—8. For what reason then was it not preferred to the rule in the text?—9. Can this case also be performed by reducing first to vulgar fractions?—10. Can you perform an example by this method?

Contraction.—1. How do you multiply, to retain any proposed number of decimals in the product?—2. Does inverting the multiplier alter the product?—3. Why do you place the units figure of the multiplier under that figure of the multiplicand which you wish the last in the product?—4. When there are several places of whole numbers in the multiplier, what is the reason that the first figure in each product by them must stand under the right hand figure of the product by the figure in units place?—5. What is the reason that the first figure of each product by the decimal places also stands under the right of the product by the figure in units place?—6. What advantage results from inverting the multiplier?—7. What is the reason you carry 1 from 5 to 15, &c. from the product of the figures on the right of the one you are multiplying by?—8. Does this method of carrying always give the last decimal place correct?—9. How would you proceed if it were absolutely necessary to have the last decimal correct?—10. Can you perform an example by this method?—11. In what cases is this contraction chiefly useful?

DIVISION.—Case 1.—1. How do you divide when the divisor is terminate?—2. On what principle is this rule founded? —3. What is the reason you annex ciphers when all the decimal places are brought down?—4. When the dividend repeats or circulates, what is the reason you annex the repeating or circulating figures instead of ciphers?—5. What is the reason that the decimal places in the quotient must be equal to the excess of those in the dividend, above those in the divisor?—6. When the decimals in the divisor and quotient are not equal to those in the dividend, what is the reason you supply the deficiency by prefixing ciphers to the quotient?—7. When the decimal places in the divisor and dividend are equal, what is the quotient?—8. When the divisor is a decimal and the dividend a whole number, how do you know where to place the decimal point in the quotient?—9. Do you know any other method of determining the decimal point in the quotient?—10. Can this case be performed by converting the decimals into vulgar fractions?

Case 2.—1. How do you divide when the divisor repeats or

circulates?—2. What is the reason of this rule?—3. What is the simplest method of performing the multiplication in this case?—4. Do you know any method purely decimal, by which we can divide when both divisor and dividend are interminate? —5. Can you illustrate this method by an example?—6. What is the reason this rule is not generally used?—7. Do you know any method of performing this case by exterminating the repeater or circulate in the divisor?—8. Can you illustrate this method by an example?—9. Can this case also be performed by vulgar fractions?—10. Can you perform an example by this method?

CONTRACTION.—1. How do you divide so as to retain any proposed number of decimals in the quotient?—2. What is the principal difficulty in this contraction?—3. What is the simplest method of obviating this difficulty?—4. Why do you carry 1 from 5 to 15, &c. in the figures cut off from the divisor?—5. Is this contraction of much practical utility?

SIMPLE PROPORTION.—1. How do you perform proportion by decimals?—2. On what principle is this rule founded?—3. Does the application of decimals abbreviate the operation?—4. Can any general rule be given to ascertain when decimals can be used with advantage.

COMPOUND PROPORTION.—1. How do you perform compound proportion by decimals?—2. On what principles is this rule founded?

GENERAL OBSERVATIONS.—1. When you multiply by a decimal, is the product greater or less than the multiplicand?—2. When you divide by a decimal, is the quotient greater or less than the dividend?—3. How is the reciprocal of any number formed?—4. What effect do you produce when you multiply by the reciprocal of any number?—5. What effect is produced when you divide by the reciprocal of any number?—6. What class of decimals is produced from vulgar fractions whose denominators are 2, 5, or any power of 2, 5, or the product of any power of 2 by any power of 5?—7. What class of decimals is produced from vulgar fractions whose denominators are 3 or 9?—8. What class of decimals is produced from vulgar fractions whose denominators are 2 or 5, or any of their powers, &c. multiplied by 3 or 9?—9. If a mixed repeater is multiplied by 3 or 9, to what class of decimals does the product belong?—10. When the denominator of a vulgar fraction is any power of 3 excepting 9, what class of decimals does it

produce?—11. How do you ascertain how many places are in the circle derived from such fractions?—12. What kind of repeaters divided by 3 give pure circles?—13. What class of decimals is derived from fractions whose denominators are 2, 5, or any power of 2, 5, &c. multiplied by any other number than 3 or 9?—14. What class of decimals is produced by vulgar fractions whose denominators are neither 3 nor 9, and not divisible by 2 or 5?—15. If a vulgar fraction is given to reduce to a decimal, do you know any rule by which you can ascertain the class to which it will belong, and also how many places, either finite or circulating, or of both, it will contain?

SIMPLE DISTRIBUTIVE PROPORTION.—1. What is distributive proportion?—2. How is distributive proportion generally divided?—3. In what cases is simple distributive proportion applied?—4. Must either the stock of the several partners, or the time of their continuance be the same?—5. By what other names is distributive proportion known?—6. Is the application of the rule the same under these names?—7. What is the general rule for simple distributive proportion?—8. On what principles is this rule founded?—9. What is the reason of this rule?—10. How do you prove simple distributive proportion?—11. What is the reason of this method of proof?—12. When both the stock and time of each partner are equal, how do you find the share of each?—13. Can the operation be performed shorter than by the general rule?—14. How many different methods of shortening it are given in the notes?—15. What is the first of these methods, and the reason of it?—16. What is the second method, and the reason of it?—17. What is the third method, and the reason of it?—18. What is the fourth method, and the reason of it?—19. What is the fifth method, and the reason of it?—20. What does the general rule require given, to find the answer?—21. If the whole stock were given, and the particular gain of each partner, how would you find their stocks?

COMPOUND DISTRIBUTIVE PROPORTION.—1. What is compound proportion?—2. Why is it called compound?—3. What is the rule for working compound distributive proportion?—4. Does this rule consider the product of each stock, by the time of its continuance, as the particular stock or time of each partner?—5. On what principles is this rule founded?—6. What is the reason of this rule?—7. How do you prove questions in compound distributive proportion?—8. Can all questions in compound distributive proportion be

performed by this rule?—9. What kind of questions are those, to which this rule is not applicable?—10. What is the rule for solving questions of this class?—11. In proving questions wrought by this rule, is it enough that the sum of the several shares be equal to the whole property to be divided?—12. What more is necessary to complete the proof?—13. By what other names is compound distributive proportion known?

COMMISSION AND BROKERAGE.—1. What is commission and brokerage?—2. How do you calculate it?—3. On what principles are the rules founded?—4. Who are called factors in the mercantile world?—5. Can factors sell their employer's goods on credit?—6. If a factor becomes bankrupt, or die insolvent with his employer's property in his hand, can it be appropriated to the payment of the factor's debts?—7. What are the necessary qualifications in a factor?—8. Who are brokers, and how are they employed?—9. What are the qualifications requisite in a broker?

SIMPLE INTEREST.—1. What is interest?—2. What is the sum lent called?—3. What is the sum given by the borrower to the lender for the use of his money called?—4. What is the sum of the principal and interest called?—5. Was exacting of interest ever prohibited by law?—6. What was the effect of these prohibitions?—7. When was interest first made legal in England?—8. What rate per cent. did the law then allow?—9. When was this rate of interest reduced?—10. When was the present rate of 5 per cent. made the legal interest?—11. By whom was this rate established?—12. What is the penalty against usury?—13. What is the legal interest in Ireland, East Indies, and New South Wales?—14. What is simple interest?—15. When money is lent, and no rate specified, what rate is understood?—16. When any specified term of credit is expired, what rate of interest is due till payment is made?—17. What is the reason that legal interest is now higher than the market rate?

CASE 1.—1. How do you calculate the interest of any sum, for any number of years, at any proposed rate per cent.?—2. On what principles is the rule founded?—3. Can all questions in this case be wrought by compound proportion?—4. What is the reason that $\frac{1}{20}$ of the principal multiplied by the years gives the interest at 5 per cent?—5. What is the reason that $\frac{1}{5}$ of the interest at 5 per cent. added to itself gives the interest at 6 per cent., and subtracted gives the interest at 4 per cent.?—6. What is the best method of ascertaining the most useful

business contractions in this case ?—7. Can decimals be introduced with advantage in calculations of interest ?

CASE 2.—1. How do you calculate the interest of any sum for months ?—2. On what principles is this rule founded ?—3. Might not this case be calculated by compound proportion ?—4. How would you discover useful contractions in this case ?—5. When the rate is 5 per cent., what is the interest of £1 for a month ?—6. Would not the principal considered as pence, and multiplied by the months, give the interest at 5 per cent. ?—7. Do you know any other short method of finding the interest at 5 per cent. for months ?—8. When the term of a bill, &c. is expressed in months, what kind of months are understood ?—9. What is the reason that interest for months is not admitted in courts of law?

CASE 3.—1. How do you calculate interest for any number of days, at any rate per cent. ?—2. On what principles is the rule for this case founded ?—3. When questions in this case are stated by compound proportion, what numbers are always in the first terms ?—4. Is not 36500 therefore the natural divisor in every question belonging to this case ?—5. May every question in this case be wrought by multiplying the principal by the rate, and days, and dividing by 36500 ?—6. What then is the reason that you multiply by double the rate, and divide by 73000 ?—7. What is the reason this gives the same result as the other ?—8. When the rate is 5 per cent., what is the reason you divide by 7300 without multiplying by the rate ?—9. What is the reason you use 73000 as a divisor in every case of this rule, instead of using a particular divisor for every rate ?—10. In finding the interest for days, whether is the day we reckon from, or the day we reckon to, included ?—11. How would you find the days from 12th January to 23d July ?—12. How would you find the days between these dates by the table in the notes ?—13. How do business men proceed with their calculations when there are shillings, &c. in the principal ?—14. How do they proceed when the interest is under a penny ?—15. What is the best method of calculating accurately when there are shillings, &c. in the principal ?—16. How do you calculate interest for days by the table in the notes ?—17. Is this method as tedious as by the general rule ?—18. How do you find the interest when the time is in weeks ?

BANKING-HOUSE CALCULATIONS.—CASE 1.—1. In what do banking-house calculations chiefly consist ?—2. How do you calculate interest when partial payments are

made ?—3. Does the bank of England give interest for money lodged in it ?—4. What is a deposit account ?—5. Why is the interest not brought out in £'s, &c. at each payment ?—6. Do you know any other method of calculating these accounts ? —7. What are the objections to this method ?—8. What are the two great requisites in a bank accountant ?

CASE 2.—1. How do you calculate interest on cash and current accounts ?—2. On what principles are these calculations founded ?—3. What is meant by a cash account ?—4. What is meant by a current account ?—5. What are the two forms given in the example wrought in the text ?—6. How are bank pass books kept ?—7. How do you prove the accuracy of the additions and subtractions in the accountant's form ?—8. How do you prove whether the days are rightly taken ?—9. In what does the method recommended in the notes for the accountant's form differ from that in general use ?—10. What are the advantages of this method ?—11. How do you calculate the interest on bonds ?—12. Is the practice of adding the interest to the principal before subtracting the payment, sanctioned by courts of justice ?—13. What is the simplest method of calculating interest on bonds ?—14. Why do we not count 29 days for February leap year when it ends within the currency of a complete year, and yet count it when it is in the odd days ?

CASE 3.—1. How do you calculate the discount on bills ?—2. On what principles is this rule founded ?—3. What is it that bankers call discount ?—4. What is the proceeds of a bill ? —5. When bankers charge commission, whether do they calculate it upon the sum specified in the bill, or upon the proceeds ?—6. What is the reason that bankers charge discount for three days beyond the written term of the bill ?—7. What are these three days called ?—8. Do the days of grace vary in different countries ?—9. When the term of a bill is expressed in months, are calendar months understood ?—10. How long do bankers discount bills before they become due ?—11. If a bill is dated on the 3d of February at four months, when might it be discounted, and till what date would the discount be calculated ?—12. Do you know any other methods which are sometimes resorted to by people in straitened circumstances, to obtain the command of money ?—13. Can you give an example of these methods ?—14. Are these methods advantageous to those who have recourse to them ?

STOCK-JOBBING CALCULATIONS.—1. What are the stocks ?—2. Are there any records kept of the loans which

constitute the funds ?—3. Why are they called funded debts? —4. At what intervals is the interest payable on these debts? —5. Can stock be transferred from one person to another ?—6. With what are the calculations in stock-jobbing connected? —7. Does the value of property in the funds fluctuate?—8. Does the interest fluctuate with the nominal capital ?—9. Is the interest payable on the nominal capital, or on the price paid for it ?—10. Why are the 3 per cent. consols so called ?—11. From what did the 3 per cents. reduced derive their name? —12. How did the imperial annuities receive their name ?—13. What are the other permanent annuities ?—14. When do the long annuities terminate ?—15. How are they bought and sold ?—16. On what days is the interest payable on the 3 per cent. consols, 3 per cent. imperial, new 4 per cents., South Sea stocks, and new South Sea annuities ?—17. When is the interest payable on the other stocks ?—18. Where is the dividends on the stocks payable ?—19. Are the transfer books always kept open ?—20. When stock is frequently transferred in the course of the half year, who receives the interest or dividend ?—21. What is the reason that the 3 per cent. consols, and the 3 per cents. reduced, never sell for the same money ?—22. Does not the same reason account for the difference of price between the 4 per cents., and the new 4 per cents. ?—23. How are new loans got up ?—24. Are these several stocks transferable together during the currency of the loan ?—25. What are they called, when transferred together? —26. When any one of these stocks are sold separately before the instalments are paid up, what is it called ?—27. Is the capital of the Bank of England, East India and South Sea Companies called stock, and also transferable ?—28. Why are they not called funds ?—29. Why are exchequer, navy, and ordnance bills called unfunded debts ?—30. What is the nature of an exchequer bill ?—31. What is the nature of a navy bill? —32. What is the nature of an ordnance bill ?—33. How and when are these bills paid ?—34. What is the nature of India bonds, and when are they payable ?—35. By whom is the business in the stocks transacted ?—36. What per centage are stock brokers allowed ?—37. Is not every man at liberty to transact his own business in the stocks?

Case 1.—1. How do you calculate the value of stock sold? —2. What is the reason you deduct $\frac{1}{8}$ from the selling price per cent. ?—3. On what principles is the rule for this case founded ?—4. Whether does the rule give the sum which the stock produces, or the sum which the broker should remit to his employer?

CASE 2.—1. How do you calculate the price of stock bought? —2. What is the reason you add $\frac{1}{8}$ to the selling price?—3. On what principle is this rule founded?—4. Whether does the rule give the sum the broker pays for the stock, or the sum for which he must draw upon his employer?

CASE 3.—1. How do you find how much stock may be bought for a given sum?—2. What is the reason this rule gives the quantity of stock which the proposed sum will purchase?—3. What is the reason that you add $\frac{1}{8}$ to the selling price in this case?—4. When there is a fraction in the divisor, what is the best method of working?—5. Whether does this rule give the sum which the broker pays for the stock, or the sum it costs the purchaser?

CASE 4.—1. How do you find the rate of interest arising from investing money in the stocks?—2. On what principles is the rule for this case founded, and why do you increase the selling price by $\frac{1}{8}$ for a divisor?—3. How do you calculate the rate of interest which any stock will yield, when the dividend due upon it at the time of purchasing is to be taken into the account?—4. Is this the customary method of calculating in these cases?—5. If it were required to find what interest would arise from money vested in the 3 per cent. consols, on the 30th November, when selling at $86\frac{7}{8}$, how would you proceed?—6. If the 3 per cent. consols were selling for $85\frac{3}{8}$, and the 3 per cents. reduced for $86\frac{5}{8}$ on the 29th September, how would you find which of these funds was the preferable investment?—7. Does this method give the answer perfectly correct?—8. By what method would you find the correct answer? —9. Why is this method not generally practised?—10. Might not this be easily remedied by the application of tables?—11. How do you find the value of an annuity in the long annuities?—12. How do you find what sum will purchase any proposed annuity in the long annuities?—13. How do you find what annuity any proposed sum will purchase in the long annuities?—14. When a man transacts his own business in the stocks, how do you make the rules for the several cases apply? —15. Suppose you saw in a newspaper 3 per cent. con. $87\frac{3}{8}$, $\frac{1}{2}$, $\frac{1}{8}$, what would you understand by it?—16. Suppose you saw in the newspapers 3 per cents. red. $86\frac{7}{8}$, 7, $86\frac{5}{8}$, what would you understand by it?—17. What would you understand by 3 per cent. cons. shut, and 3 per cent. red.—?—18. What would you understand by exchequer bills 25, 26, P. M. and by 3, 2 dis.? 19. What would you understand by long ann. $16\frac{3}{8}$, $16\frac{1}{2}$?

INSURANCE CALCULATIONS.—1. What is insurance? 2. Who is the insurer?—3. Who is the insured?—4. What is

the sum paid to the insurer called ?—5. What is the contract between the parties called ?—6. What is policy duty ?

CASE 1.—How do you calculate the premium and policy duty on land insurances ?—2. On what principles is this rule founded ?—3. By whom are land insurances effected ?—4. How is policy duty charged ?—5. When there is only part of the value of a property insured, and a total loss ensues ; what must the insurers pay ?—6. When insurance is effected to the whole value of the property, and either a partial or total loss ensues; how are the insurers obliged to make it good ?—7. When only part of a property is insured, and the rest risked, and a partial loss is sustained, how should it be borne ?

CASE 2.—1. How do you calculate sea insurance ?—2. By whom is sea insurance carried on ?—3. What is an insurance broker allowed for effecting an insurance ?—4. How do brokers proceed when they receive orders to effect an insurance ?—5. When the broker has obtained underwriters to the amount required, what is it then necessary for him to do ?—6. How are insurances effected in time of war ?—7. If a ship deviate from the course prescribed in the policy without legal reasons, what is the consequence ?—8. When merchants act as agents, what do they charge for effecting an insurance, and what for settling a loss ?—9. What do brokers charge for settling a loss ?—10. When the value of the goods shipped is less than the value insured, what is the difference called ?—11. Of what is the return made for short interest ?—12. What do underwriters deduct from the returns for short interest, for their trouble ?—13. Who are prohibited from acting as underwriters ?

CASE 3.—1. How do you calculate how much must be insured to cover a given sum ?—2. On what principles is this rule founded ?—3. Is it necessary for the insured to run part of the risk ?—4. Are the calculations in this case opposed to the spirit of the law ?—5. Do merchants always insure so as to cover their property and expense of insurance ?—6. Is the rule in the text perfectly correct in its results ?—7. In what is it deficient ?—8. How then would you proceed to obtain the result perfectly correct in every respect ?—9. How do you find how much must be insured to cover property on a voyage out and home ?

CASE 4.—1. How do you calculate averages ?—2. How do you calculate the average loss per cent. ?—3. On what principles are the rules for this case founded ?—4. From what do the difficulties in this case principally arise ?—5. By whom are general or gross averages borne ?—6. By whom are particular averages borne ?—7. When masts are cut away, or goods

thrown overboard in a storm for the preservation of the ship and cargo, is the loss borne by general average ?—9. What is the cutting away of masts, &c. and throwing goods overboard for the preservation of the remainder called ?—10. What is necessary to render jettison legal ?—11. When a ship is damaged in defending her against an enemy, whether is the loss sustained by general or particular average ?—12. Do you know any other expenses borne by general average ?—13. What do you understand by charges of salvage ?—14. Does salvage belong to general average ?—15. What salvage is allowed for retaking a ship from the enemy ?—16. Are ransoms admitted into general averages ?—17. Are averages always paid, however small ?—18. What are the best data on which to calculate general average ?—19. When computing averages for masts, rigging, &c. is the whole expense admitted into the calculation ?—20. Whence do particular averages arise ?—21. Can you give a case which would fall under particular average ?—22. When goods are damaged so as to sell under the market rate of sound goods, how is the particular average settled ?—23. Under what per cent. are underwriters exempted generally from particular averages ?—24. Under what per cent. are underwriters exempted from particular averages on tobacco, sugar, hemp, flax, hides, &c. ?—25. What is necessary to bring underwriters in liable for particular averages on corn, seed, flour, fruit, fish and salt ?—26. When a sum is paid for particular average, is the premium upon that sum also returned ?—27. What is the reason that particular averages under 3 per cent. are never paid ?—28. May not the insured suffer considerable loss from this regulation ?—29. Can you state a case by which this may be exemplified and made intelligible ?—30. Do you know any method of guarding against such extensive losses ?—31. What is barratry of the master and crew ?—32. How should this be guarded against ?—33. What documents are considered necessary to recover a loss ?—34. If fraud can be proved against the insured, what is the consequence ?—35. At what time are losses generally paid in Britain ?

COMPOUND INTEREST.—1. What is compound interest ?—2. How do you find the compound interest and amount of any sum ?—3. On what principle is the rule for compound interest founded ?—4. Is compound interest permitted in Britain ?—5. Is there any possibility of realizing the advantages of compound interest in a legal manner ?—6. Is compound interest justifiable on the principles of equity ?—7. Is the rule given in the text for calculating compound interest, a simple method ?

DISCOUNT.—1. What is discount ?—2. What is the present value of a sum due some time hence ?—3. How do you calculate discount ?—4. Is this properly called true discount ?—5. Can you prove this, and illustrate the principle of the rule by an example ?—6. Is the difference between this and bankers' discount considerable ?—7. In what proportion do they differ ?—8. By what rules are the operations in discount performed ?—9. How would you perform discount by decimals ?—10. Can you illustrate this rule by an example ?

EQUATION OF PAYMENTS.—1. What is equation of payments ?—2. How do you perform it ?—3. Is this rule in general use ?—4. On what principle is it founded ?—5. Is this principle correct ?—6. What is the objection to it ?—7. Do you know any rule for equation of payments founded on correct principles ?—8. Who invented that rule ?—9. Can you illustrate the rule by an example ?—10. Is this rule easily applied ?—11. Do you know any method of obtaining the advantages of this rule without submitting to its drudgery ?

BARTER.—1. What is barter ?—2. How do you calculate barter ?—3. Which of these methods is the simplest ?—4. Can you prove this, and illustrate the rules by an example ?—5. Is either party understood to be a gainer by bartering ?

PROFIT AND LOSS.—1. What is profit and loss ?—2. By what rule is profit and loss performed ?—3. Has profit and loss always been divided into particular cases ?—4. When the prime cost and selling price are given, how do you find the gain or loss per cent. ?—5. What is the reason of this rule ?—6. When the cost and the gain or loss per cent. are given, how do you find the selling price ?—7. What is the reason of this rule ?—8. When the selling price, and the gain or loss per cent. are given, how do you find the prime cost ?—9. What is the reason of this rule ?—10. When two selling prices are given, and the gain or loss by one of them, how do you find it on the other ?—11. What is the reason of this rule ?—12. How do you advance the cash price, to allow a proposed discount ?—13. What is the reason of this rule ?—14. When the whole gain or loss and the rate are given, how do you find the cost and selling price ?—15. What is the reason of this rule ?

MERCANTILE COMPOSITIONS.—1. What is mercantile compositions ?—2. When the quantity and rate of several ingredients are given, how do you find the rate of any proposed

part of it?—3. What is the reason of this rule?—4. Can you illustrate this by an example?—5. How do you find how much of each simple is in any proposed quantity of a given composition?—6. What is the reason of this rule?—7. How do you increase or diminish simples in any proposed ratio?—8. What is the reason of this rule?—9. When the rate of the simples, and the mixture are given, how do you find the quantity of the several simples?—10. What do you understand by linking?—11. What name was given to the rule from this circumstance?—12. Is linking absolutely necessary in this case?—13. Why is it called mercantile compositions?—14. What is the reason that the differences placed according to the rule give the required quantity of the several simples?—15. Can you illustrate this by an example?—16. Are the quantities of the simples confined to what is obtained by linking?—17. How would you find more answers?—18. How many answers can you find in this way?—19. Is it necessary to link according to the rule?—20. In what other way can this be done to give correct results?—21. May you not by this method have a variety of infinite series of answers?—22. Can you give an example of this?—23. When we multiply or divide the quantities obtained by linking, is it necessary that we should multiply or divide them all?—24. May we multiply any one result and divide any other?—25. Under what restrictions can we multiply one and divide another?—26. Can you illustrate the whole of this by an example?—27. Why does the rule restrict to a particular way of linking?—28. Does the endless varieties of answers, of which this case admits, render it less useful?—29. When the rate of the simples, and of the mixture are given, and one of the simples limited, how do you find the quantity of the other simples?—30. What is the reason of the first part of this rule?—31. What is the reason of the second part of it?—32. Can you illustrate this by an example?—33. When more than one simple is limited, how do you proceed?—34. Can you illustrate this by an example?—35. When do questions of this class become impossible?—36. When the rate of the simples, and of the mixture are given, but the quantity of the mixture limited, how do you find the quantity of the simples?—37. Can you illustrate this by an example?—38. On what principles is this operation founded?—39. When any simple is of little or no value when compared with the others, what is done?

EXCHANGE.—1. What is exchange?—2. By whom and how is it transacted?—3. What is the object of exchange?—4. What is the par of exchange?—5. How is par estimated?—

6. Does par always remain the same?—7. Under what circumstances does the par fluctuate?—8. What is the course of exchange?—9. When is the course of exchange at par?—10. Do you know any causes which tend to prevent the course differing much from the par?—11. What is agio?—12. What is usance?—13. What is double and half usance?—14. What are days of grace?—15. What is understood by the fixed and variable sum in exchange?—16. When the exchange is high, whether is the country which gives the fixed, or the variable sum, the gainer?—17. Is the exchange with the rest of Great Britain always in favour of London?—18. What is the reason of this?—19. At what date do bankers in Edinburgh give drafts on London?—20. What do they charge for inland remittances?—21. Is the exchange between IRELAND and Britain always unfavourable to Ireland?—22. What is the reason of that?—23. Does Ireland give or receive the fixed sum?—24. Does the course of exchange with Ireland vary much?—25. What is the par?—26. How many days of grace are allowed?—27. What is the par between JAMAICA currency and sterling?—28. Do bills on London sell at a premium or discount?—29. Is there any fixed par between currency and sterling in the other West India Islands?

AMERICA.—1. How are accounts kept in North America?—2. Are bills on London at a premium or discount?—3. How are accounts kept in the United States?—4. Repeat the table of American money?—5. Is the current value of the dollar the same in all the states?—6. In which of the states is it current for 6/?—7. What proportion does it bear to sterling in these states?—8. In what states is it current for 8/, and what ratio does it then bear to sterling?—9. In what states is it current for 7/6, and what ratio does it then bear to sterling?—10. In what states is it current for 4/8, and what ratio does it then bear to sterling?—11. How does America exchange with Britain?—12. What is the par of exchange?

FRANCE.—1. How are accounts kept in France?—2. How does France exchange with Britain?—3. What is the par, usance, and days of grace with France?—4. Is the franc and liver the same value?—5 How did this difference arise?—6. Repeat the tables of money, weights and measures by the old system?—7. Repeat the tables of money, weights and measures by the new system?—8. Whether is a high or low course of exchange with France, the best for Britain?

HOLLAND.—1. What are the different kinds of money used in Holland?—2. Which is the most valuable?—3. What is their difference called?—4. How are accounts kept in Hol-

land?—5. How does Amsterdam exchange with Britain?—6. What is the par, usance, and days of grace in Holland?—7. Repeat the tables of money, weights and measures used in Holland.—8. Whether is a high or low course of exchange with Holland the best for Britain?

FLANDERS AND BRABANT.—1. How does Flanders exchange with Britain?—2. What is the usance and days of grace here? 3. Repeat their tables of money, weights and measures?

GERMANY.—1. Are not accounts kept differently in different parts of Germany?—2. Repeat the tables of money, weights and measures used in the different places?—3. How does Hamburg exchange with Britain?—4. How many rix-dollars Hambro' banco are equal to £1 Fl. banco?—5. What is the par, usance, and days of grace in Hambro'?—6. How does Austria, &c. exchange with Britain, and what is the par?—7. How does Brunswick, &c. exchange with Britain, and what is the par, usance, and days of grace?—8. Whether is a high or low course of exchange with Germany, the best for Britain?

RUSSIA.—1. How are accounts kept in Russia?—2. Repeat the tables of money, weights and measures used in Russia.—3. How is the rouble divided?—4. Are these the same value?—5. How does Russia exchange with Britain?—6. What is the par, usance, and days of grace with Russia?—7. Is the course generally stated in silver, or bank note roubles?—8. Whether is a high or low course of exchange with Russia, the best for Britain?

SWEDEN.—How are accounts kept in Sweden?—2. Repeat the tables of money, weights and measures used in Sweden?—3. How does Sweden exchange with Britain?—4. What is the par, usance, and days of grace with Sweden?—5. Whether is a high or low course of exchange, the best for Britain?

DENMARK AND NORWAY.—1. How are accounts kept in Denmark?—2. How does Denmark exchange with Britain?—3. What is the par, usance, and days of grace with Denmark?—4. Is the course of exchange specified in silver or paper money?—5. Is a high or low course with these countries the best for Britain?

PRUSSIA.—1. Are accounts kept differently in different places of Prussia?—2. Repeat the tables of money, weights and measures used in Prussia?—3. How are accounts kept in Berlin and Stettin?—4. How does Prussia exchange with Britain?—5. What is the par, usance, and days of grace in Prussia?—6. Whether is a high or a low course of exchange with Prussia the best for Britain?

POLAND.—1. How are exchanges with Poland transacted?—2. How are accounts kept in Poland?—3. Repeat the tables of money, weights and measures used in Poland?—4. What is the course of exchange between Great Poland, and Holland?

SPAIN.—1. Repeat the tables of money, weights and measures used in Spain.—2. How are accounts kept in Spain?—3. How does Spain exchange with Britain?—4. What is the par, usance, and days of grace in Spain?—5. What is the ratio of vellon to old plate?—6. What are the different kinds of plate money?—7. Which of these is used in exchanges with Britain?—8. What precaution is necessary in drawing bills of exchange on Spain, to prevent their being paid in exchequer bills?—9. Whether is a high or low course here the best for Britain?

PORTUGAL.—1. How are accounts kept in Portugal?—2. Repeat the tables of money, weights and measures used in Portugal.—3. How does Portugal exchange with Britain?—4. What is the par, usance and days of grace in Portugal?—5. Whether is a high or low course best for Britain?

ITALY.—1. Repeat the tables of money, weights and measures used in Rome.—2. How does Rome exchange with Britain?—3. What is the par, usance and days of grace here?—VENICE.—4. Repeat the tables of money, weights and measures used in Venice.—5. How does Venice exchange with Britain?—6. What is the par, usance and days of grace in Venice?—NAPLES.—7. Repeat the tables of money, weights and measures used in Naples.—8. How do they exchange with Britain?—9. What is the par, usance and days of grace in Naples?—MILAN.—10. Repeat the tables of money, weights and measures used in Milan.—11. How do they exchange with Britain?—12. What is the par, usance and days of grace in Milan?—GENOA.—13. Repeat the tables of money, weights, and measures used in Genoa.—14. How do they exchange with Britain?—15. What is the par, usance and days of grace in Genoa?—LEGHORN AND FLORENCE.—16. Repeat the tables of money, weights and measures used in Leghorn.—17. How do they exchange with Britain?—18. What is the par, usance and days of grace in Leghorn?—19. With which of the Italian states is a low course of exchange the most profitable for Britain?—20. With which of them is a high course the most profitable for Britain?

TURKEY.—1. How are accounts kept in Turkey?—2. Repeat the tables of money, weights, and measures used in Turkey.—3. How does Turkey exchange with Britain?—4. What

is the par, usance, and days of grace in Turkey.—4. Whether is a high or low course the best for Britain?

East Indies.—1. How are accounts kept in Bombay?—2. Repeat the table.—3. What is the current value of a rupee?—4. How are accounts kept in Madras?—5. Repeat the table.—6. What is the value of the star pagoda?—7. How are accounts kept in Bengal?—8. Repeat the table.—9. How are the company's accounts kept here?—10. What batta do these bear against current rupees?—11. How many sicca rupees make a gold mohur?—12. How many rupees make a lack, and how many make a crore?—13. How are accounts kept in the Mysore country?—14. Repeat the table.—15. How are accounts kept on the Malabar coast?—16. How are accounts kept in China?—17. Repeat the table.—18. What is the cash in China composed of?—19. What are a thousand cash worth in sterling?

Arbitration of Exchange.—1. How is arbitration generally divided?—2. What rule is used to calculate arbitrations on the continent of Europe?—3. Repeat the chain rule.—4. Are not the weights and measures of different countries generally compared by this rule?—5. What is the reason merchants engaged in foreign trade should be well acquainted with the arbitration of exchanges?—6. By how much should the circular exchange exceed the direct to make it profitable?

INVOLUTION.—What is involution?—2. What is the power of a number, and how is it found?—3. How is the power expressed?—4. How are high powers best found?—5. Is the appellation of square, cube, &c. properly applied to abstract numbers?—5. What is the reason of this?

EVOLUTION.—1. What is evolution?—2. What is meant by extracting the square root of any power?—3. How do you extract the square root of a whole number?—4. How do you extract the square root of a vulgar fraction?—5. What is the reason you divide the given number into periods of two figures?—6. Does the root always consist of as many figures as there are periods in the number whose root is required?—7. On what principle is this rule founded?—8. What is the reason you reject the right hand place in the increased remainders when finding next figure of the root?—9. How do you prove the square root?—10. Can you illustrate the reasons and principles you have given, by an example?—11. In what department is a knowledge of the square root indispensable?—12. How do you find the side of a square equal to any given super-

sides.—13. How do you find the side of a square greater or less than a given square, in any assigned proportion ?—14. How do you find the side of a square equal to several given squares ? —15. How would you find the diameter of a circle bearing any assigned proportion to a circle whose diameter is known ? —16. When any two sides of a right angled triangle are given, how do you find the third side ?—17. How do you find a mean proportional between two numbers ?

CUBE ROOT.—What is the cube root of a number ?—2. How do you extract the cube root of any number ?—3. What is the reason you divide the number into periods of three figures ?—4. What is the reason of the other part of the process ?—5. What is the reason you annex two ciphers to the first part, and one to the second part of the divisor ?—6. What is the reason that adding the last complete divisor to its second part, and double its third part, gives the new trial divisor ?—7. Can we extract the roots of higher powers by the combined application of the square and cube root ?—8. How do you know when they can be thus applied ?—9. What is the general rule for extracting the roots of all powers ?—10. Repeat the squares and cubes of the nine digits.—11. How do you find the side of a cube whose solidity is known ?—12. How do you find the side of a cube equal to any given solid ?—13. When the dimensions of a solid are given, how do you find the dimensions of a similar solid, greater or less in any given proportion ? —14. When the dimensions of two similar solids are given, and the weight of one, how do you find the weight of the other ? —15. How do you find two mean proportionals between two given numbers ?

POSITION.—1. What is position ?—2. What other names is position known by ?—3. What kind of questions belong to single position ?—4. Why is it called single position ?—5. How do you work single position ?—6. On what principle is this rule founded ?—7. Can the questions under this case be solved by double position ?

DOUBLE POSITION.—1. What kind of questions belong to double position ?—2. Why is it called double ?—3. How do you work double position ?—4. On what principle is this rule founded ?—5. What is Bonnycastle's rule for this case ?—6. What is the case to which it does not apply ?—14. In what circumstances does position give only an approximation to the required number ?—15. How do you approximate to the required number ?—16. What is the use of this approximation ?

ARITHMETICAL PROGRESSION.—1. When are numbers said to be in arithmetical progression?—2. What are the first and last terms called?—3. What are the other terms called?—4. What is the number by which they increase or decrease called?—5. Is this properly named arithmetical progression?—6. What is the sum of the extremes equal to?—7. What is an arithmetical mean between any two numbers?—8. How do you find the sum of the series when the extremes and number of terms are given?—9. How do you find the common difference when the extremes and number of terms are given?—10. How do you find the number of terms, when the extremes and common difference are given?—11. When one extreme, the common difference, and the number of terms are given, how do you find the other extreme?

GEOMETRICAL PROGRESSION.—1. When are numbers said to be in geometrical progression?—2. What is the product of the two extremes equal to?—3. How is a geometrical series produced?—4. What is the last extreme in an infinite decreasing series equal to?—5. What is the common ratio?—6. When one extreme, the number of terms, and the common ratio are given, how do you find the other extreme?—7. When the extremes and common ratio are given, how do you find the number of terms?—8. How do you find the ratio when the extremes, and the number of terms are given?—9. How do you find the sum of the terms of a geometrical series?—10. Is this rule equally applicable to an increasing and decreasing series?

PERMUTATIONS, &c.—1. What is permutation?—2. What is combination?—3. How do you find the number of permutations which can be made with any given number of articles?—4. How do you find how many permutations can be made out of a given number of articles, of which there are several of several sorts?—5. When any number of different articles are given, how do you find the various combinations which may be formed, by taking any proposed number of them at a time?—6. How do you find how many permutations can be made out of a given number of different articles, by taking any proposed number of them at a time?—7. How do you find how many compositions can be formed out of a given number of sets of different articles, having as many articles in each composition as there are sets?—8. How do you find the number of combinations which any number of different articles will admit of, taking them by twos, threes, &c. up to the given

number ?—9. How do you find all the possible combinations and permutations of which any proposed number of articles is susceptible, when taken by twos, threes, &c. to the given number of articles ?

CHANCES.—1. In what part of arithmetic is the calculation of chances of importance ?—2. When there is but one trial, and one chance of success, how is the chance of success and failure expressed ?—3. When there are several trials, and only one point to be gained, how do you calculate the chance of success ?—4. When there is only one trial, and several points to be gained, how do you find the chance of success and failure ?

COMPOUND INTEREST AND ANNUITIES.—1. What is compound interest ?—2. What is an annuity ?—3. How is table 1 constructed ?—4. How is it applied ?—5. How is table 2 formed, and how is it applied ?—6. How is table 3 formed, and how is it applied ?—7. How is table 4 formed, and how is it applied ?—8. How is table 5 formed, and how is it applied ?—9. How are table 6 and 7 constructed ?—10. How do you find the compound interest and amount of any sum ?—11. When the years given are not in the table, how do you find the tabular number ?—12. How do you find the present value or principal which will amount to a given sum, in a given time, at a proposed rate ?—13. How do you find the time, when the principal, amount, and rate are given ?—14. How do you find the rate when the principal amount and time are given ?—LIMITED ANNUITIES.—15. How do you find the amount, when the annuity, time, and rate are given ?—16. How do you find the annuity, when the amount, rate, and time are given ?—17. How do you find the time, when the annuity, amount, and rate are given ?—18. When the annuity, amount, and time are given, how do you find the rate ?—19. How do you find the present value, when the annuity, time, and rate are given ?—20. How do you find the annuity, when its present value, time of continuance, and rate are given ?—PERPETUAL ANNUITIES.—21. What is a perpetual annuity ?—22. When the value in perpetuity, and the rate are given, how do you find the annuity ?—23. When a perpetual annuity and its rate are given, how do you find its value ?—24. When the annuity and its value are given, how do you find the rate ?—25. How do you find how many years purchase a perpetuity is worth, when the rate is given ?—26. How do you find the rate when the years purchase are given ?—ANNUITIES IN REVERSION.—27. What is an annuity in reversion ?—28.

When the annuity, rate, the time before the reversion, and the time of continuance are given, how do you find the value of the reversion?—29. When the value of a reversion, the time before it commences, the time of continuance, and the rate are given, how do you find the annuity?—LIFE ANNUITIES.—30. What are life annuities?—31. How do you find the present value of an annuity, at a given rate, to continue during the life of a person whose age is known?—32. How do you find the value of an annuity, secured on the joint continuance of two given lives?—33. How do you find the value of an annuity to continue during the currency of two given lives?—34. How do you find the value of an annuity during the joint continuance of three given lives?—35. When one or both lives are not found in the tables, what do you do?—36. How do you find the value of an annuity to terminate with the longest survivor of three given lives?—37. How do you find the present value of the reversion of an annuity, after the death of 1, 2, or 3 persons whose ages are given?—38. How do you find the present value of an annuity to be enjoyed for life by the purchaser, after the death of the present possessor?—39. How do you find what sum must be paid by a husband at once, or in annual instalments during life, to secure a proposed annuity to his widow during the remainder of her life?—40. How do you find what sum must be paid annually by a husband during marriage, to entitle a widow to any proposed annuity?—41. How do you find how much must be paid present, and also by annual instalments during life, to insure any proposed sum to heirs at death?

DUODECIMALS.—1. How do duodecimals increase and decrease?—2. By whom are they used?—3. How is duodecimal multiplication performed?—4. What other name are they known by?—5. How many square inches are equal to a duodecimal inch?—6. How many cubic inches are equal to a duodecimal solid inch?—7. When the number of feet in the factors is great, how do you proceed with them?—8. When there are yards given, how would you proceed with them?—9. How is the area of a rectangular surface found by duodecimals?—10. How is the solid content of rectangular bodies found by them?—11. How is the content of round timber found?—12. How do you measure logs of wood, &c. when they are not the same breadth and depth at both ends?—13. When round trees taper irregularly, how would you measure them?—14. Is the rule in the text for measuring round timber correct?—15. When trees are girt with the bark on, what deduction is made

for it ?—16. What are the rules given in the notes for measuring round timber ?—17. Where would you cut a tree to make it measure the most possible by the common rule ?—18. How would you cut a tree to make the greater end measure the most possible by the common rule ?—19. How would you cut a tree, so that the greater end may measure as much as the whole tree uncut ?

TONNAGE OF SHIPS.—1. What is the rule in the text for finding the tonnage ?—2. How do you find the tonnage by taking the distance between the light mark, and the full draught of water ?—3. How do shipwrights in London calculate the tonnage ?—4. What method is used in the Royal Navy ?

TABLES OF WEIGHTS AND MEASURES.—It would be superfluous to annex questions on these tables. Their importance in business is such that they should be made a part of every day's exercise during the whole time the pupil is studying arithmetic. There should, therefore, be some portion of them given, along with a few of the preceding questions, as a daily exercise for the pupil's leisure hours. It is not intended that the tables should be committed as they are printed; it is sufficient that the pupil learn how many of a lower denomination make one of the next higher; thus, in wine measure, 4 gills = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon, &c. The other parts of the tables are chiefly intended as references, and for converting the old weights and measures into the imperial, and the reverse; or the weights and measures of different countries into British, and the reverse; and it is hoped they will be found of importance in that way.

FINIS.

ERRATA.

Page 12, title, for "division," read "multiplication."—No. 69, last Ans. 99990.——P. 13, last line, read "three billion four hundred thousand million and twelve."——P. 29. No. 12, in Ans. read 15/.——P. 36, No. 81, Ans. 17 gal. 4. 4F℥. 5FO℥. 19♏.—No. 84, in Ans. read 10117 yds.——P. 37, No. 89, in Ans. read 10sk. 2 bu. 1 pk.——P. 50, No. 8, Ans. 280233 lb. 5 oz. 18 dwt. 12 gr.——P. 66, No. 8, Ans. £174—No. 25, in Ans. for 5/, read /5.——P. 69, in example, 1st method, blot out /8=$\frac{1}{6}$.——P. 72, No. 18, in question, read 1/1$\frac{1}{2}$.—No. 19, Ans. £573 1s. 4d.—No. 20, Ans. £541.——P. 83, No. 46, Ans. £14918 13s. 4d.——P. 85, No. 62, Ans. 21d. 4h.—No. 66, in question, read 102 frails.—No. 68, Ans. 4$\frac{1}{5}$.—No. 75, Ans. 600 ells.——P. 89, No. 22, Ans. 164250.—No. 25, Ans. 1307$\frac{22}{133}$.—No. 27, Ans 6$\frac{2}{13}$.——P. 95, No. 6, Ans. $\frac{8000}{3000}$, $\frac{3600}{3000}$, $\frac{2342}{3000}$.——P. 97, No. 12, Ans. $\frac{53}{72}$.——P. 103, last line, for "man," read "many."——P. 107, No. 3, Ans. .7822916̇.——P. 109, No. 17, in Ans. read 26m. 38.4s.——P. 112, Case 8, No. 4, for 186, read 185.——P. 114, No. 2, Ans. 78.19558̇.——P. 119, No. 3, Ans. 6.40̇84̇0.—No. 4, Ans. .588̇94̇0.——P. 135, No. 3. first Ans. £379 0s. 1d.$\frac{1}{5}$.——P. 140, No. 2, in Ans. read 10$\frac{1}{2}$d.——P. 143, No. 2, read, discounted at 4 per cent.—No. 3, in Ans. read $\frac{3}{4}$.——P. 148, No. 2, in Ans. read $\frac{1}{2}\frac{1}{5}$.——P. 155, No. 2, in Ans. read 0$\frac{1}{2}$d $\frac{4}{4}\frac{6}{5}$.——P. 169, No. 3, for 15/, read 16/.——P. 170, No. 2, for 1/8, read 1/10.——P. 186, Prussia par 6 rxd. 16 grosh.——P. 200, No. 1, for 200, read 164.——P. 204, No. 6, Ans. £5500.——P. 207, No. 3, read, least extreme be 16.——P. 212, Case 4, No. 2, for 6, read 5 letters.——P. 225, Case 4, No. 1, Ans. 3$\frac{1}{2}$.—Case 5, No. 4, in Ans. for 16/, read 14/.——P. 236, Case 10, in rule, for "multiplied by the perpetuity," read "multiplied by the proposed sum."——P. 238, timber measure, No. 3, Ans. 70f. 6″.——P. 239, No. 10, Ans. 38f. 3′ 10″ 2‴, &c.——P. 240, No. 2, Ans. £48939—No. 16, in Ans. read 23$\frac{2}{11}$ sec.——P. 241, No. 24, Ans. 600 ells.——P. 320, third line, for $\frac{4\times 12}{4}$ 12 =, read $\frac{4\times 12}{4} = 12$.